The role of the rate of change of power (RoCoP) in low inertia systems

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Frequency and Power Variations

- What is the link between frequency and power variations at network buses?
- Is the current definition of ”frequency” adequate?

Common definition of Frequency

- The IEEE Std. IEC/IEEE 60255-118-1 define the frequency of an ac signal as follows:

$$f(t) = \frac{1}{2\pi}\dot{\vartheta}(t) = \frac{1}{2\pi}\dot{\theta}(t) + f_o\,,$$

- This definition works well only if the magnitude of the ac signal is constant!

Power Injections at Buses

- Let us consider the power injection at network buses:

$$\bar{\boldsymbol{s}}(t) = \boldsymbol{p}(t) + \jmath \boldsymbol{q}(t) = \bar{\boldsymbol{v}}(t) \circ \bar{\boldsymbol{\imath}}^*(t) \, ,$$

- where voltages and currents are Park's vectors, i.e., are valid in transient conditions:

$$\bar{\boldsymbol{v}}(t) = \boldsymbol{v}_{\rm d}(t) + \jmath \boldsymbol{v}_{\rm q}(t) \, .$$

Assumption

- Let us assume that transmission line dynamics are fast, hence:

$$\bar{\boldsymbol{\imath}}(t) \approx \bar{\mathbf{Y}}\, \bar{\boldsymbol{v}}(t)\,,$$

- Hence the power injections can be rewritten as:

$$\bar{\boldsymbol{s}}(t) = \bar{\boldsymbol{v}}(t) \circ [\bar{\mathbf{Y}}\, \bar{\boldsymbol{v}}(t)]^*\,.$$

System Model

- Let consider the conventional DAE model for transient stability analysis:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \,, \qquad \boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \,,$$

$$\Rightarrow$$

$$\dot{\boldsymbol{y}} = \frac{\partial \boldsymbol{\phi}}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\phi}(\boldsymbol{x})) \,.$$

Derivatives of Voltages and Powers

- In the previous DAE model, voltages and power are algebraic variables.
- We can write them as follows:

$$\dot{\bar{\boldsymbol{s}}} = \frac{\partial \bar{\boldsymbol{s}}}{\partial \bar{\boldsymbol{v}}} \, \dot{\bar{\boldsymbol{v}}} + \frac{\partial \bar{\boldsymbol{s}}}{\partial \boldsymbol{x}} \, \dot{\boldsymbol{x}} \, .$$

Derivatives of Voltages and Powers

- We want to find an ”usable” expression that links together the time derivatives of the voltage, state variables and the rate of change of (complex) power.
- To do so, let us define a *new* quantity

Complex Frequency

- Let $u_h \equiv \ln(v_h)$
- Then, one has: $du_h = \dfrac{dv_h}{v_h}$
- Then let define: $\bar{\zeta}_h \equiv u_h + \jmath\,\theta_h$
- And, finally, let $\bar{\boldsymbol{\eta}} = \dot{\bar{\boldsymbol{\zeta}}} = \dot{\boldsymbol{u}} + \jmath\dot{\boldsymbol{\theta}}$
- The complex frequency is: $\boxed{\bar{\boldsymbol{\eta}} \equiv \boldsymbol{\varrho} + \jmath\,\boldsymbol{\omega}}$

Link with the Voltage

- It is possible to demonstrate that, based on the definition of the complex frequency one has:

$$\dot{\boldsymbol{v}} = \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\eta}}$$

- Hence: $\bar{\boldsymbol{\imath}}(t) \approx \bar{\mathbf{Y}}\, \bar{\boldsymbol{v}}(t)$ ⟹ $\boxed{\dot{\boldsymbol{\imath}} = \bar{\mathbf{I}}\, \bar{\boldsymbol{\eta}}}$
- where $\bar{\mathbf{I}} = \bar{\mathbf{Y}}\, \mathrm{diag}(\bar{\boldsymbol{v}})$

Link with the Power

- Then taking the conjugate and multiplying by the voltage

$$\bar{\boldsymbol{v}} \circ \dot{\bar{\boldsymbol{i}}}^{*} = \bar{\mathbf{S}}\,\bar{\boldsymbol{\eta}}^{*}$$

- Where $\bar{\mathbf{S}}$ is a matrix whose elements are the complex power flow in the branches of the grid.

