

Frequency Divider: How to Calculate Frequency during Electromechanical Transients

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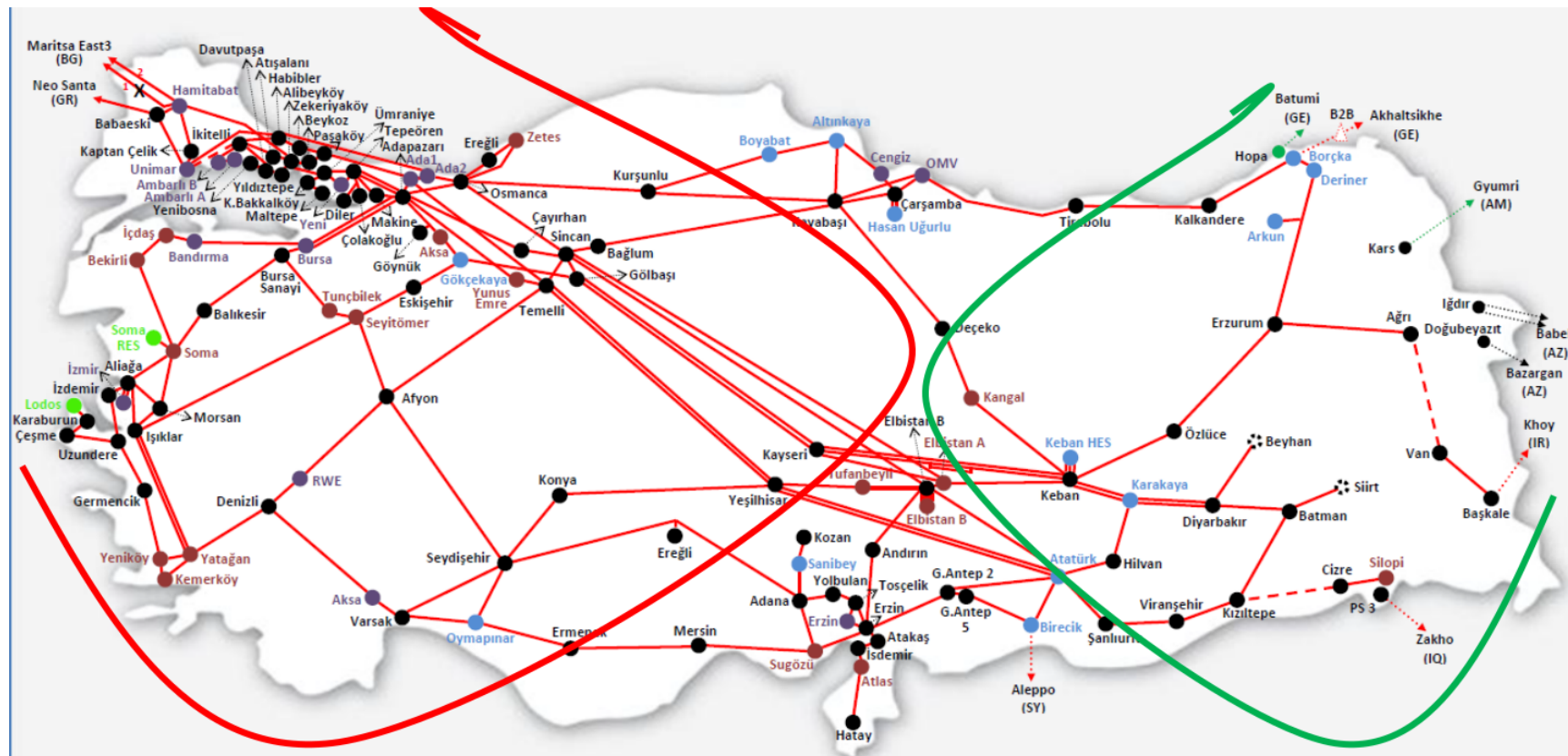
Dublin, Ireland

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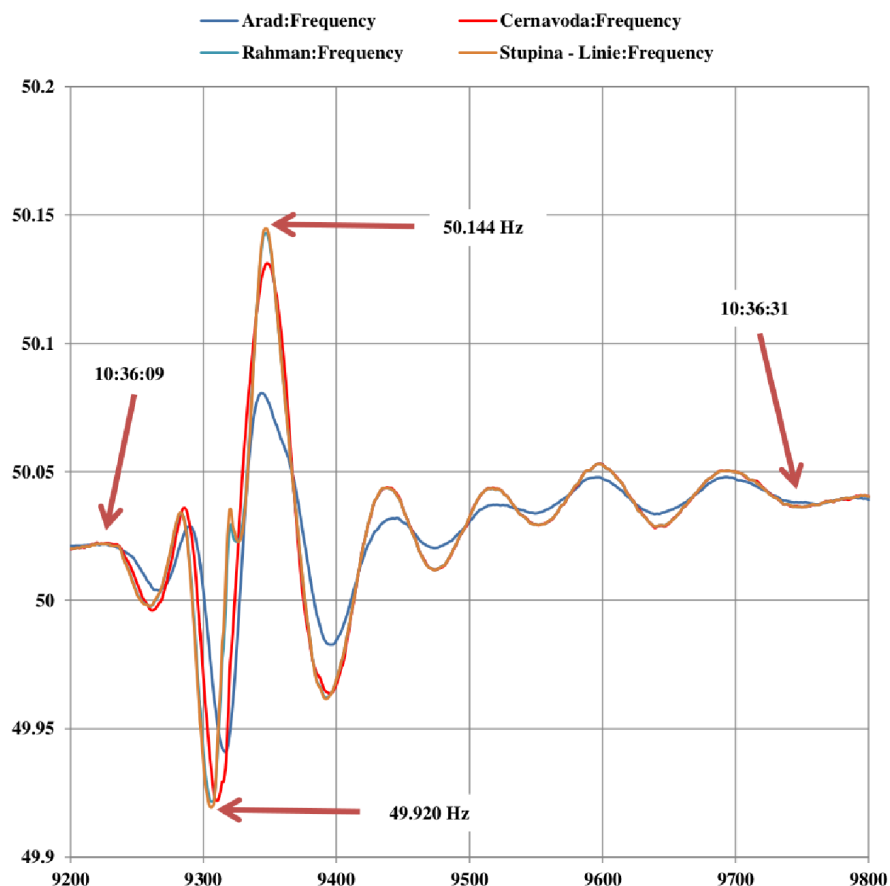
- Motivation
- Existing methods
- Frequency divider formula
- Examples

Turkey Blackout on 31st of March 2015 – I

- The blackout in Turkey led to the outage of 32 GW.



Turkey Blackout on 31st of March 2015 – II



- As a consequence of the line outages and the blackout in Turkey, the Romanian system experienced severe frequency oscillations.
- Bigger oscillations were measured at locations geographically closer to Turkey.

