Steady-state Solution of Hybrid Power and Power Electronic Systems

<u>Federico Bizzarri</u> and Angelo Brambilla

Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano
P.za Leonardo da Vinci 32, I-20133 Milano, Italy
e-mail: federico.bizzarri@polimi.it

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The main question is …

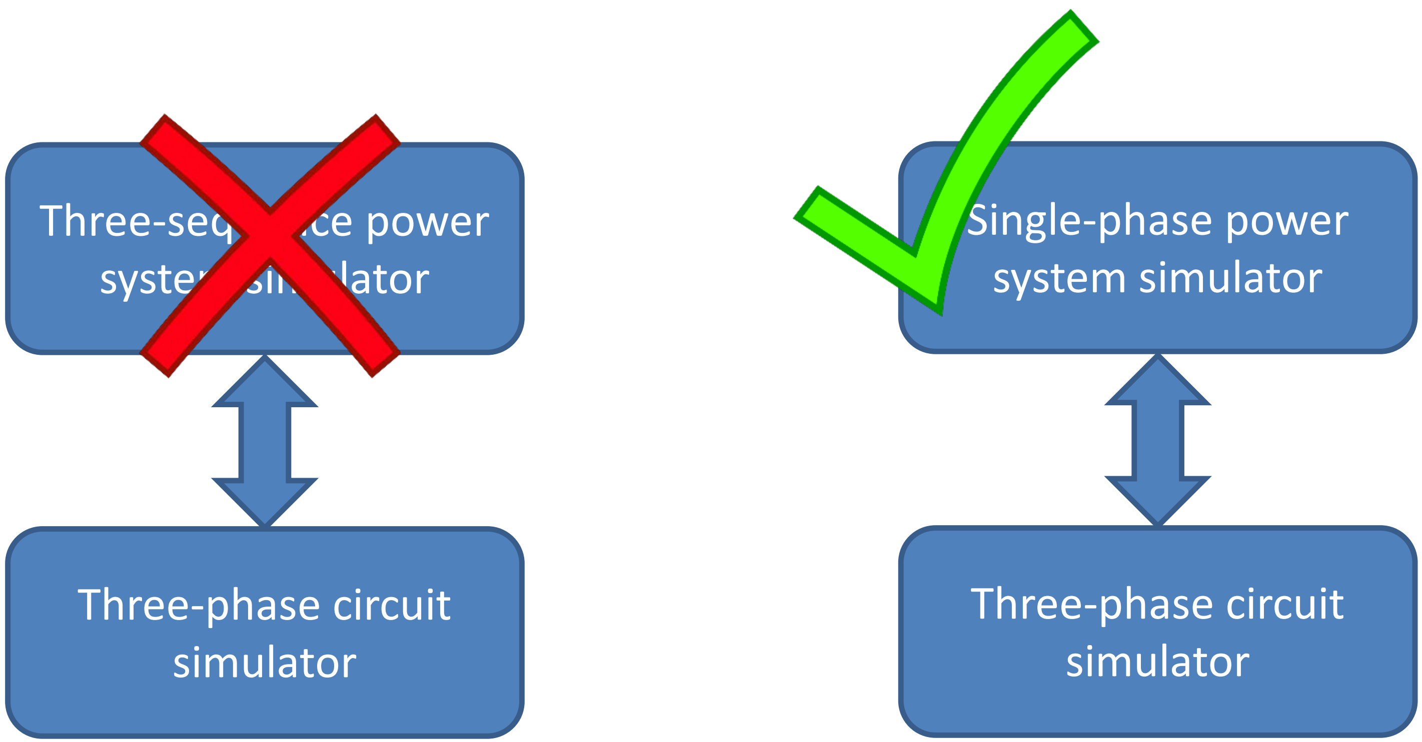
… how to compute the **steady-state solution** of an electrical system that is made up of some parts modeled as a three-phase electr(on)ic circuit and some others according to the power system model? Are we able to study the **stability** of such a solution?

Some typical examples are found in solving

- Integrated Transmission and Distribution (T&D) systems;
- Transmission systems including transmission lines that need to be modelled as linear dynamical systems instead of as constant impedances;
- AC-DC networks with high voltage direct current (HVDC) links where voltage source converters (VSCs) are used to manage the bidirectional power flow from the AC power grid to the DC link and vice versa.

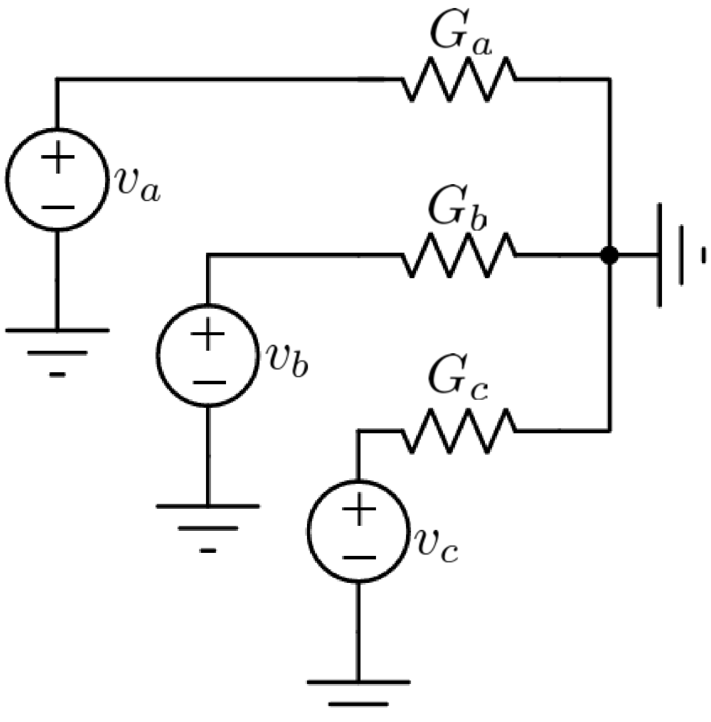
On the whole we are interested in

… those systems that in the literature are referred to as **hybrid power systems** (HPSs), since they mix what till a few years ago we may had defined as “conventional” and “unconventional” power systems.



Let's break the ice

An easy-peasy 3-phase circuit



$$v_a(t) = E_o \cos\left(\Omega t + \delta_0\right)$$

$$v_b(t) = E_o \cos\left(\Omega t + \delta_0 - \frac{2}{3}\pi\right)$$

$$v_c(t) = E_o \cos\left(\Omega t + \delta_0 + \frac{2}{3}\pi\right)$$

(positive or abc sequence)

$$G_a = G_b = G_c \equiv G$$

Balance Y load

$$P_e(t) = \sum_{k=a,b,c} \underbrace{E_o^2 G \cos^2\left(\Omega t + \delta_0 + \psi_k\right)}_{P_k(t)} = \frac{3}{2} G E_o^2$$

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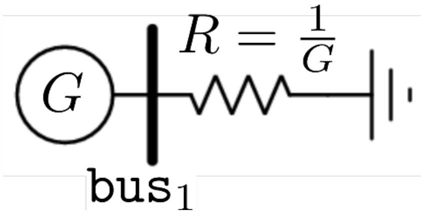
Let's break the ice

An easy-peasy power-system example: a synchronous (swing) machine is coupled to a resistive load.

$$\widehat{v}_d = E_o \cos(\delta(t))$$
$$\widehat{v}_q = E_o \sin(\delta(t))$$



$$\dot{\delta}(t) - \Omega\,(\omega(t) - 1) = 0$$

$$H\dot{\omega}(t) + D\,(\omega(t) - 1) - P_m + P_e(t) = 0$$

$$P_e(t) = \frac{3}{2} G E_o^2$$

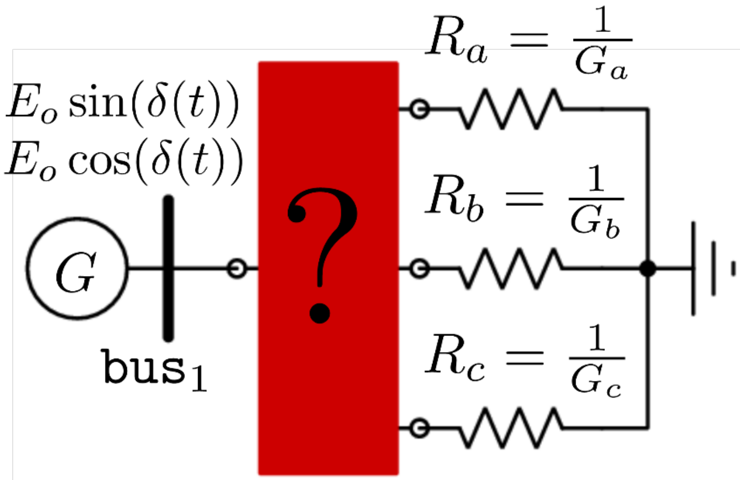
$$\begin{aligned}
\hat{A}_R &= \frac{3}{2}(\widehat{v}_d + \jmath\widehat{v}_q)(\widehat{\imath}_d + \jmath\widehat{\imath}_q)^* \\
&= \frac{3}{2}(\widehat{v}_d + \jmath\widehat{v}_q)(G\widehat{v}_d + \jmath G\widehat{v}_q)^* \\
&= \frac{3}{2}G(\widehat{v}_d^2 + \widehat{v}_q^2) \\
&= \frac{3}{2}G E_o^2
\end{aligned}$$

$$P_m = P_e$$
$$\omega(t) = 1$$
$\delta(t) = \delta_0$ (arbitrary)