Link with the RoCoP

- And finally, we note that:

$$\begin{aligned}\dot{\boldsymbol{s}} &= \frac{d}{dt}(\bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}^*) \\ &= \dot{\bar{\boldsymbol{v}}} \circ \bar{\boldsymbol{\imath}}^* + \bar{\boldsymbol{v}} \circ \dot{\bar{\boldsymbol{\imath}}}^* \\ &= \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\eta}} \circ \bar{\boldsymbol{\imath}}^* + \bar{\boldsymbol{v}} \circ \dot{\bar{\boldsymbol{\imath}}}^* \\ &= \bar{\boldsymbol{s}} \circ \bar{\boldsymbol{\eta}} + \bar{\boldsymbol{v}} \circ \dot{\bar{\boldsymbol{\imath}}}^* \,,\end{aligned}$$

- So we obtain the expression:

$$\boxed{\dot{\bar{\boldsymbol{s}}} - \bar{\boldsymbol{s}} \circ \bar{\boldsymbol{\eta}} = \bar{\mathbf{S}}\, \bar{\boldsymbol{\eta}}^*}$$

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Components of the RoCoP

- From the definition of complex frequency we can define the following components of the RoCoP:

$$\dot{\boldsymbol{s}}' = \jmath \bar{\boldsymbol{s}} \circ \boldsymbol{\omega} - \jmath \bar{\mathbf{S}} \, \boldsymbol{\omega} \,,$$
$$\dot{\boldsymbol{s}}'' = \bar{\boldsymbol{s}} \circ \boldsymbol{\varrho} + \bar{\mathbf{S}} \, \boldsymbol{\varrho} \,.$$

- where $\dot{\boldsymbol{s}} = \dot{\boldsymbol{s}}' + \dot{\boldsymbol{s}}''$.

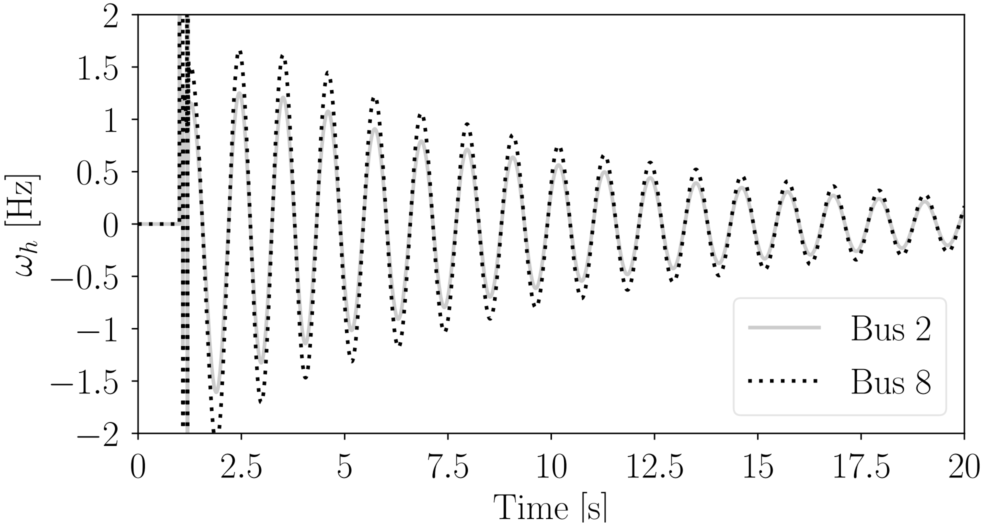
Approximated Expressions

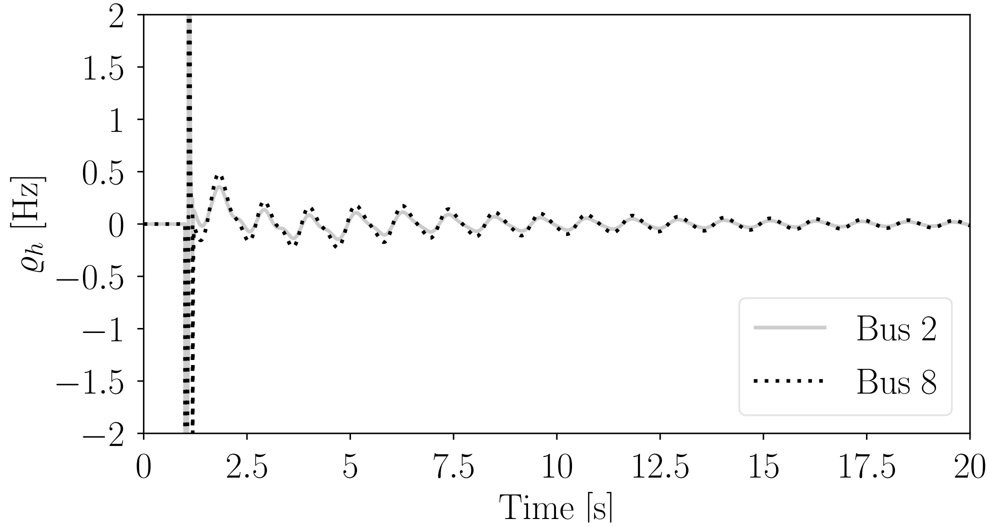
- Then, one can define some approximated expressions:

$$\dot{\boldsymbol{p}}' \approx \mathbf{B}'\boldsymbol{\omega}\,, \qquad \dot{\boldsymbol{q}}' \approx \mathbf{G}'\boldsymbol{\omega}\,,$$

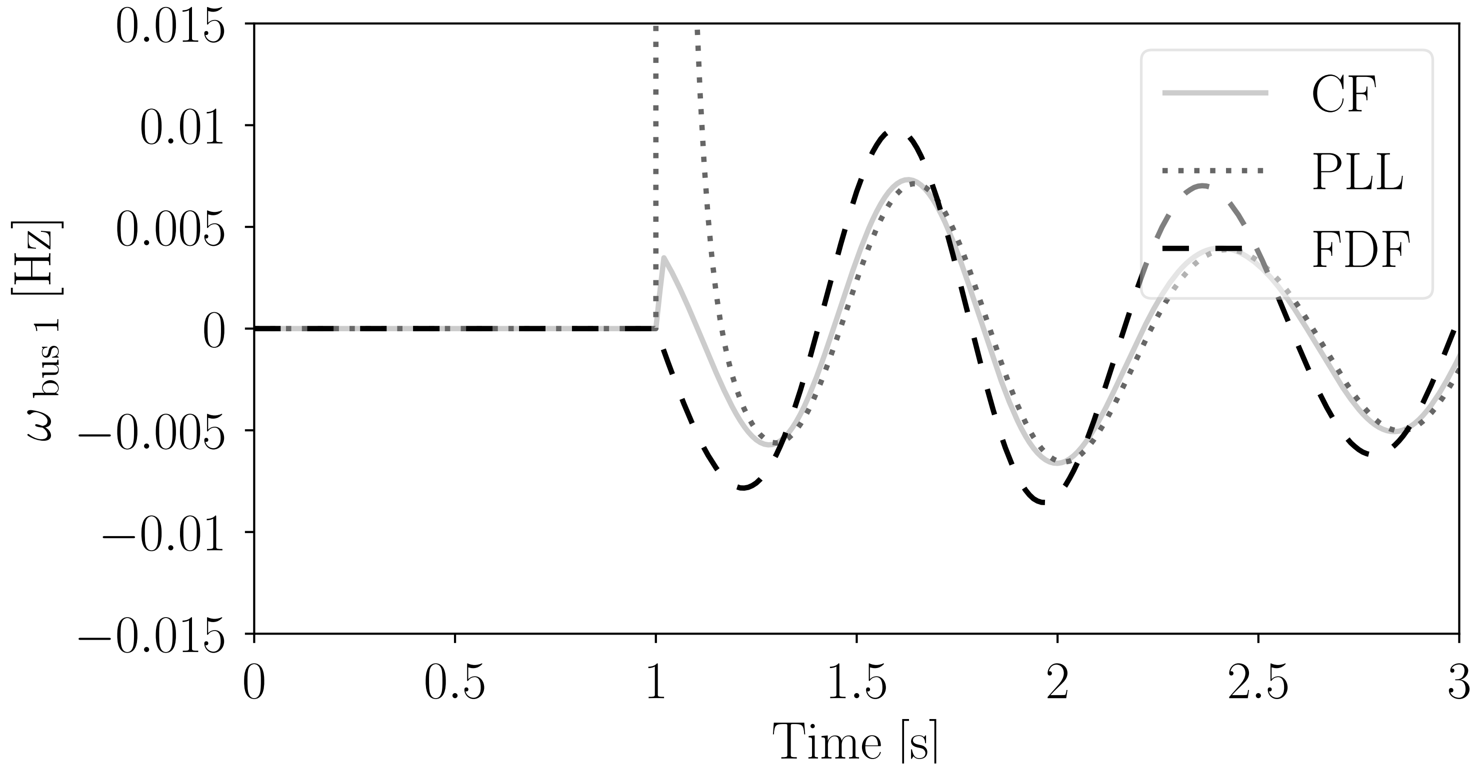
$$\dot{\boldsymbol{p}}'' \approx \mathbf{G}''\boldsymbol{\varrho}\,, \qquad \dot{\boldsymbol{q}}'' \approx \mathbf{B}''\boldsymbol{\varrho}\,,$$