Existing Methods

Center of Inertia

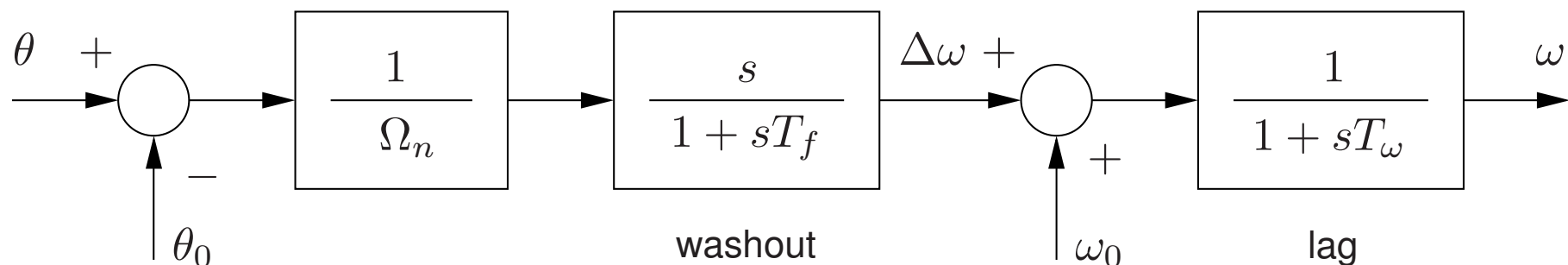
- The *center of inertia* (COI) is a weighted arithmetic average of the rotor speeds of synchronous machines that are connected to a transmission system:

$$\omega_{\text{COI}} = \frac{\sum_{j \in \mathcal{G}} H_j \omega_j}{\sum_{j \in \mathcal{G}} H_j}$$

where ω_j and H_j are the rotor speed and the inertia constant, respectively, of the synchronous machine j and \mathcal{G} is the set of synchronous machines belonging to a given cluster.

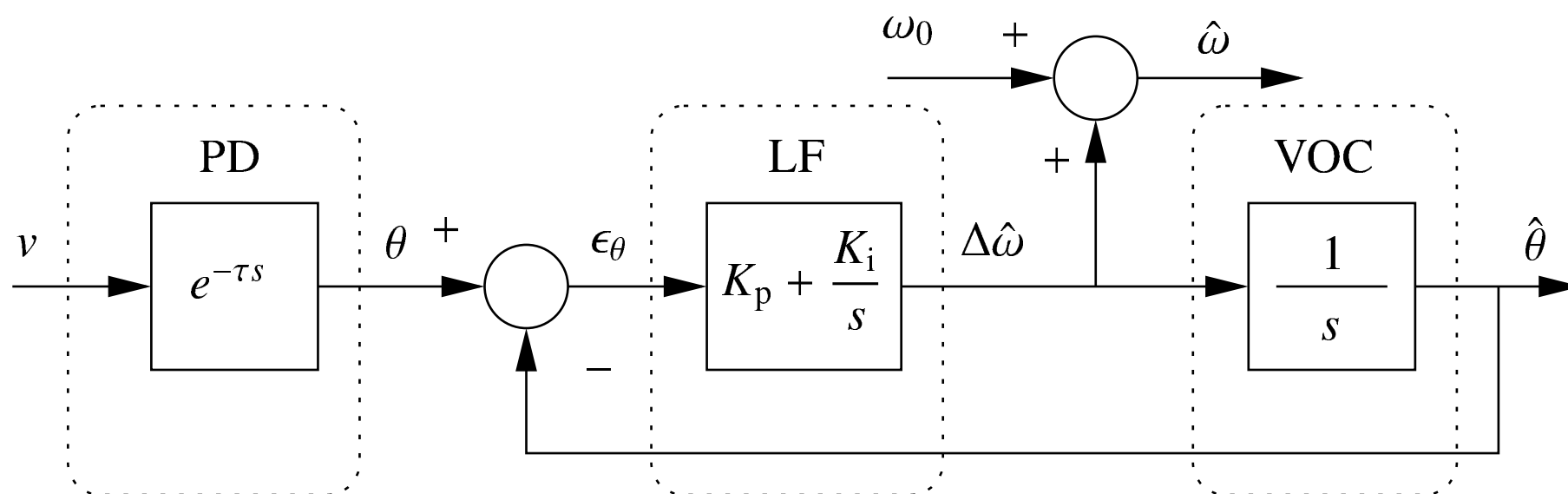
Derivative of the Bus Voltage Phase Angle (θ)

- The frequency estimation is obtained by means of a washout and a low-pass filter.
- The washout filter approximates the derivative of the input signal.
- $T_f = 3/\Omega_n$ s and $T_w = 0.05$ s are used as default values for all simulations.



Phase-Locked Loop

- The phase-locked loop (PLL) is the most common device utilized in power converters to track the phase of the ac voltage at the bus where the converter is connected.
- It is composed of a phase detector (PD); a loop filter (LF); and a voltage oscillator control (VOC).



Frequency Divider Formula

Nodal Equations – I

- The very starting point is the augmented admittance matrix, with inclusion of synchronous machine internal impedances as it is commonly defined for fault analysis.
- System currents and voltages are linked as follows:

$$\begin{bmatrix} \bar{i}_G \\ \bar{i}_B \end{bmatrix} = \begin{bmatrix} \bar{Y}_{GG} & \bar{Y}_{GB} \\ \bar{Y}_{BG} & \bar{Y}_{BB} \end{bmatrix} \begin{bmatrix} \bar{e}_G \\ \bar{v}_B \end{bmatrix} \quad (1)$$

where \bar{v}_B and \bar{i}_B are bus voltages and current injections, respectively, at network buses; \bar{i}_G are generator current injections; \bar{e}_G are generator emfs behind the internal generator impedance; \bar{Y}_{BB} is the standard network admittance matrix plus a diagonal matrix that accounts for the internal impedances of the synchronous machines at generator buses, i.e., $\bar{Y}_{BB} = \bar{Y}_{bus} + \bar{Y}_{G0}$; \bar{Y}_{GG} , \bar{Y}_{GB} and \bar{Y}_{BG} are admittance matrices obtained using the internal impedances of the synchronous machines.

Nodal Equations – II

- To further elaborate on (1), let us assume that load current injections \bar{i}_B can be neglected in (1).
- Let's rewrite (1) as follows:

$$\begin{bmatrix} \bar{i}_G \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{GG} & \bar{Y}_{GB} \\ \bar{Y}_{BG} & \bar{Y}_{BB} \end{bmatrix} \begin{bmatrix} \bar{e}_G \\ \bar{v}_B \end{bmatrix} \quad (2)$$

- Bus voltages \bar{v}_B are thus a function of generator emfs and can be computed explicitly:

$$\bar{v}_B = -\bar{Y}_{BB}^{-1} \bar{Y}_{BG} \bar{e}_G = \bar{D} \bar{e}_G \quad (3)$$

Time Derivative in the dq-frame – I

- Let's consider the time derivative – indicated with the functional $p(\cdot)$ – of the bus voltage phasors in a dq-frame rotating with frequency ω_0 :

$$p\bar{v}_{dq,h} = \frac{d}{dt}\bar{v}_{dq,h} + j\omega_0\bar{v}_{dq,h} \quad (4)$$

where $\bar{v}_{dq,h} = v_{d,h} + jv_{q,h}$.