Steady state solution
(Power Flow solution)

Let's break the ice

$$\widehat{v}_d = E_o \sin(\delta(t))$$
$$\widehat{v}_q = E_o \cos(\delta(t))$$



$$v_a(t) = E_o \cos\left(\Omega t + \delta(t)\right)$$

$$v_b(t) = E_o \cos\left(\Omega t + \delta(t) - \frac{2}{3}\pi\right)$$

$$v_c(t) = E_o \cos\left(\Omega t + \delta(t) + \frac{2}{3}\pi\right)$$

Balance Y load

$$G_a = G_b = G_c \equiv G$$

$$P_e(t) = \sum_{k=a,b,c} \underbrace{G E_o^2 \cos^2\left(\Omega t + \delta(t) + \psi_k\right)}_{P_k(t)} = \frac{3}{2} G E_o^2$$

$$\dot{\delta}(t) - \Omega\left(\omega(t) - 1\right) = 0$$

$$H\dot{\omega}(t) + D\left(\omega(t) - 1\right) - P_m + P_e(t) = 0$$

$$\delta(t) = \underbrace{\delta(t_0)}_{\delta_0} + \int_{t_0}^{t} \Omega(\omega(\tau) - 1)d\tau$$

$$P_m = P_e$$
$$\omega(t) = 1$$
$$\delta(t) = \delta_0$$

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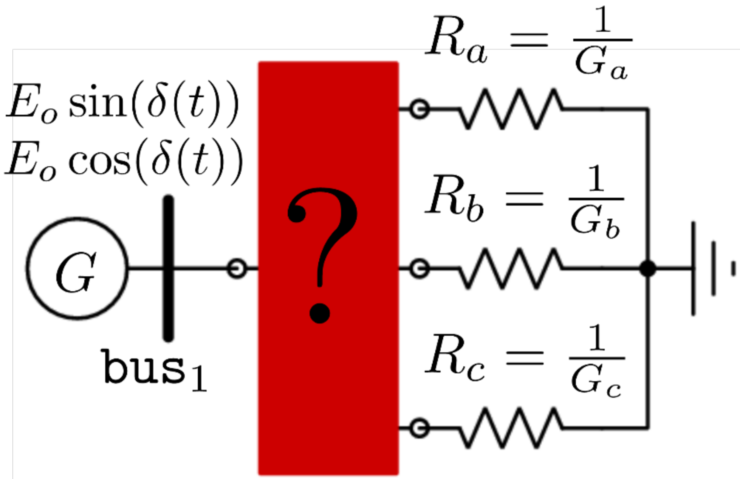
Let's break the ice

$$\widehat{v}_d = E_o \sin(\delta(t))$$
$$\widehat{v}_q = E_o \cos(\delta(t))$$



Balance Y load

$$G_a = G_b = G_c \equiv G$$

$$P_m = P_e$$
$$\omega(t) = 1$$
$$\delta(t) = \delta_0$$

$$\dot{\delta}(t) = \Omega\,(1 - 1) = 0$$

$$H\dot{\omega}(t) = -D\,(1 - 1) + P_e(t) - P_e(t) = 0$$

$$\delta(t) = \underbrace{\delta(t_0)}_{\delta_0} + \int_{t_0}^{t} \Omega(\omega(\tau) - 1)d\tau = \delta_0$$

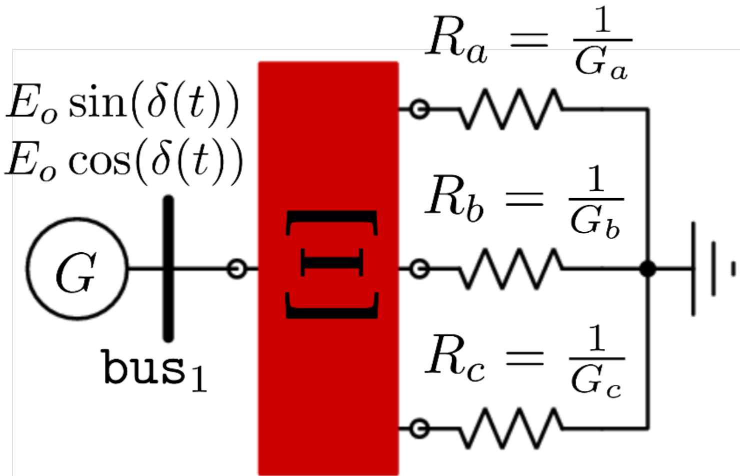
$$v_a(t) = E_o \cos\left(\Omega t + \delta_0\right)$$

$$v_b(t) = E_o \cos\left(\Omega t + \delta_0 - \frac{2}{3}\pi\right)$$

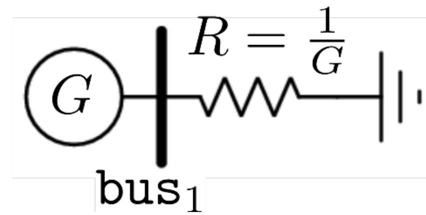
$$v_c(t) = E_o \cos\left(\Omega t + \delta_0 + \frac{2}{3}\pi\right)$$

Let's break the ice

$$\widehat{v}_d = E_o \sin(\delta(t))$$
$$\widehat{v}_q = E_o \cos(\delta(t))$$



$$G_a = G_b = G_c \equiv G$$



Balance Y load

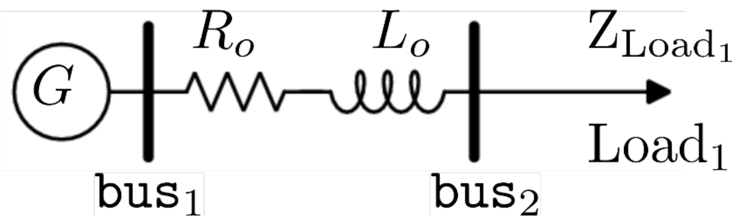
$$G_a = G_b = G_c \equiv G$$

$$\begin{bmatrix} \widehat{v}_d \\ \widehat{v}_q \\ \widehat{v}_0 \end{bmatrix} = \overbrace{\frac{2}{3}\begin{bmatrix} \cos(\Omega t) & \cos\left(\Omega t - \frac{2\pi}{3}\right) & \cos\left(\Omega t + \frac{2\pi}{3}\right) \\ -\sin(\Omega t) & -\sin\left(\Omega t - \frac{2\pi}{3}\right) & -\sin\left(\Omega t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}^{\Xi} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

a-phase to *d*-axis alignment Park transform
(preserving the amplitude)

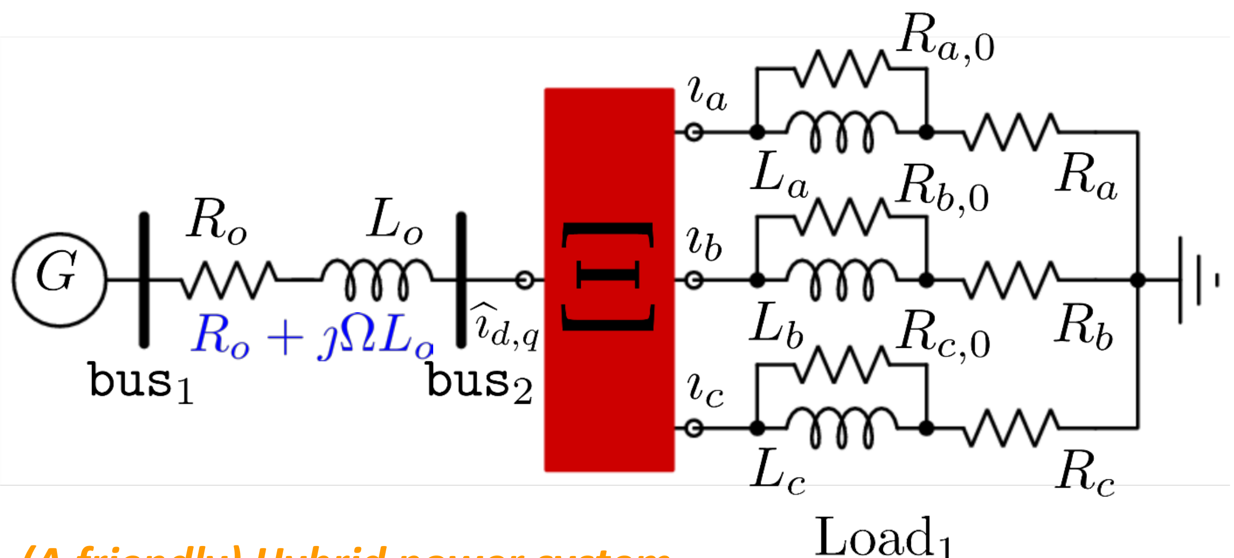
$$(\widehat{\imath}_d, \widehat{\imath}_q) \underset{\Xi^{-1}}{\overset{\Xi}{\leftrightharpoons}} (\imath_a, \imath_b, \imath_c) \qquad (\widehat{v}_d, \widehat{v}_q) \underset{\Xi^{-1}}{\overset{\Xi}{\leftrightharpoons}} (v_a, v_b, v_c)$$

A little more in general …



A generator, a line, two buses, and a constant impedance load modeled with the **single-phase** representation in the DQ frame (**positive sequence** representation).