Example: rho and omega

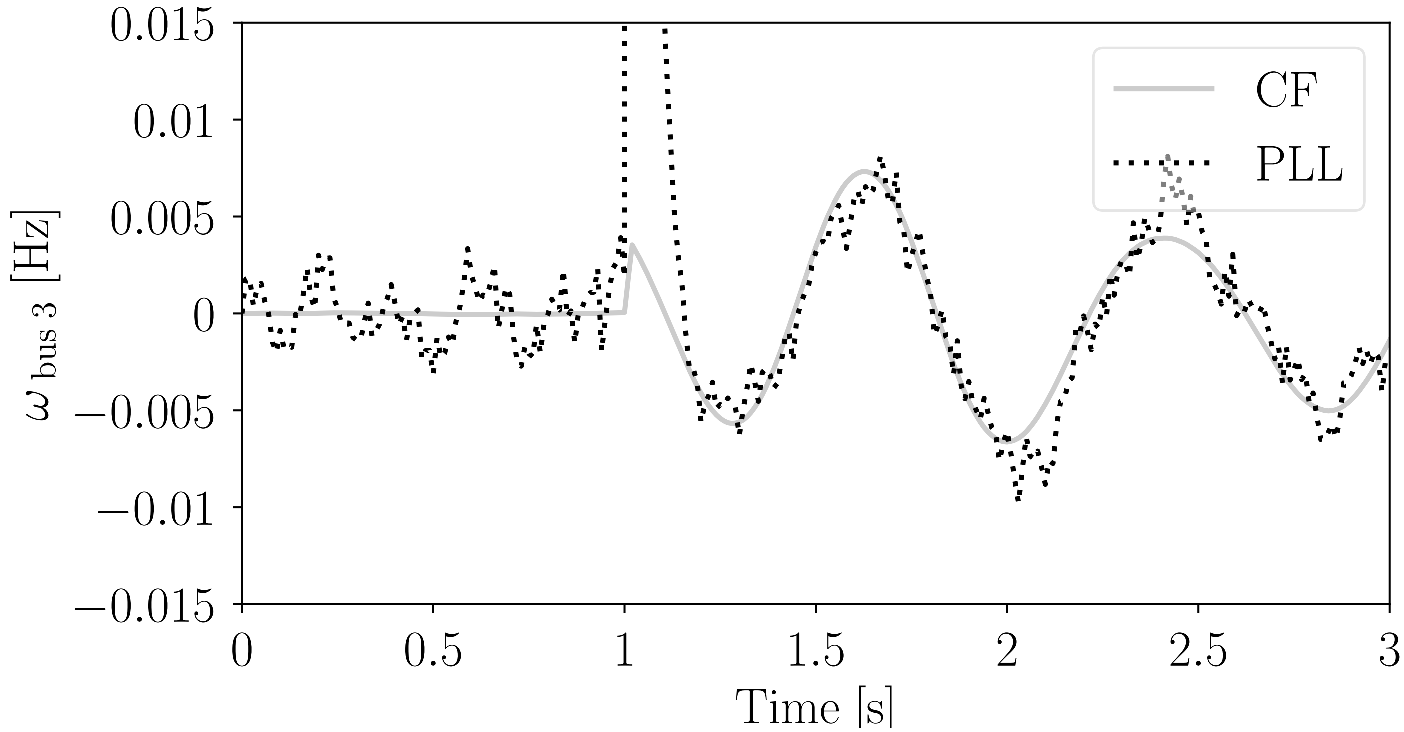




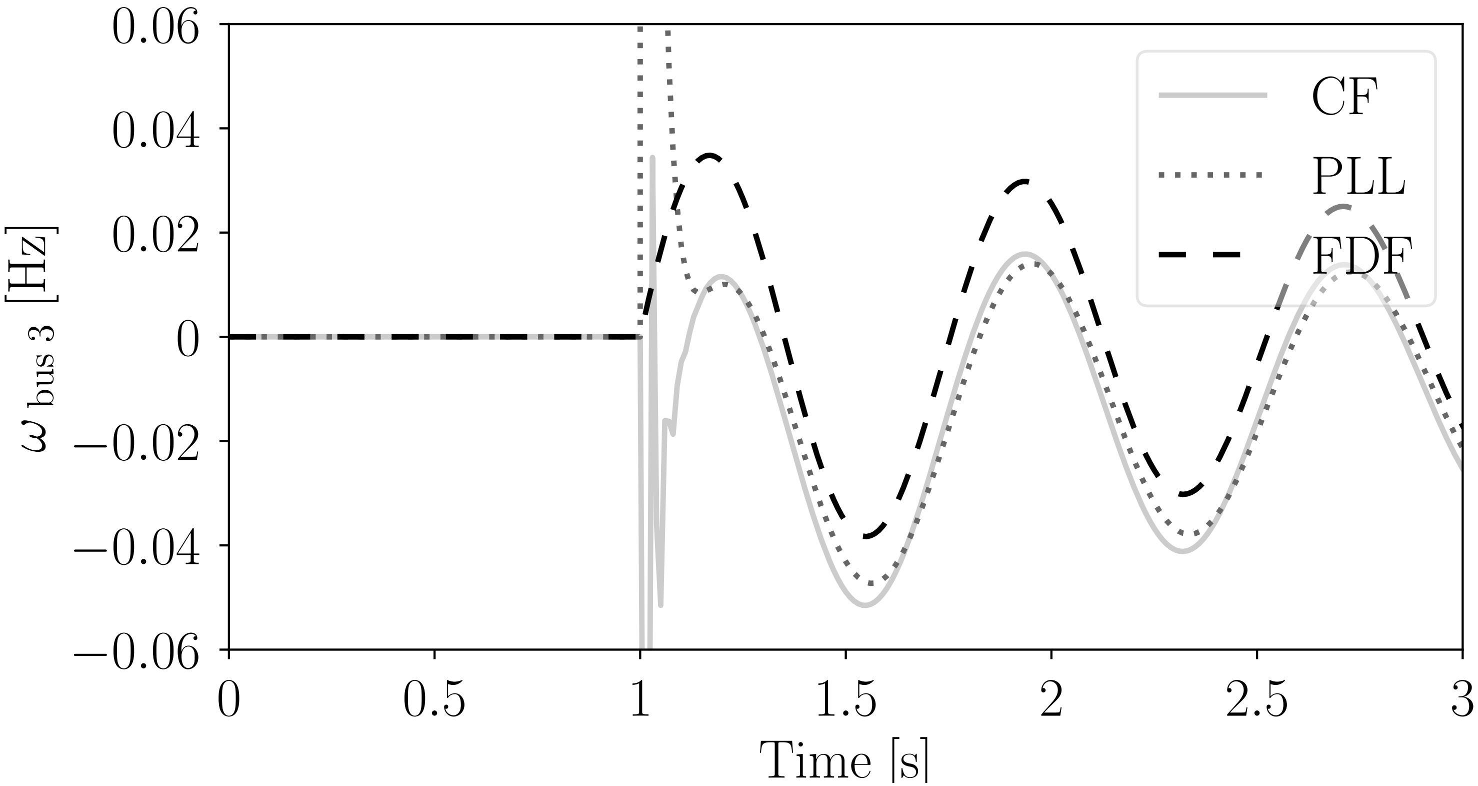
Example: synchronous machine bus



Example: effect of noise

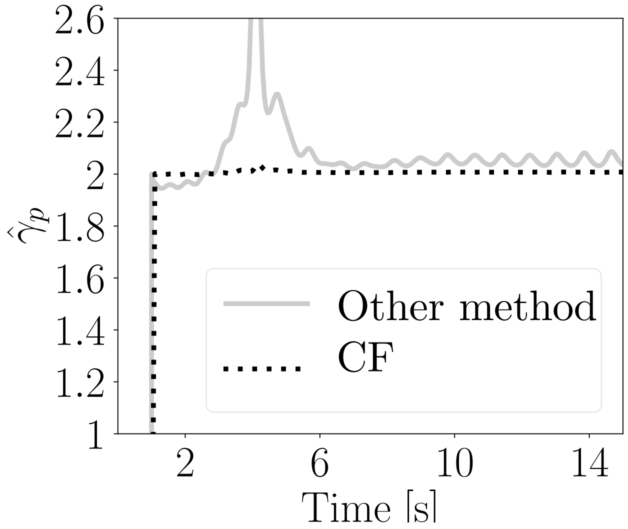


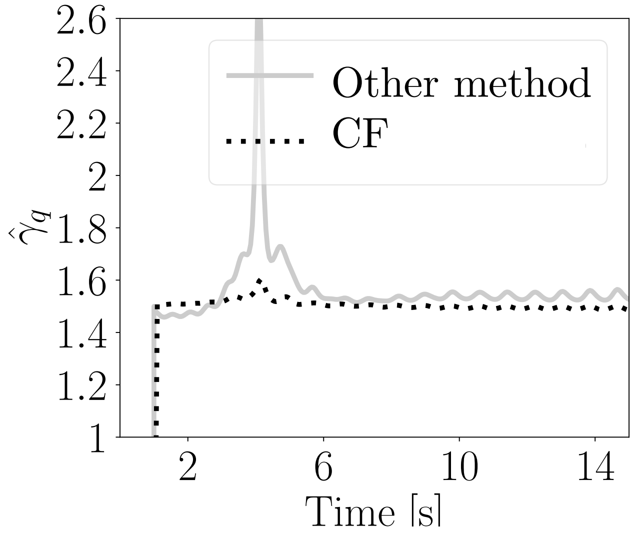
Example: DER


0.06
0.04
0.02
0
−0.02
−0.04
−0.06
$\omega_{\text{bus 3}}$ [Hz]