- Assuming “slow” electromechanical transient, (4) can be approximated as:

$$p\bar{v}_h \approx j\omega_h\bar{v}_h \quad (5)$$

where $\Delta\omega_h = \omega_0 + \Delta\omega_h$ is the frequency at bus h .

Other Approximations

- The following approximations and assumptions are applied:
 - $\bar{v}_B \approx 1$ pu and $\bar{e}_G \approx 1$ pu;
 - The conductances of the elements of all admittance matrices utilized to compute \bar{D} are negligible, e.g., $\bar{Y}_{BB} \approx j\mathbf{B}_{BB}$;
- Finally, let us define bus and generator frequency variations as:

$$\Delta\omega_B = \omega_B - \omega_0 \cdot 1 \quad (6)$$

$$\Delta\omega_G = \omega_G - \omega_0 \cdot 1$$

where, usually, $\omega_0 = 1$ pu.

Frequency Divider Formula

- After applying all approximations above, we obtain **the proposed frequency divider formula**:

$$\mathbf{B}_{BB} \Delta\omega_B = -\mathbf{B}_{BG} \Delta\omega_G \quad (7)$$

or, alternatively:

$$\Delta\omega_B = \mathbf{D} \Delta\omega_G \quad (8)$$

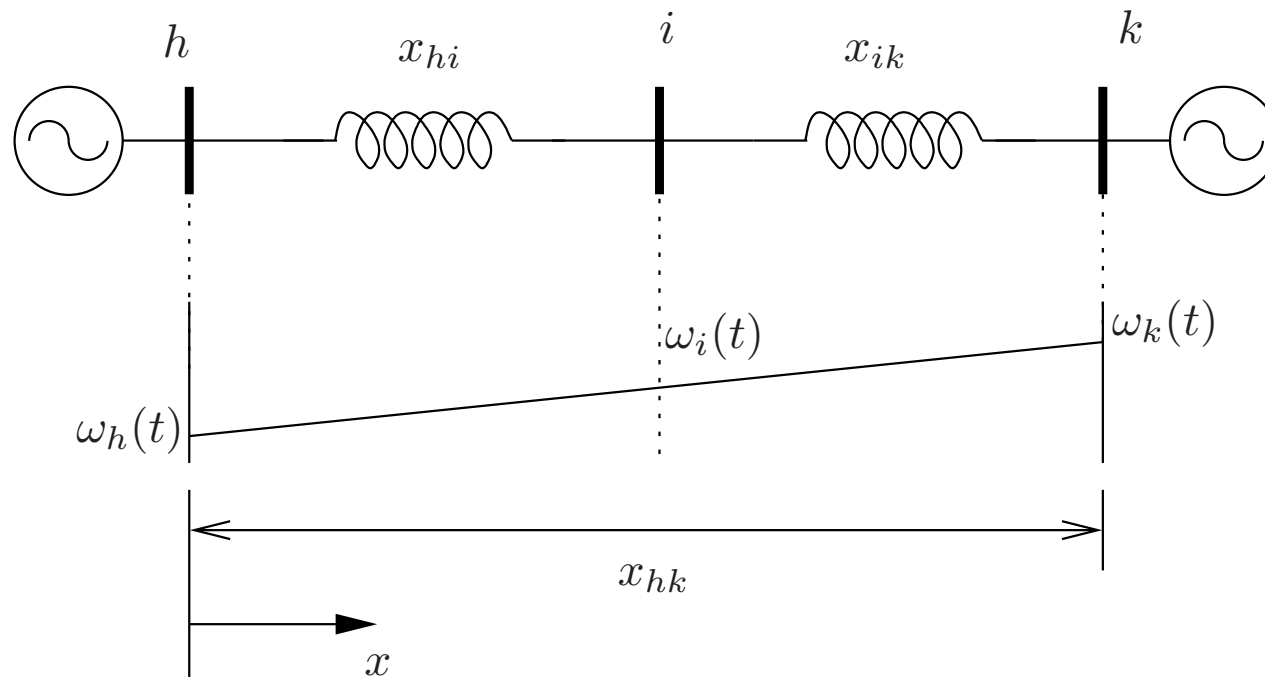
where $\mathbf{D} = -\mathbf{B}_{BB}^{-1} \mathbf{B}_{BG}$.

- The latter formula has the same formal structure of voltage dividers in resistive dc circuits, hence the proposed name.

Examples

Radial System – I

- Let assume a lossless connection, with total reactance $x_{hk} = x_{hi} + x_{ik}$.
- The frequencies at buses h and k , say ω_h and ω_k , respectively, are the rotor speeds of the synchronous generators.



Radial System – II

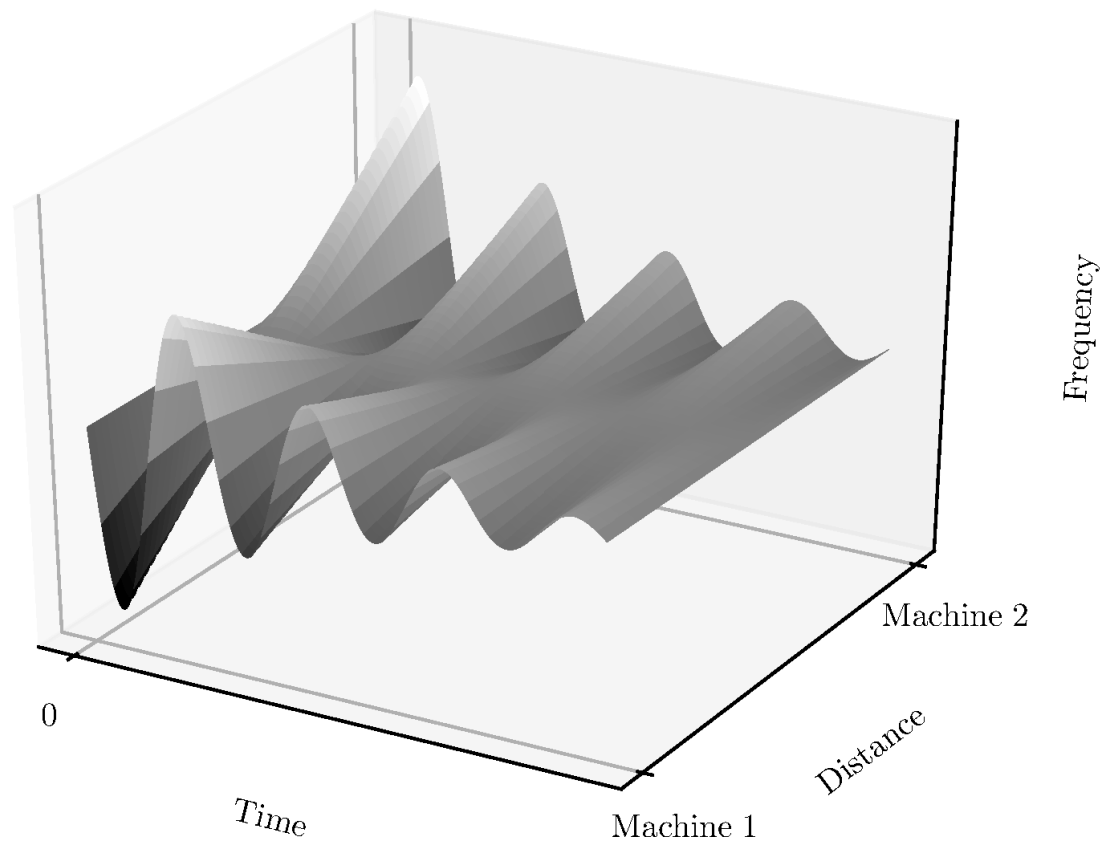
- Applying the frequency divider formula (8), we obtain:

$$\begin{aligned}
 \omega_i(t) &= \mathbf{D} \cdot \begin{bmatrix} \omega_h(t) \\ \omega_k(t) \end{bmatrix} = -\mathbf{B}_{BB}^{-1} \mathbf{B}_{BG} \cdot \begin{bmatrix} \omega_h(t) \\ \omega_k(t) \end{bmatrix} \\
 &= \left[\frac{1}{x_{hi}} + \frac{1}{x_{ik}} \right]^{-1} \begin{bmatrix} \frac{1}{x_{hi}} & \frac{1}{x_{ki}} \end{bmatrix} \cdot \begin{bmatrix} \omega_h(t) \\ \omega_k(t) \end{bmatrix} \\
 &= \frac{x_{ik}}{x_{hk}} \cdot \omega_h(t) + \frac{x_{hi}}{x_{hk}} \cdot \omega_k(t)
 \end{aligned} \tag{9}$$

- The instantaneous frequency $\omega_i(t)$ at a generic point i between the boundaries h and k is a linear interpolation between $\omega_h(t)$ and $\omega_k(t)$.

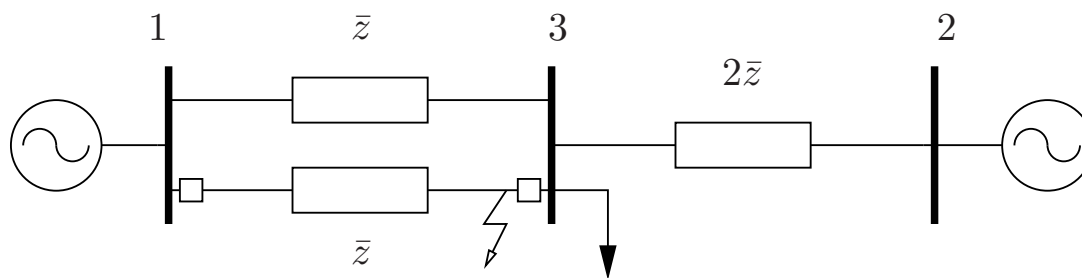
Two-machine System – II

- Graphical representation of the frequency divider.



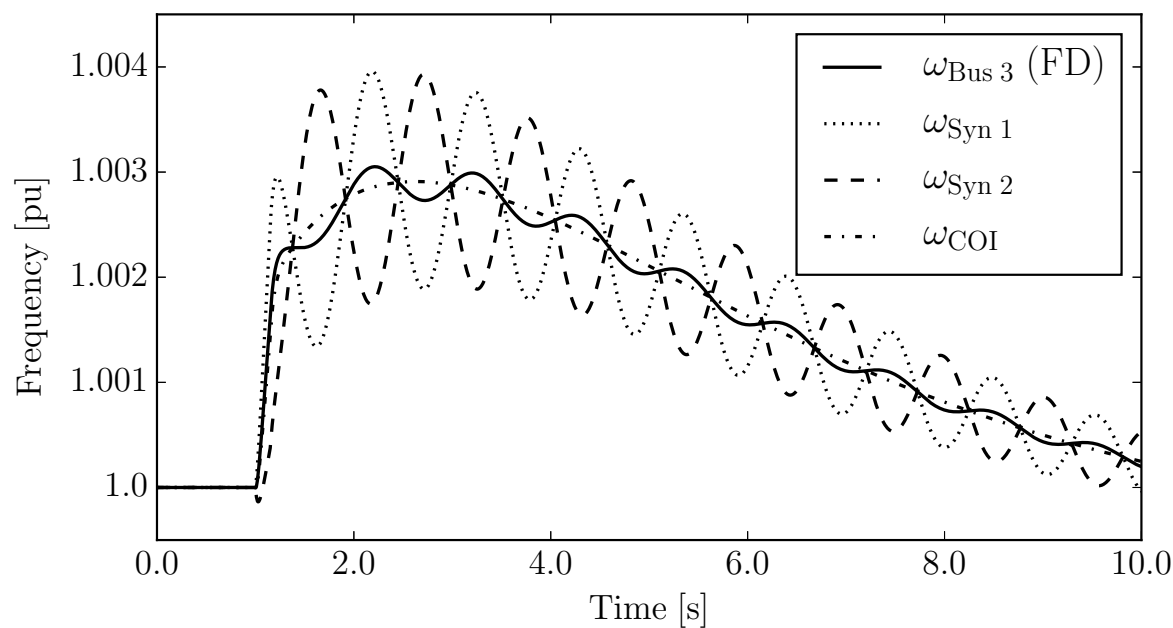
Example – I

- Let's first consider a standard model for transient stability analysis where transmission lines are lumped and modeled as constant impedances and generator flux dynamics are neglected.
- Generators are equal and are modeled as a 6th order synchronous machine with AVRs and turbine governors.
- The load is modeled as a constant admittance. The disturbance is a three-phase fault that occurs at bus 3 at $t = 1$ s and is cleared after 150 ms by opening one of the two lines connecting buses 1 and 3.



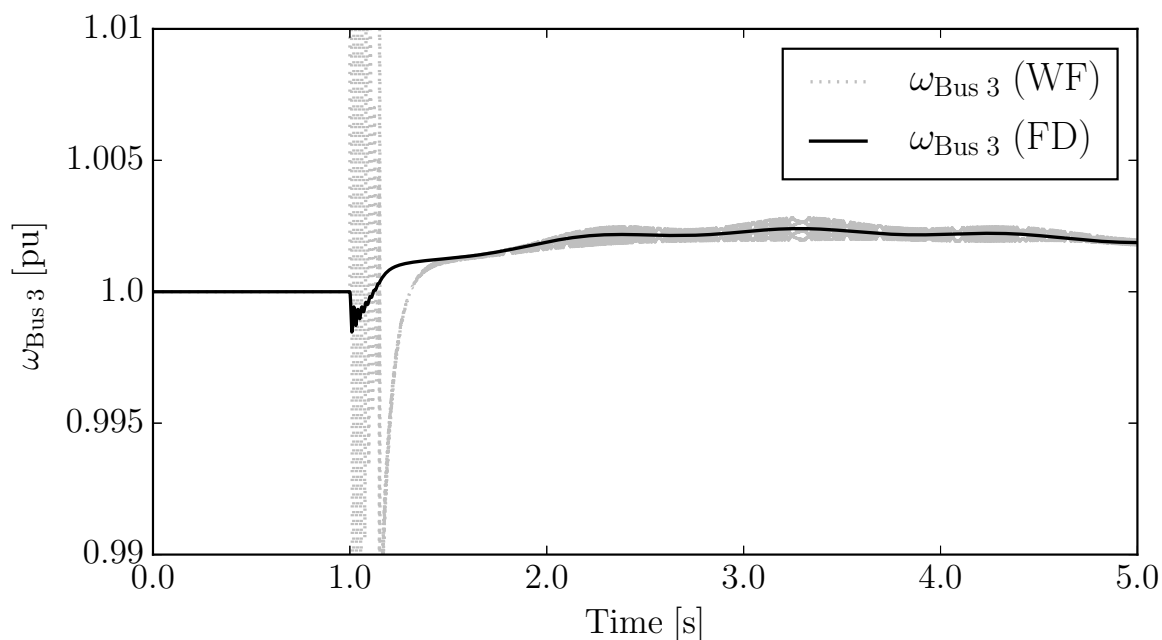
Example – II

- Transient behavior of synchronous machine rotor speeds, the frequency of the COI (ω_{COI}), and the estimated frequency at the load bus using the proposed frequency divider approach.