Classical power system



(A friendly) Hybrid power system

The line is modelled as a constant impedance in the power system framework. The load is fully dynamical.

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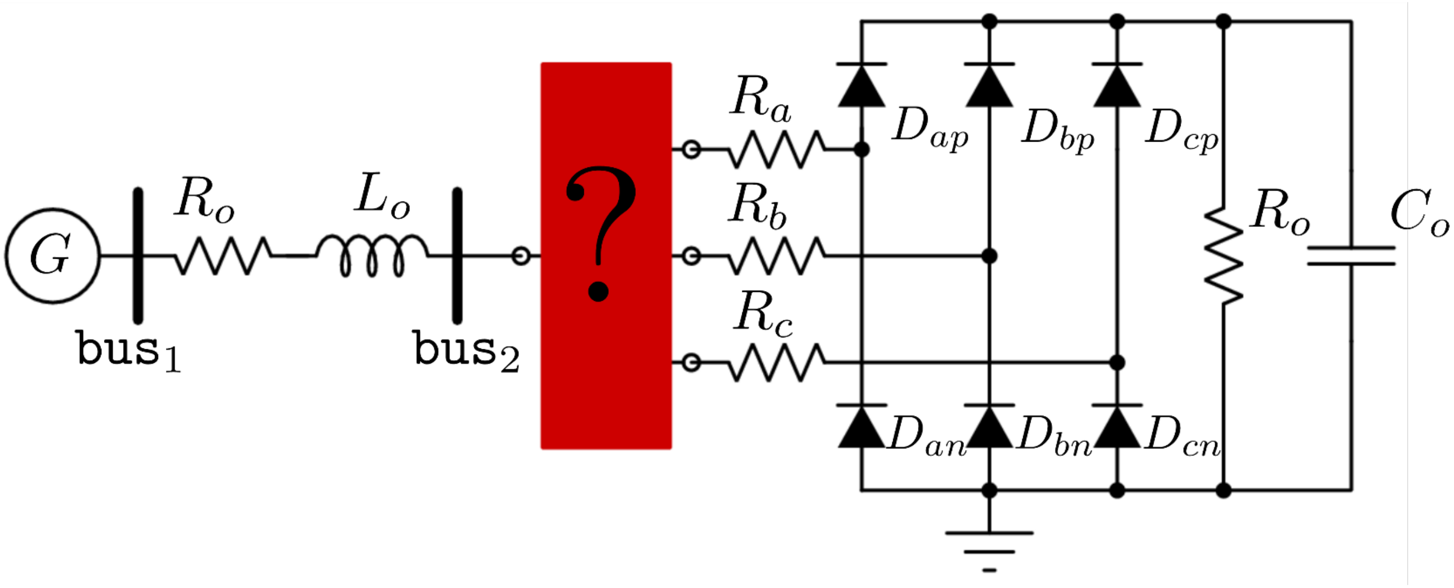
… and more …



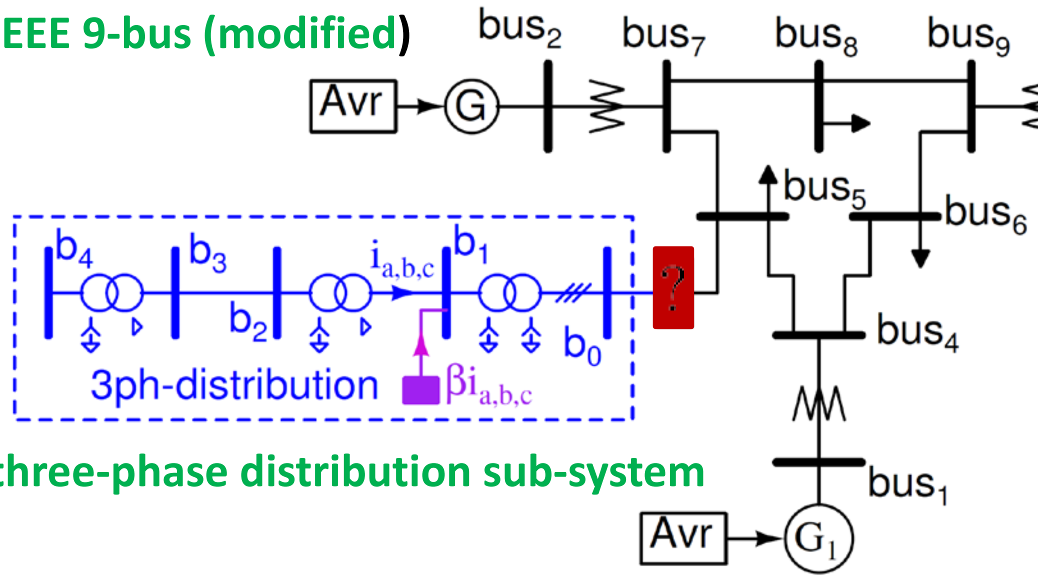
A Hybrid power system

The rectifier is a **non linear** three-phase circuit … how to model the red box?

… and more!

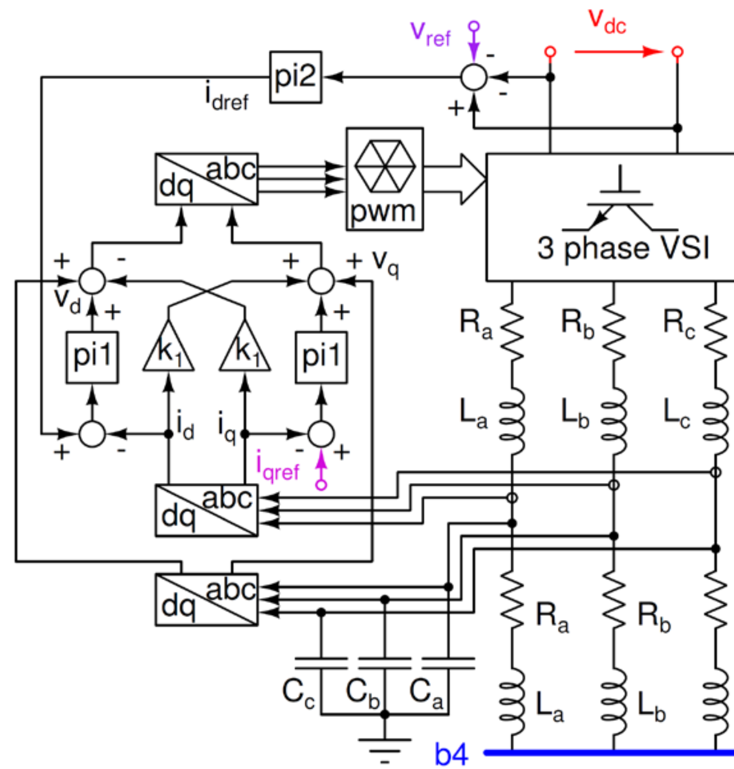
A complex Hybrid power system

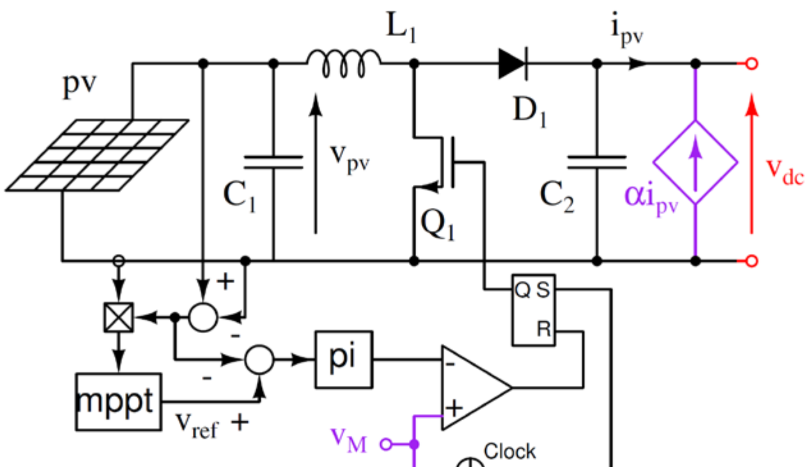
IEEE 9-bus (modified)