0
0.5
1
1.5
2
2.5
3
Time [s]
CF
PLL
FDF

Application: State Estimation

- Estimation of voltage dependent load parameters:





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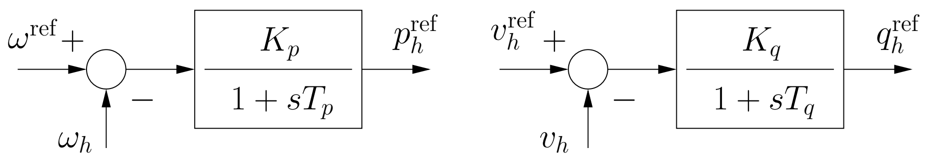
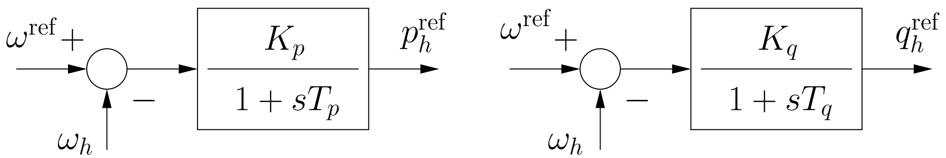
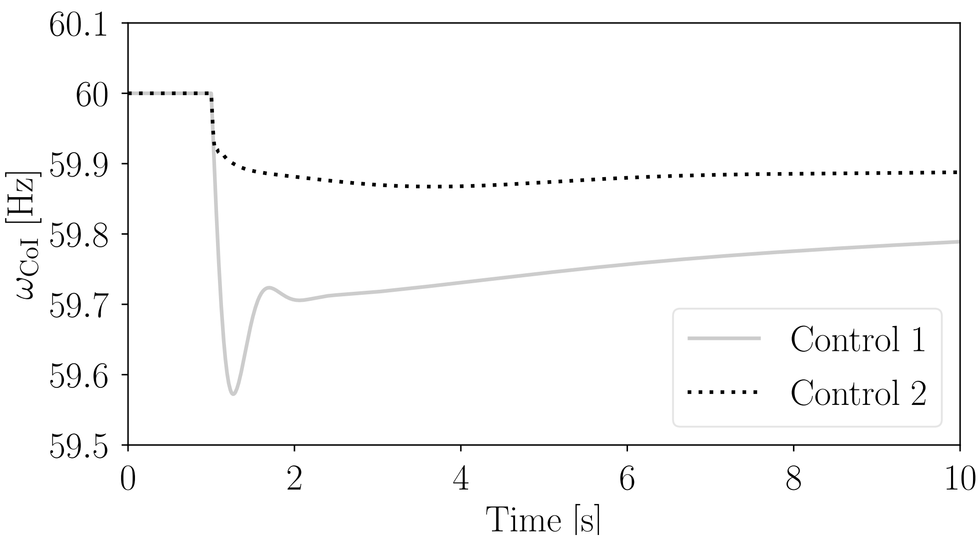
Application: Control of DERs

Control 1



Control 2





Conclusions

- The definition of the complex frequency allows defining a precise link between frequency and voltage variations
- This new quantity overcomes some limitations of the current definition of frequency and suggests a wide range of applications

Thank you!