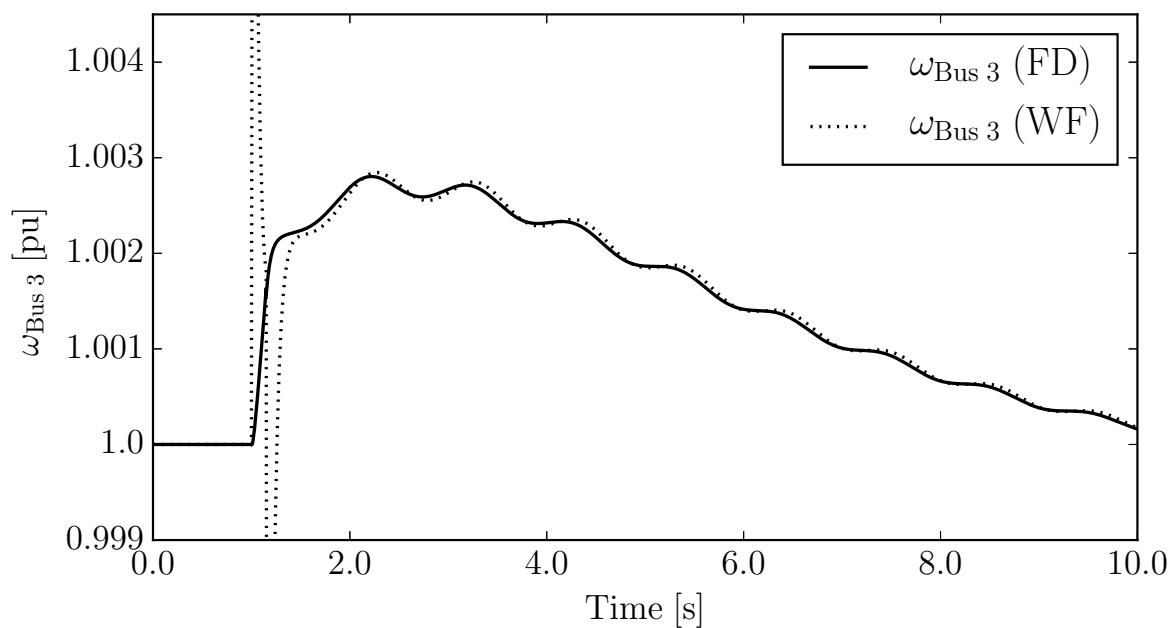
Example – III

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The system is simulated using the fully-fledged dq-axis model.



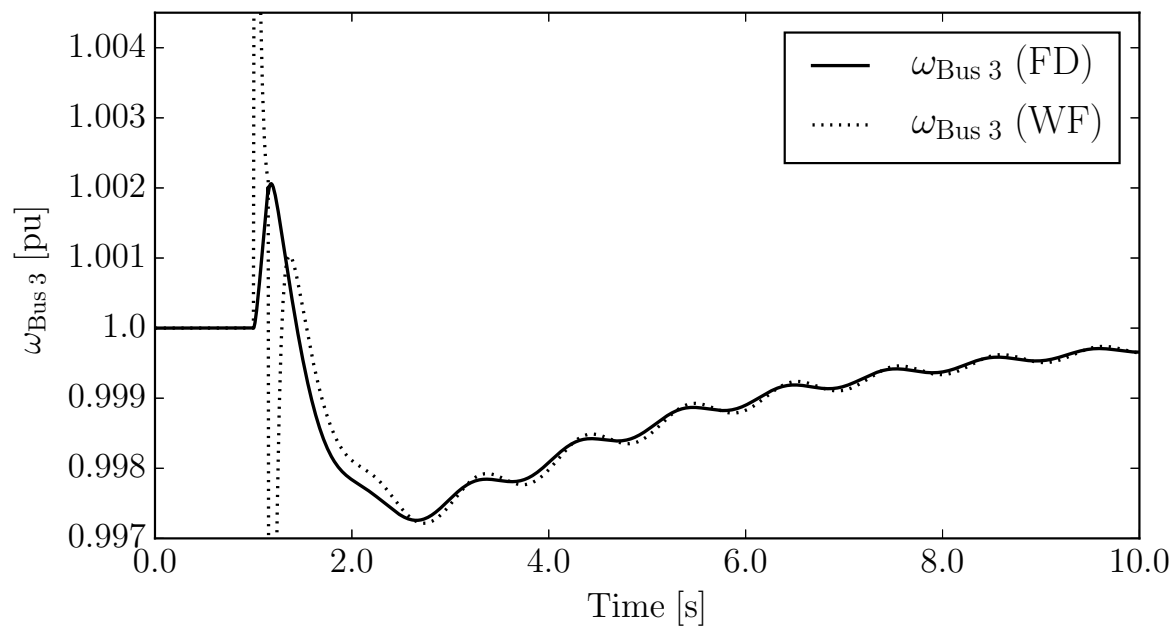
Example – IV

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The load is modelled as a frequency-dependent load representing an aluminum plant



Example – V

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The load is a squirrel cage induction motor with a 5th-order dq-axis model.