three-phase distribution sub-system

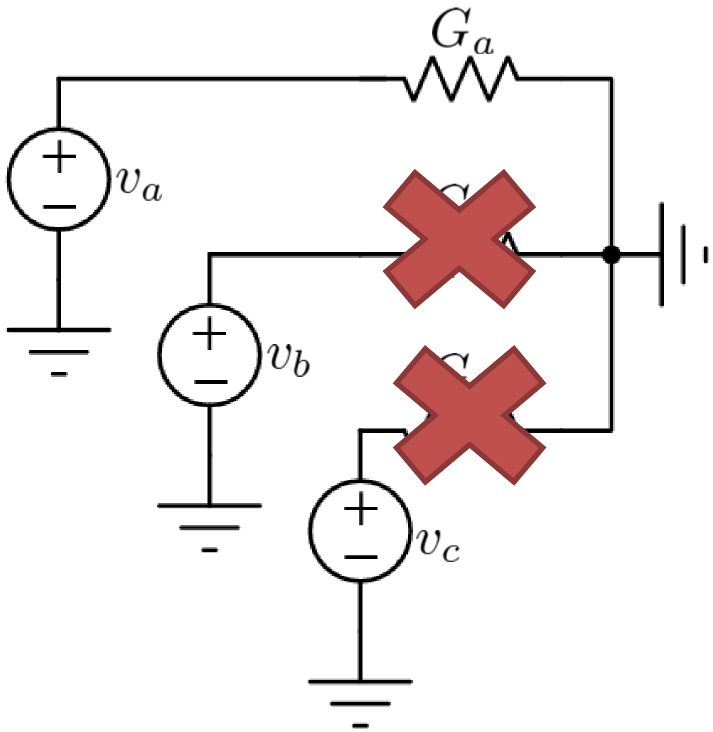
three-phase LCL VSC





DC-DC converter with MPPT connected to a PV plant

Let's go back to the easy-peasy example



$$G_a \equiv G \qquad G_b = G_c = 0$$

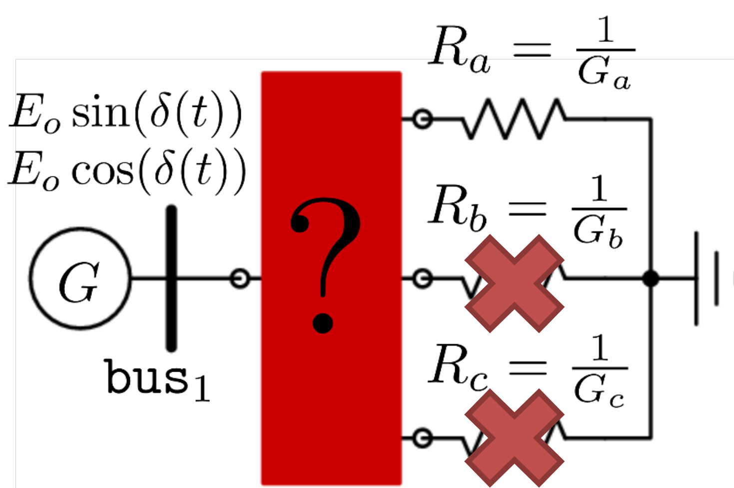
$$\imath_a = G E_o \cos(\Omega t + \delta_0)$$

$$\imath_b = \imath_c = 0$$

$$P_e(t) = G E_o^2 \cos^2(\Omega t + \delta_0)$$

$$\widehat{v}_d = E_o \sin(\delta(t))$$

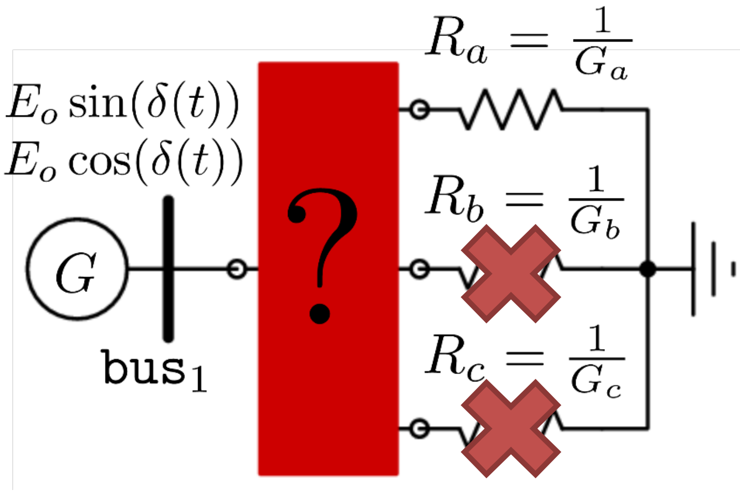
$$\widehat{v}_q = E_o \cos(\delta(t))$$



Does it still work?

Let's go back to the easy-peasy example

$$\widehat{v}_d = E_o \sin(\delta(t))$$
$$\widehat{v}_q = E_o \cos(\delta(t))$$



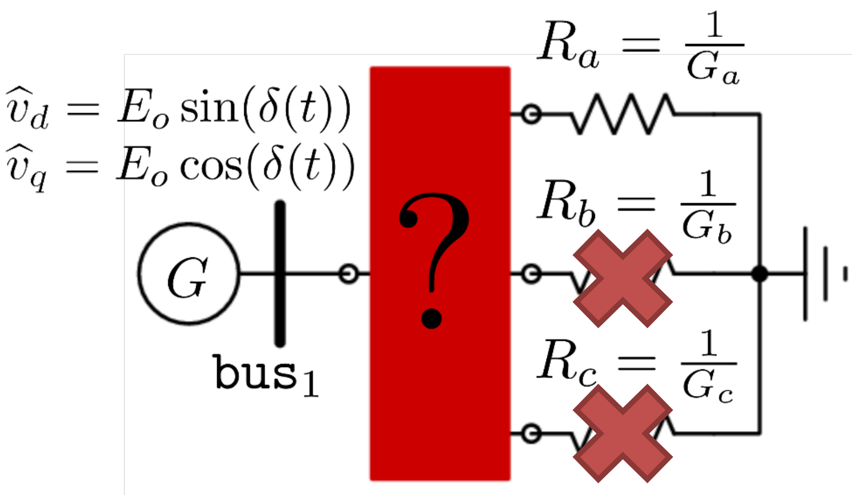
$$\imath_a = GE_o \cos(\Omega t + \delta(t))$$
$$\imath_b = \imath_c = 0$$

$$\begin{aligned} P_e(t) &= GE_o^2 \cos^2(\Omega t + \delta(t)) \\ &= \frac{GE_o^2}{2}(1 + \cos(2\Omega t + 2\delta(t))) \end{aligned}$$

$$\dot{\delta}(t) - \Omega(\omega(t) - 1) = 0$$
$$H\dot{\omega}(t) + D(\omega(t) - 1) - P_m + P_e(t) = 0$$

$$P_m = \frac{1}{2}GE_o^2$$
$$\omega(t) = ?$$
$$\delta(t) = ?$$

Let's go back to the easy-peasy example



$$\dot{\delta}(t) - \Omega\,(\omega(t) - 1) = 0$$

$$H\dot{\omega}(t) + D\,(\omega(t) - 1) - \frac{1}{2}E_o^2 G + P_e(t) = 0$$

Let us imagine the structure of the solution …

$$\delta(t) = \frac{\beta}{2}\sin(\omega_m t)$$

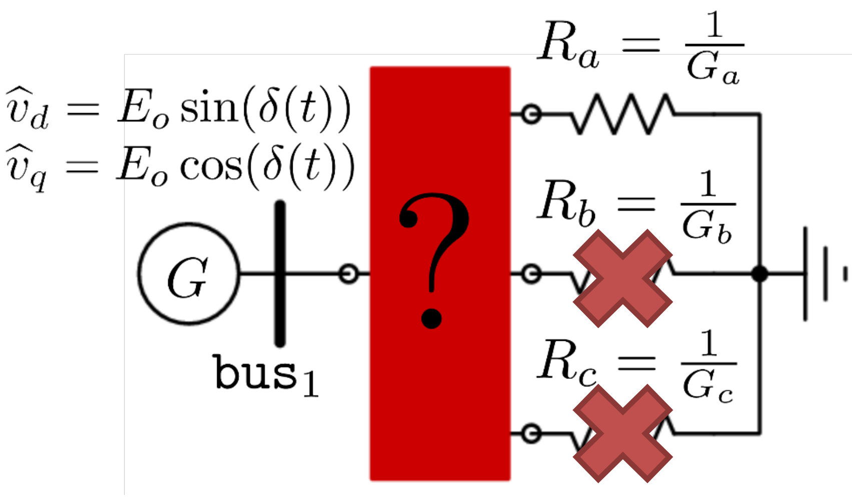
$$P_e(t) = G_a E_o^2 \cos^2\left(\Omega t + \frac{\beta}{2}\sin(\omega_m t)\right)$$

$$= \frac{G_a E_o^2}{2}\left(1 + \cos\left(2\Omega t + \beta\sin(\omega_m t)\right)\right)$$

$$= \frac{G_a E_o^2}{2}\left(1 + \sum_{n=-\infty}^{\infty} J_n(\beta)\cos((2\Omega + n\omega_m)t)\right)$$

Bessel functions of the first kind of order *n*

Let's go back to the easy-peasy example



$$\dot{\delta}(t) - \Omega\left(\omega(t) - 1\right) = 0$$

$$H\dot{\omega}(t) + D\left(\omega(t) - 1\right) - \frac{1}{2}E_o^2 G + P_e(t) = 0$$