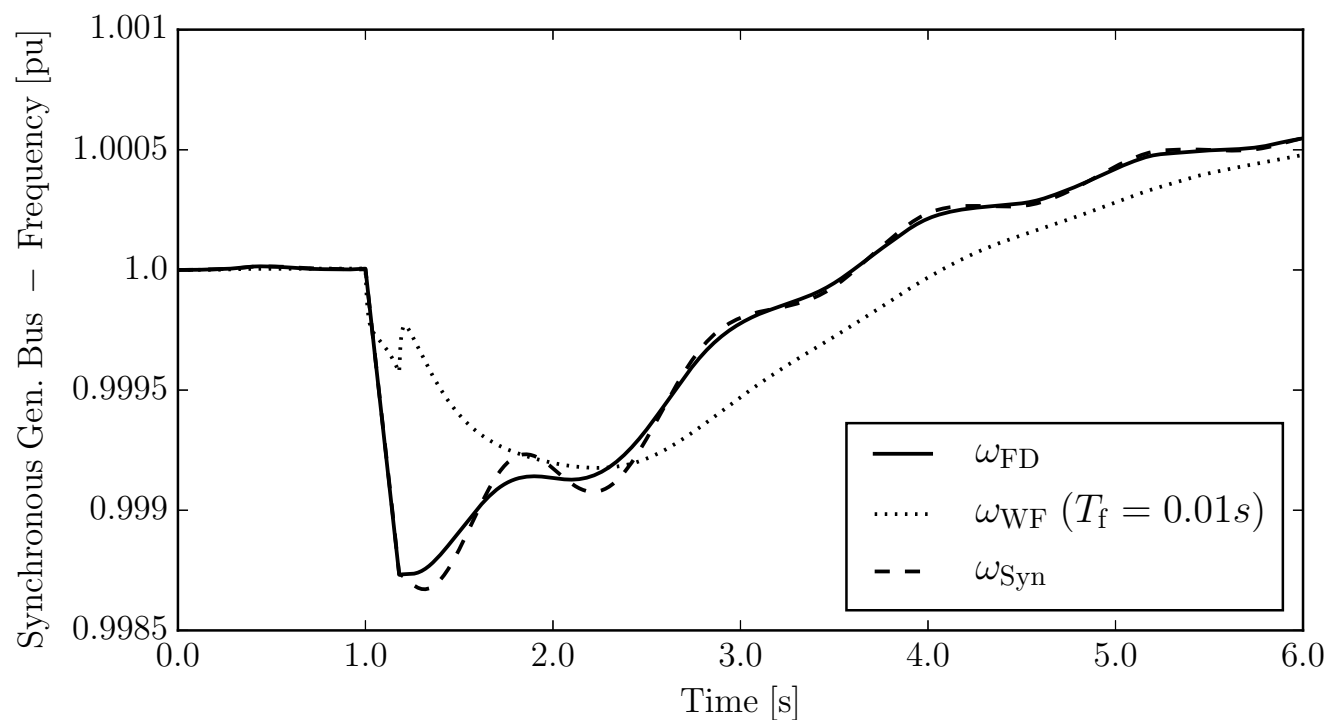


Irish Transmission System – I

- This system includes 1,479 buses, 1,851 transmission lines and transformers, 245 loads, 22 conventional synchronous power plants modeled with 6th order synchronous machine models with AVRs and turbine governors, 6 PSSs and 176 wind power plants, of which 34 are equipped with constant-speed (CSWT) and 142 with doubly-fed induction generators (DFIG).
- The large number of non-conventional generators based on induction machines and power electronics converters makes this system an excellent test-bed to check the accuracy of the proposed frequency divider.

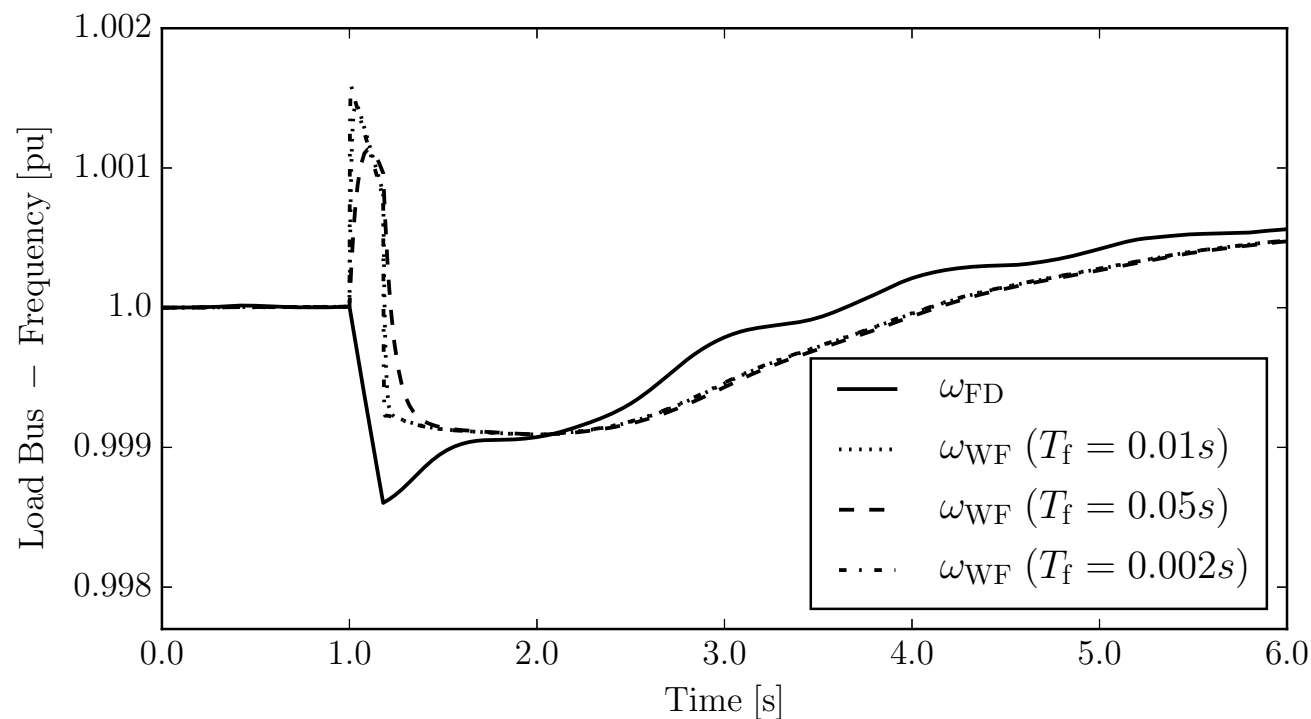
Irish Transmission System – II

- Fault close to a synchronous machine and a load. Frequency at the generator bus.



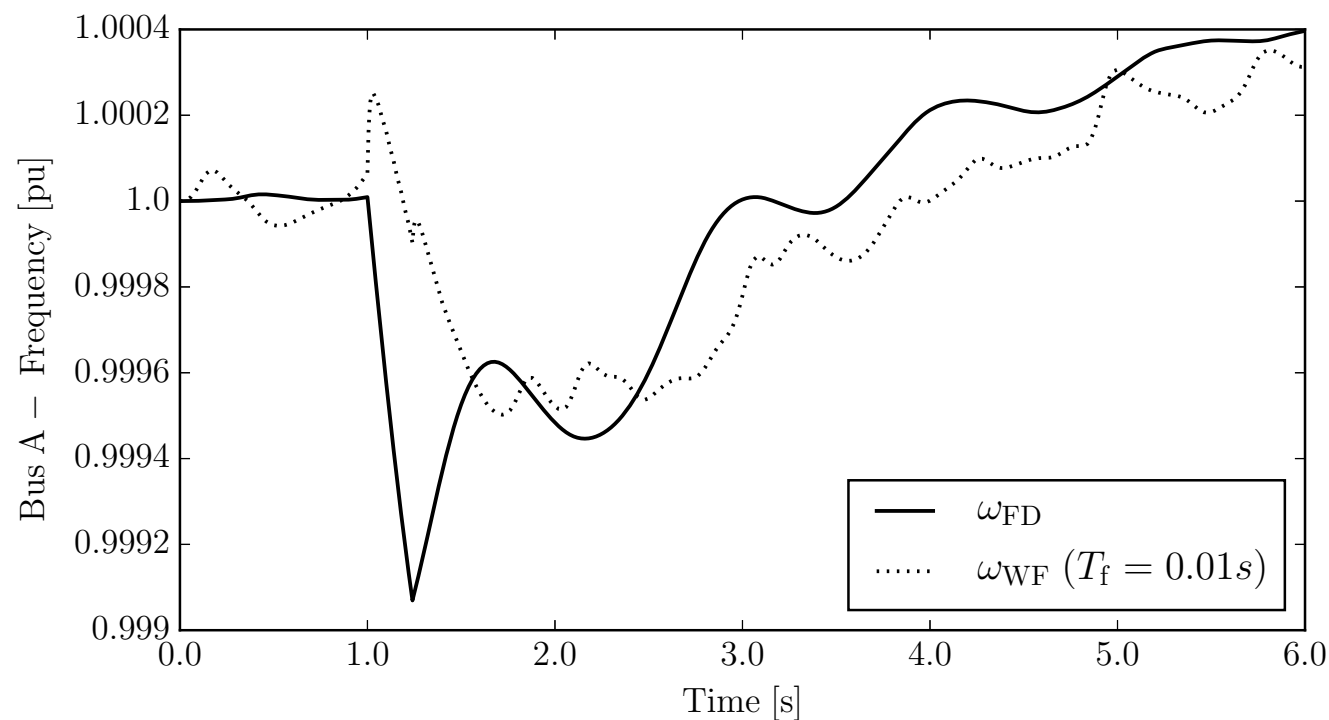
Irish Transmission System – III

- Fault close to a synchronous machine and a load. Frequency at the load bus.



Irish Transmission System – IV

- Frequency response of the Irish transmission system facing a three-phase fault close to a wind power plant.

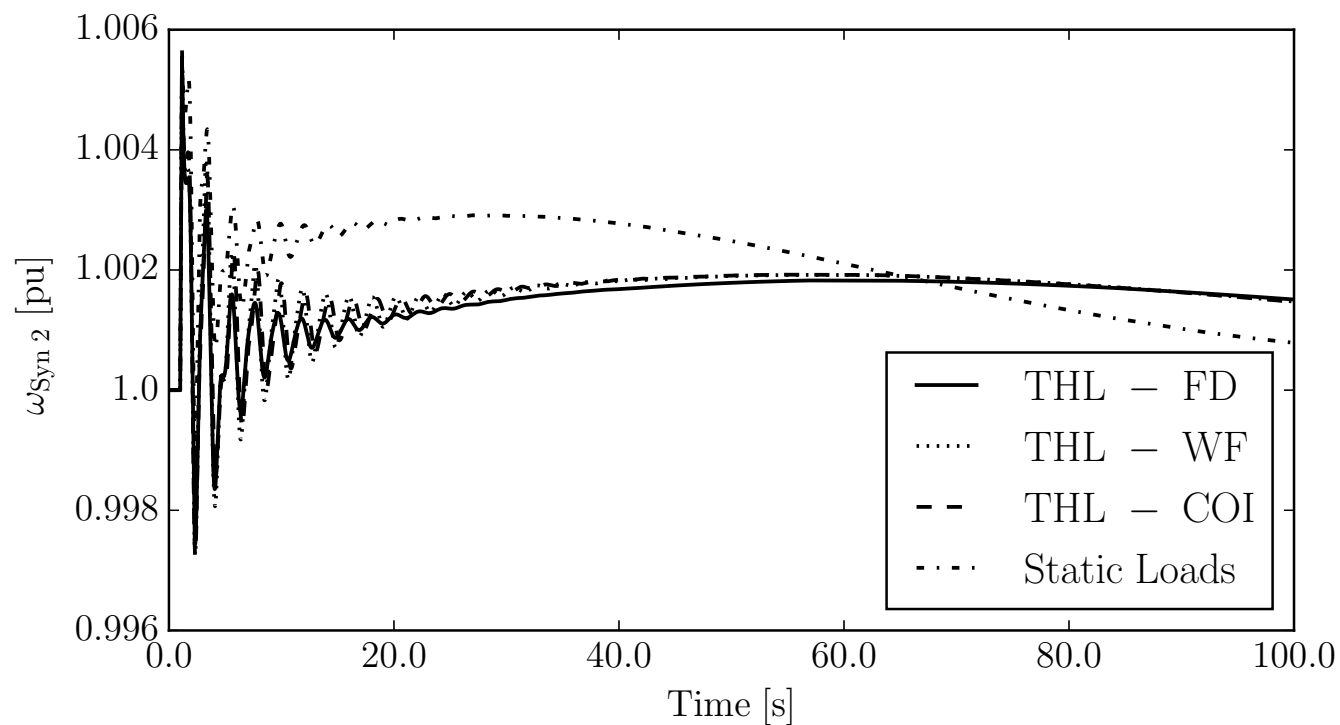


New England 39-bus, 10-machine System – I

- This benchmark network contains 19 loads totaling 7,316.5 MW and 1,690.9 MVar of active and reactive power, respectively (20% load increase with respect to the base case is assumed).
- The system model also includes generator controllers such as primary voltage regulators, as well as both primary and secondary frequency regulation (turbine governors and AGC).
- The system includes also thermostatically controlled loads, which are the 20% of the total load.
- The contingency is a three-phase fault at bus 21, cleared by the opening of the line connecting buses 16 and 21 after 160 ms.

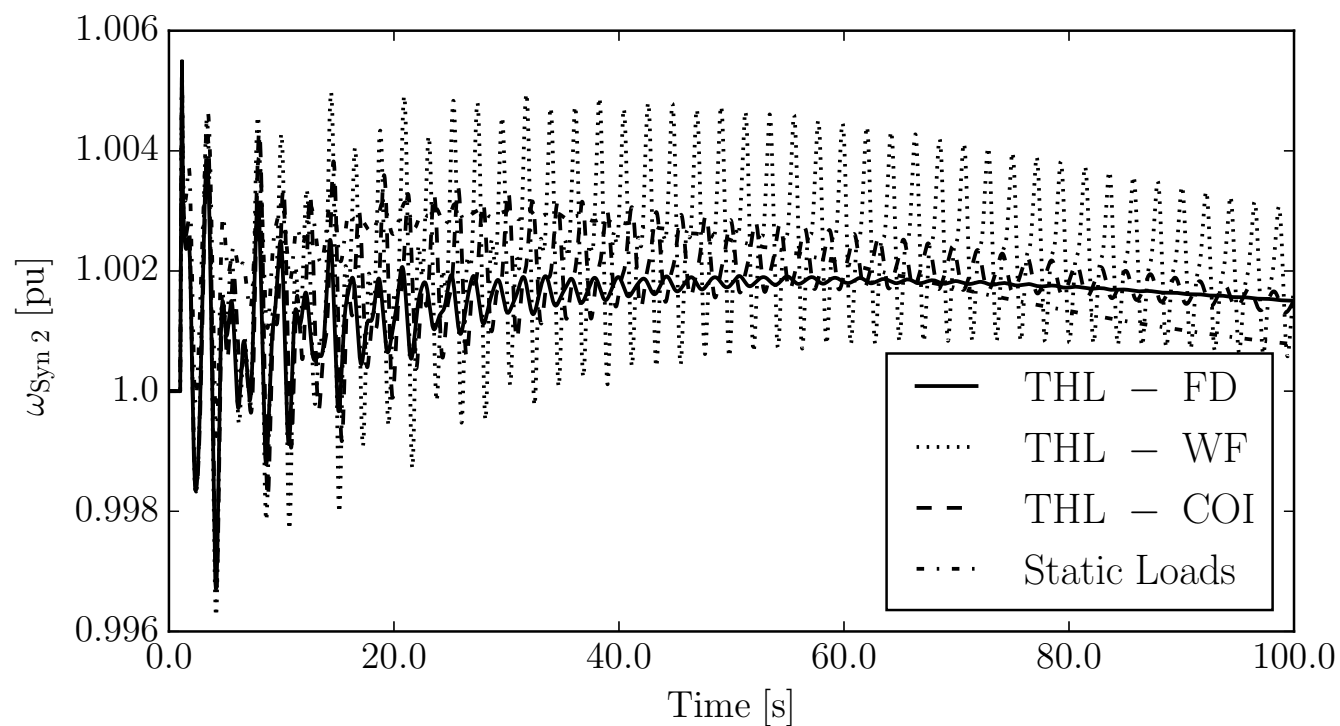
New England 39-bus, 10-machine System – II