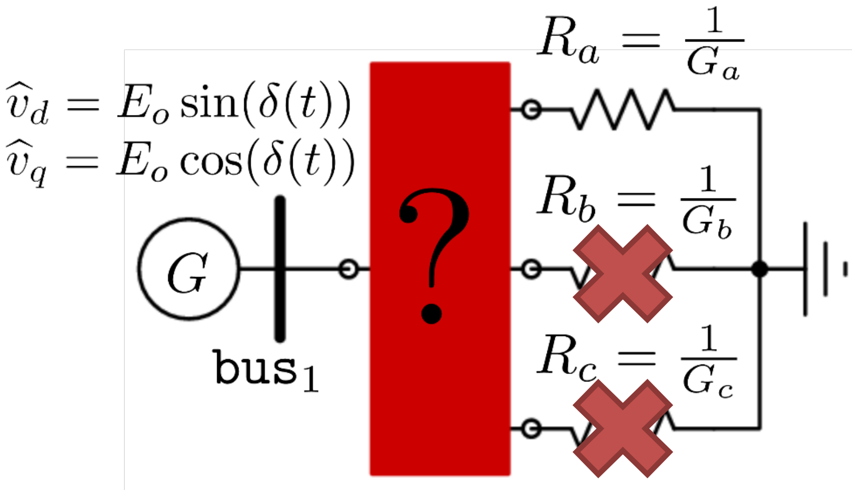
Let us imagine the structure of the solution …

$$\delta(t) = \frac{\beta}{2}\sin(\omega_m t)$$

$$P_e(t) = \frac{G_a E_o^2}{2}\left(1 + \sum_{n=-\infty}^{\infty} J_n(\beta)\cos((2\Omega + n\omega_m)t)\right)$$

We postulated a single tone in the solution … but we derived an infinite spectrum for the electrical power.

Let's go back to the easy-peasy example



$$\dot{\delta}(t) - \Omega\,(\omega(t) - 1) = 0$$

$$H\dot{\omega}(t) + D\,(\omega(t) - 1) - \frac{1}{2}E_o^2 G + P_e(t) = 0$$

Let us imagine the structure of the solution …

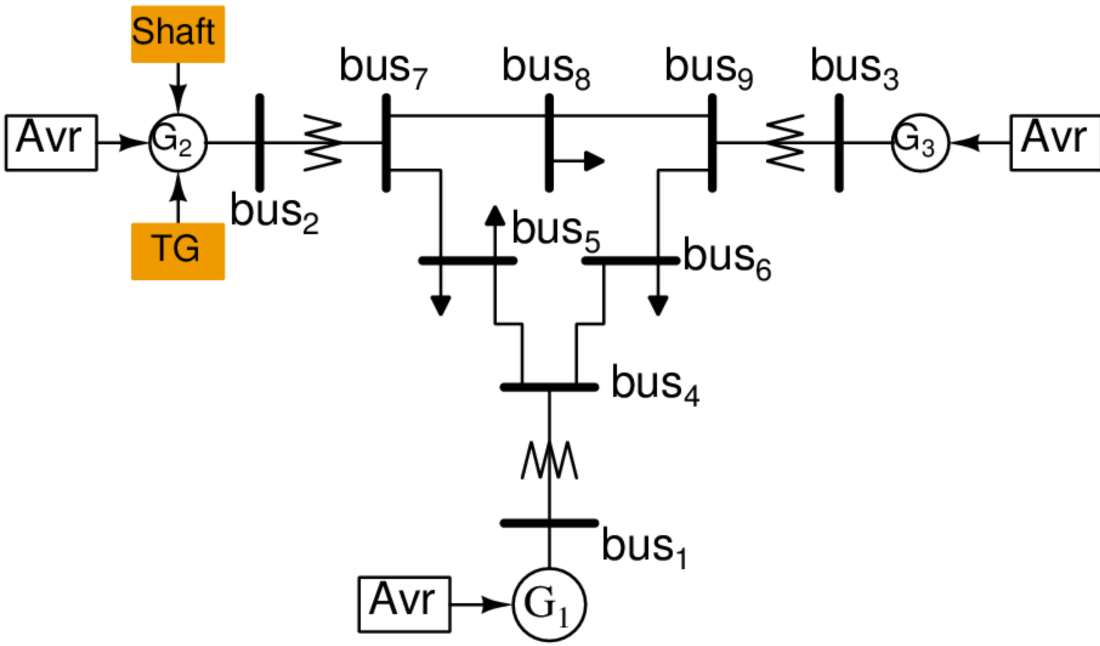
$$\delta(t) = \frac{\beta}{2}\sin(\omega_m t)$$

$$P_e(t) = \frac{G_a E_o^2}{2}\left(1 + \sum_{n=-\infty}^{\infty} J_n(\beta)\cos((2\Omega + n\omega_m)t)\right)$$

An infinite spectrum of the electrical power reflects on $\omega(t)$ and consequently on $\delta(t)$ …

In this unbalanced situation the steady-state solution of the PS model is not a constant but a periodic waveform (with a frequency spectrum spanning the whole frequency axis.

A numerical result

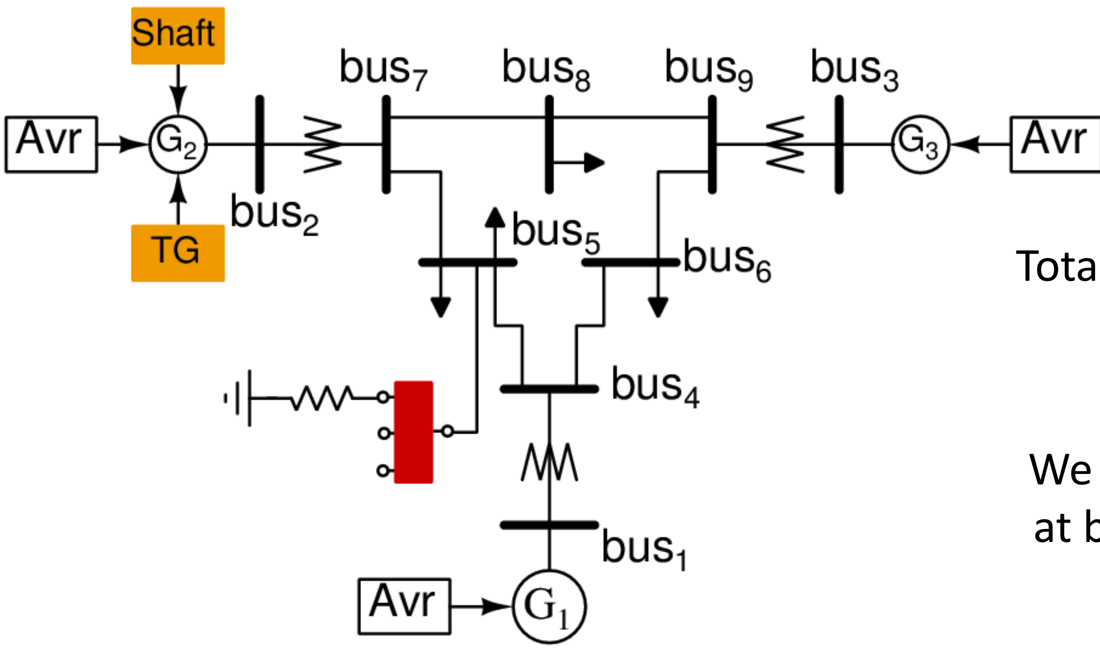


We performed a PF solution of the system and we verified its stability.

$Load_5$ = 125MW + j50MVA
$Load_6$ = 90MW + j30MVA
$Load_8$ = 100MW + j35MVA

Total load power
315MW + j115MVA
Total apparent power (approx.)
335MVA

A numerical result

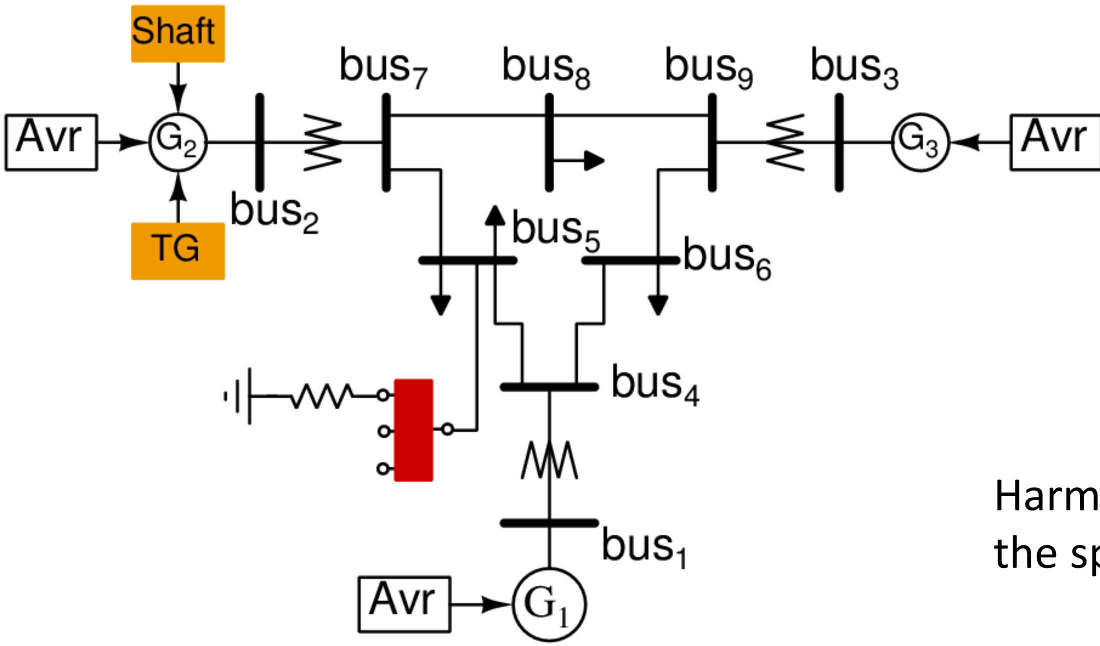


Total load power of the balanced system
315MW + j115MVA

We added a 10MW unbalanced extra-load at bus_5 and performed an EMT simulation of the whole system

Note that the connection of the unbalanced
load alters the total load active power by only 3.1%

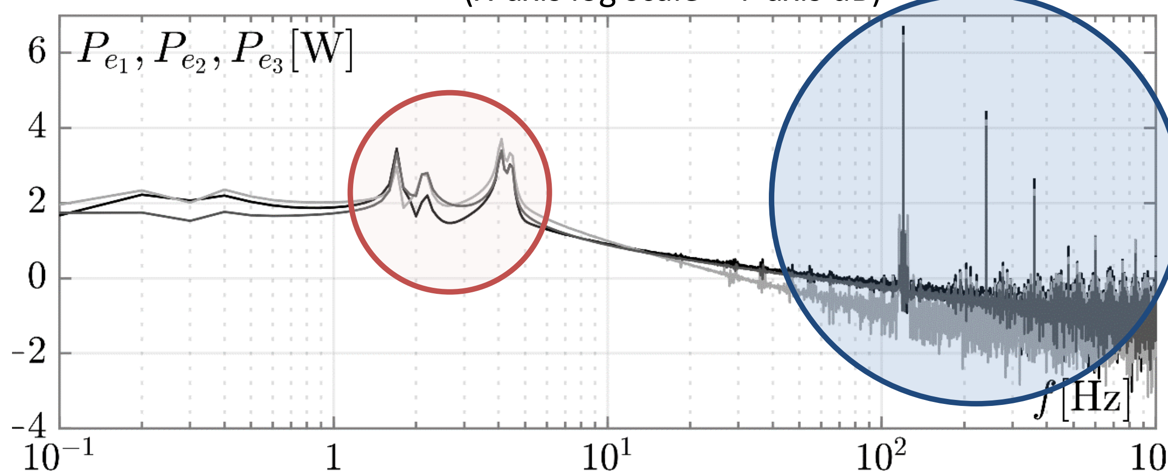
A numerical result



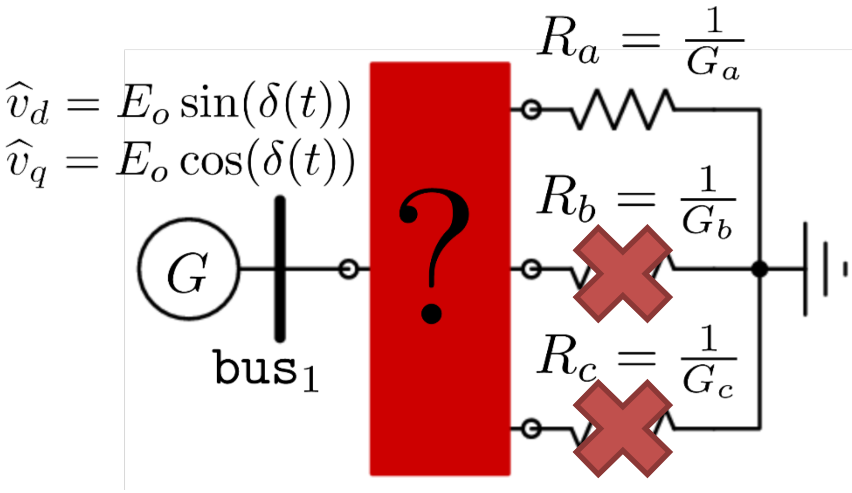
Harmonics are clearly marked by peaks in the spectra at several multiples of 60Hz

The resonance of the G_2 shaft is marked by the increase of the magnitude of power spectra in the [1,6]Hz interval

(X-axis log scale – Y-axis dB)



Let's go back to the easy-peasy example



If one wants to properly interface the three-phase side and the power system model one **the Park transform is not enough** …

It is necessary to setup an automatic interface that necessarily introduces an error and the latter should be quantified in runtime.

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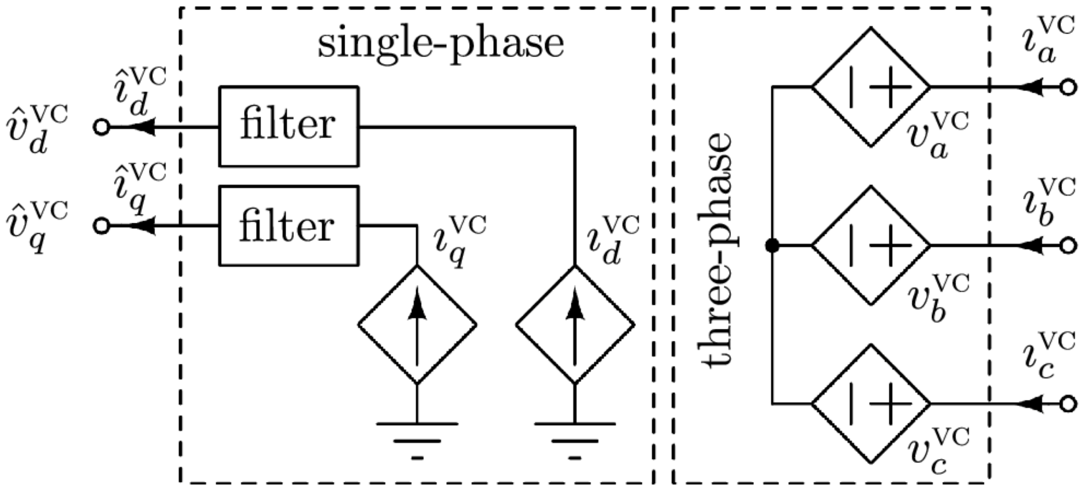
The Virtual Connector



To adequately link the single- and three-phase models we introduce a coupling element that we refer to as ***virtual connector***.

It plays the same role of the well-known *pseudo*-analog-to-digital and *pseudo*-digital-to-analog converters inserted during mixed analog/digital simulations to couple the analog and digital model paradigms of the circuit.

The words *pseudo* and *virtual* refer to the fact that such components do not exist in the *real* design but are introduced as simulation expedients.

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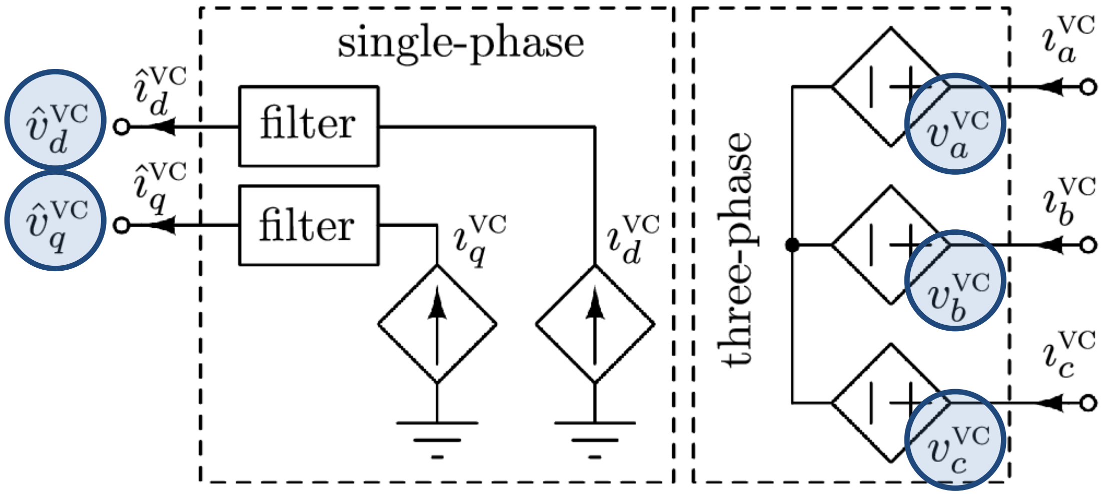
The Virtual Connector