- Rotor speed of the synchronous generator in bus 31 (Gen 2) using 3rd order synchronous machine models.



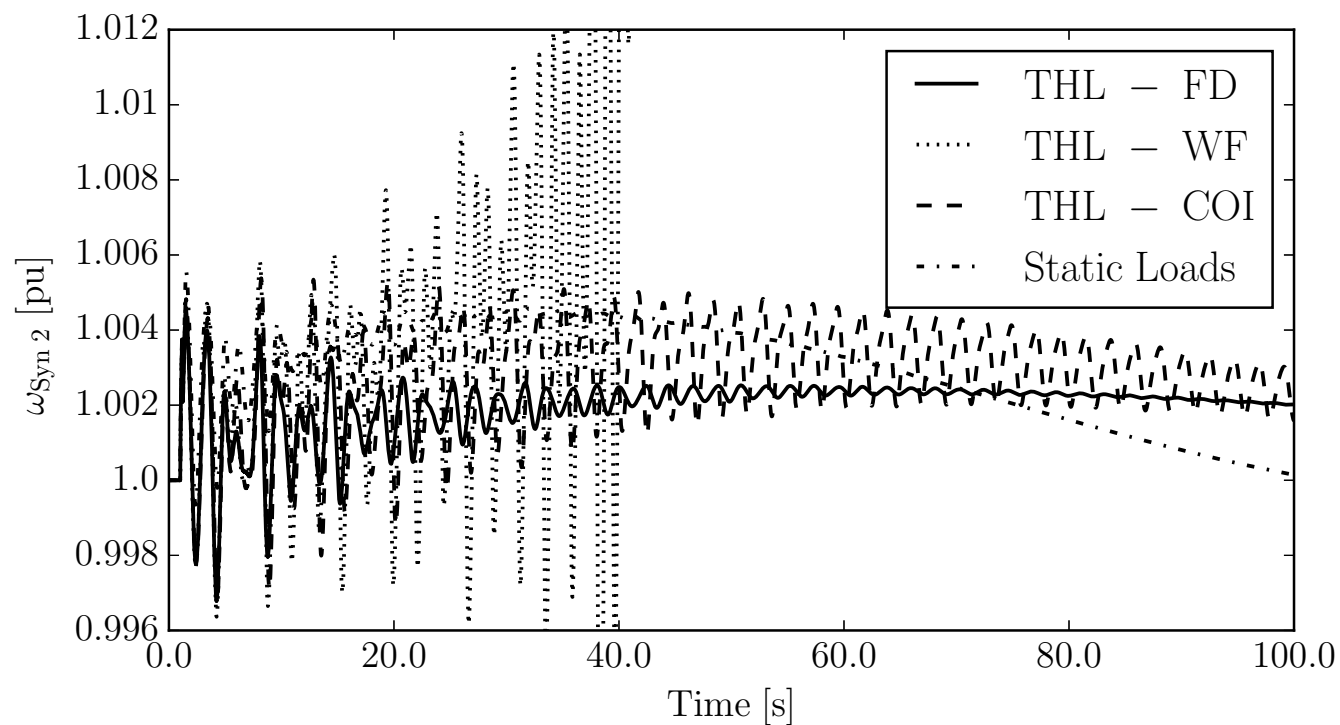
New England 39-bus, 10-machine System – III

- Rotor speed of the synchronous generator in bus 31 (Gen 2) using 6th order synchronous machine models.



New England 39-bus, 10-machine System – IV

- Rotor speed of the synchronous generator in bus 31 (Gen 2) using 8th order synchronous machine models.



ENTSO-E Transmission System – I

- The model includes 21,177 buses (1,212 off-line); 30,968 transmission lines and transformers (2,352 off-line); 1,144 coupling devices, i.e., zero-impedance connections (420 off-line); 15,756 loads (364 off-line); and 4,828 power plants.
- Of these power plants, 1,160 power plants are off-line. The system also includes 364 PSSs.

ENTSO-E Transmission System – II

- Size and number of non-zeros (NNZ) elements of matrices \mathbf{B}_{BB} , \mathbf{B}_{BG} and \mathbf{D} for the ENTSO-E system.

Matrix	Size	NNZ	NNZ %
\mathbf{B}_{BG}	$21,177 \times 4,832$	4,832	0.0047
\mathbf{B}_{BB}	$21,177 \times 21,177$	72,313	0.0161
\mathbf{D}	$21,177 \times 4,832$	86,169,456	84.2

ENTSO-E Transmission System – III

- The inverse of a sparse matrix can be very dense ...
- ... and trying to actually compute \mathbf{D} leads to memory overflow for the ENTSO-E system!
- Hence, the most efficient implementation of the frequency divider formula is the following *acausal* expression:

$$\mathbf{0} = \mathbf{B}_{\text{BB}} \cdot (\omega_{\text{B}} - \mathbf{1}_n) + \mathbf{B}_{\text{BG}} \cdot (\omega_{\text{G}} - \mathbf{1}_m)$$

or, in compact notation:

$$\mathbf{0} = \mathbf{B}_{\text{BB}} \Delta\omega_{\text{B}} + \mathbf{B}_{\text{BG}} \Delta\omega_{\text{G}}$$

Conclusions, Future Work and References

Conclusions (for now ...)

- A general expression to estimate frequency variations during the transient of electric power systems has been deduced.
- The proposed expression is derived based on standard assumptions of power system models for transient stability analysis and can be readily implemented in power system software tools for transient stability analysis.
- The formula is aimed at improving the accuracy of bus frequency estimation in traditional electromechanical power system models.
- Simulation results show that the proposed formula is accurate, numerically robust and computationally efficient.
- The formula has relevant applications in dynamic state estimation.

Open Challenges

- How to take into account fast flux transients and wave propagation?
- Effect of loads?
- Frequency makers vs. frequency takers?
- What if there is **no** synchronous machine?
- What does *frequency* mean when the signal is actually non-periodic?

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Thanks much for your attention!