$$(v_a, v_b, v_c) \xleftarrow[\Xi^{-1}]{} (\widehat{v}_d, \widehat{v}_q, 0)$$

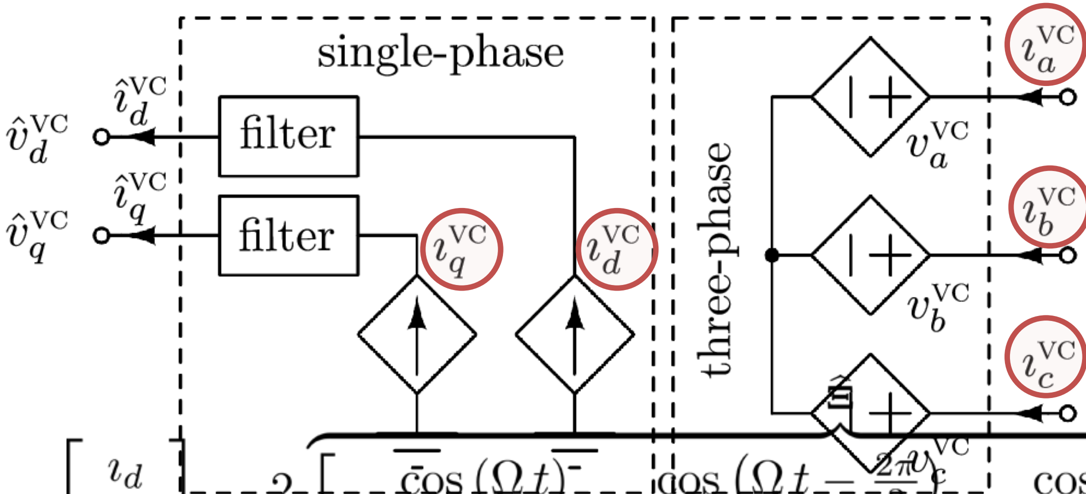
Notice that in the power system model the “zero”-component of the dq0 framework is assumed to be always null.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \overbrace{\begin{bmatrix} \cos(\Omega t) & -\sin(\Omega t) & 1 \\ \cos\left(\Omega t - \frac{2\pi}{3}\right) & -\sin\left(\Omega t - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\Omega t + \frac{2\pi}{3}\right) & -\sin\left(\Omega t + \frac{2\pi}{3}\right) & 1 \end{bmatrix}}^{\Xi^{-1}} \begin{bmatrix} \widehat{v}_d \\ \widehat{v}_q \\ 0 \end{bmatrix}$$

a-phase to *d*-axis alignment inverse Park transform
(preserving the amplitude)

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \overbrace{\begin{bmatrix} \cos(\Omega t) & -\sin(\Omega t) \\ \cos\left(\Omega t - \frac{2\pi}{3}\right) & -\sin\left(\Omega t - \frac{2\pi}{3}\right) \\ \cos\left(\Omega t + \frac{2\pi}{3}\right) & -\sin\left(\Omega t + \frac{2\pi}{3}\right) \end{bmatrix}}^{\widehat{\Xi}^{T}} \begin{bmatrix} \widehat{v}_d \\ \widehat{v}_q \end{bmatrix}$$

The Virtual Connector



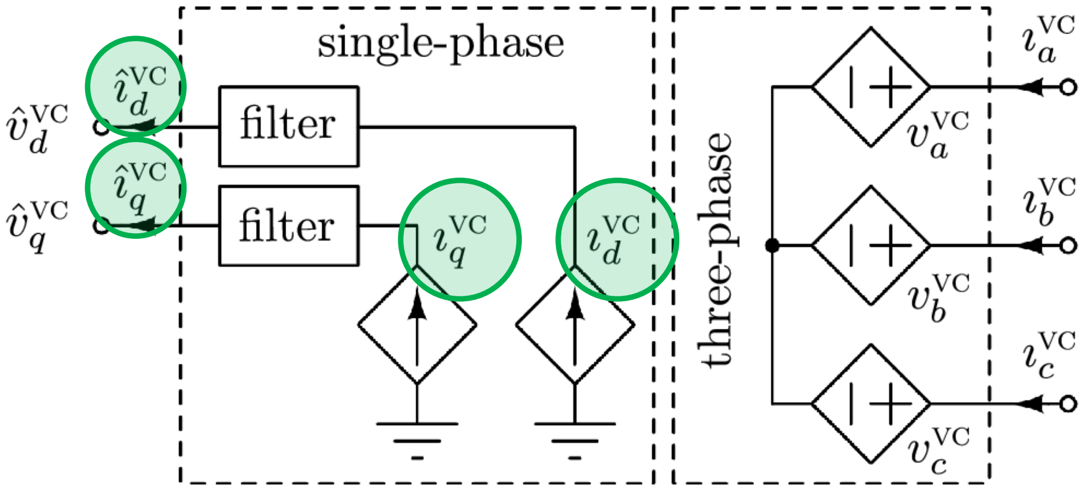
Notice the KCL

$$\imath_a + \imath_b + \imath_c = 0$$

$$(\imath_d, \imath_q, 0) \xleftarrow[\hat{\Xi}]{} (\imath_a, \imath_b, \imath_c)$$

$$\begin{bmatrix} \imath_d \\ \imath_q \end{bmatrix} = \frac{2}{3} \overbrace{\begin{bmatrix} \cos(\Omega t) & \cos\left(\Omega t - \frac{2\pi}{3}\right) & \cos\left(\Omega t + \frac{2\pi}{3}\right) \\ -\sin(\Omega t) & -\sin\left(\Omega t - \frac{2\pi}{3}\right) & -\sin\left(\Omega t + \frac{2\pi}{3}\right) \end{bmatrix}}^{\hat{\Xi}} \begin{bmatrix} \imath_a \\ \imath_b \\ \imath_c \end{bmatrix}$$

The Virtual Connector



In principle the virtual connector must neither generate nor absorb electric power.

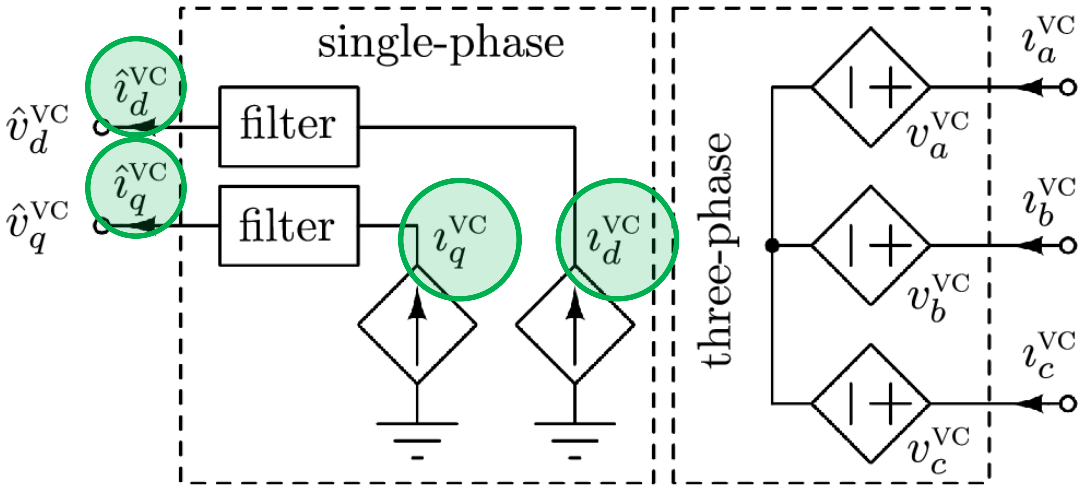
In practice the filter is as much **dissipative** as the three-phase side of the circuit is unbalanced/nonlinear

$$\hat{\imath}_d^{VC} = \frac{\Omega}{2\pi} \int_{t_0}^{t_0+\frac{2\pi}{\Omega}} \imath_d^{VC}(\tau) d\tau$$

$$\hat{\imath}_q^{VC} = \frac{\Omega}{2\pi} \int_{t_0}^{t_0+\frac{2\pi}{\Omega}} \imath_q^{VC}(\tau) d\tau$$

The filtering operation is mandatory since the filer input may be not the constant envelope of a three-phase sinusoidal positive sequence

The Virtual Connector



Accuracy index

$$\widetilde{\imath} = \left(\imath_d^{VC}, \jmath \imath_q^{VC}\right) \in \mathbb{C}$$

complex spectrum obtained using the complex Fourier transform

$$\widetilde{I} \in \mathbb{C}^{2K+1}$$

It is a bilateral spectrum truncated at the K-th harmonic.

$$\mathcal{E}_{\mathrm{hd}} = \frac{\sum_{k=-K}^{K} \widetilde{I}_k \widetilde{I}_k^* - |\widetilde{I}_0|^2}{\sum_{k=-K}^{K} \widetilde{I}_k \widetilde{I}_k^*} = 1 - \frac{|\widetilde{I}_0|^2}{\sum_{k=-K}^{K} |\widetilde{I}_k|^2}$$

$$\left(\widehat{\mathcal{E}}_{\mathrm{hd}} = 1 - \mathcal{E}_{\mathrm{hd}}\right)$$

A few words on the complex Fourier transform

$$x_R(t) \in \mathbb{R}$$

$$X_R(f) = \mathcal{F}\{x_R(t)\} \in \mathbb{C}$$

$$x_I(t) \in \mathbb{R}$$

$$X_I(f) = \mathcal{F}\{x_I(t)\} \in \mathbb{C}$$

$$x(t) = x_R(t) + \jmath x_I(t)$$

$$x(t) \leftrightarrow \mathcal{F}\{x(t)\} = X(f)$$

$$x^*(t) \leftrightarrow X^*(-f)$$

Typical odd/even symmetry of the spectrum are lost.

$$x_R(t) = \frac{1}{2}(x(t) + x^*(t))$$

$$X_R(f) = \frac{1}{2}(X(f) + X^*(-f))$$

$$x_I(t) = -\jmath\frac{1}{2}(x(t) - x^*(t))$$

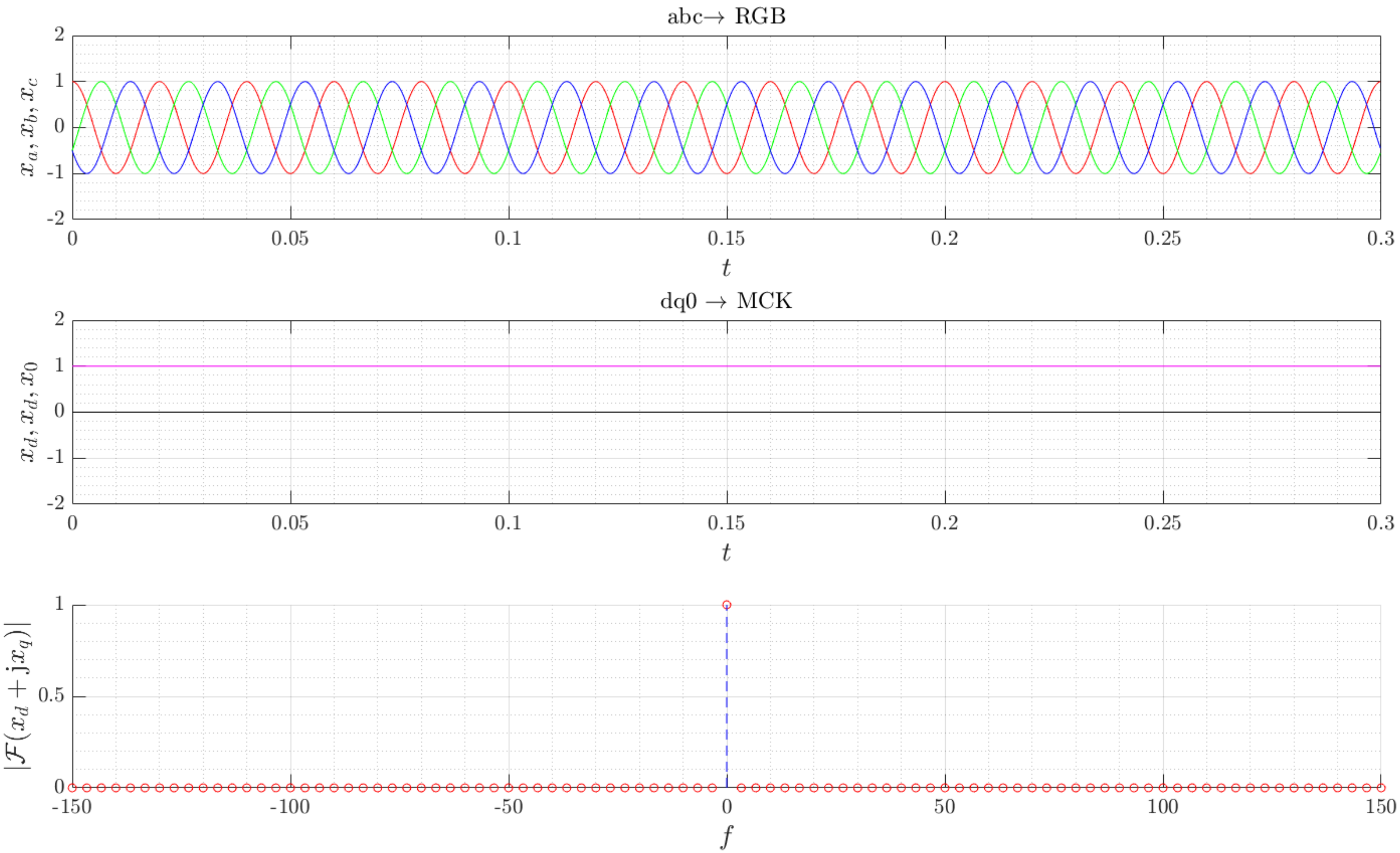
$$X_I(f) = -\jmath\frac{1}{2}(X(f) - X^*(-f))$$

$$\text{real}\{X(f)\} = \text{real}\{X_R(f)\} - \text{imag}\{X_I(f)\}$$

$$\text{imag}\{X(f)\} = \text{imag}\{X_R(f)\} + \text{real}\{X_I(f)\}$$

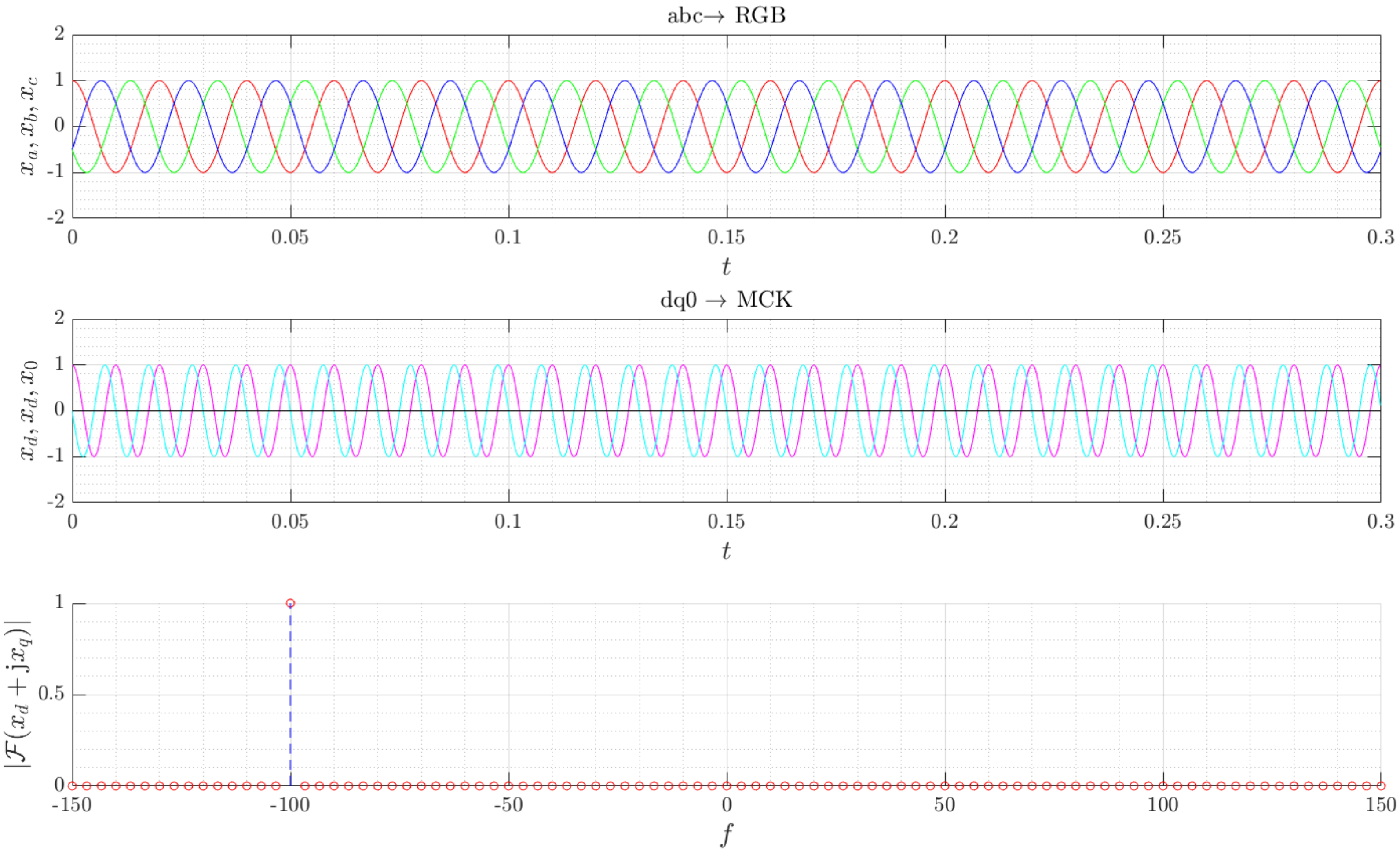
A few words on the complex Fourier transform

$$x_a(t) = \cos(\Omega t)$$
$$x_b(t) = \cos(\Omega t - \tfrac{2\pi}{3})$$
$$x_c(t) = -x_a(t) - x_b(t) \rightarrow x_0(t) \equiv 0$$



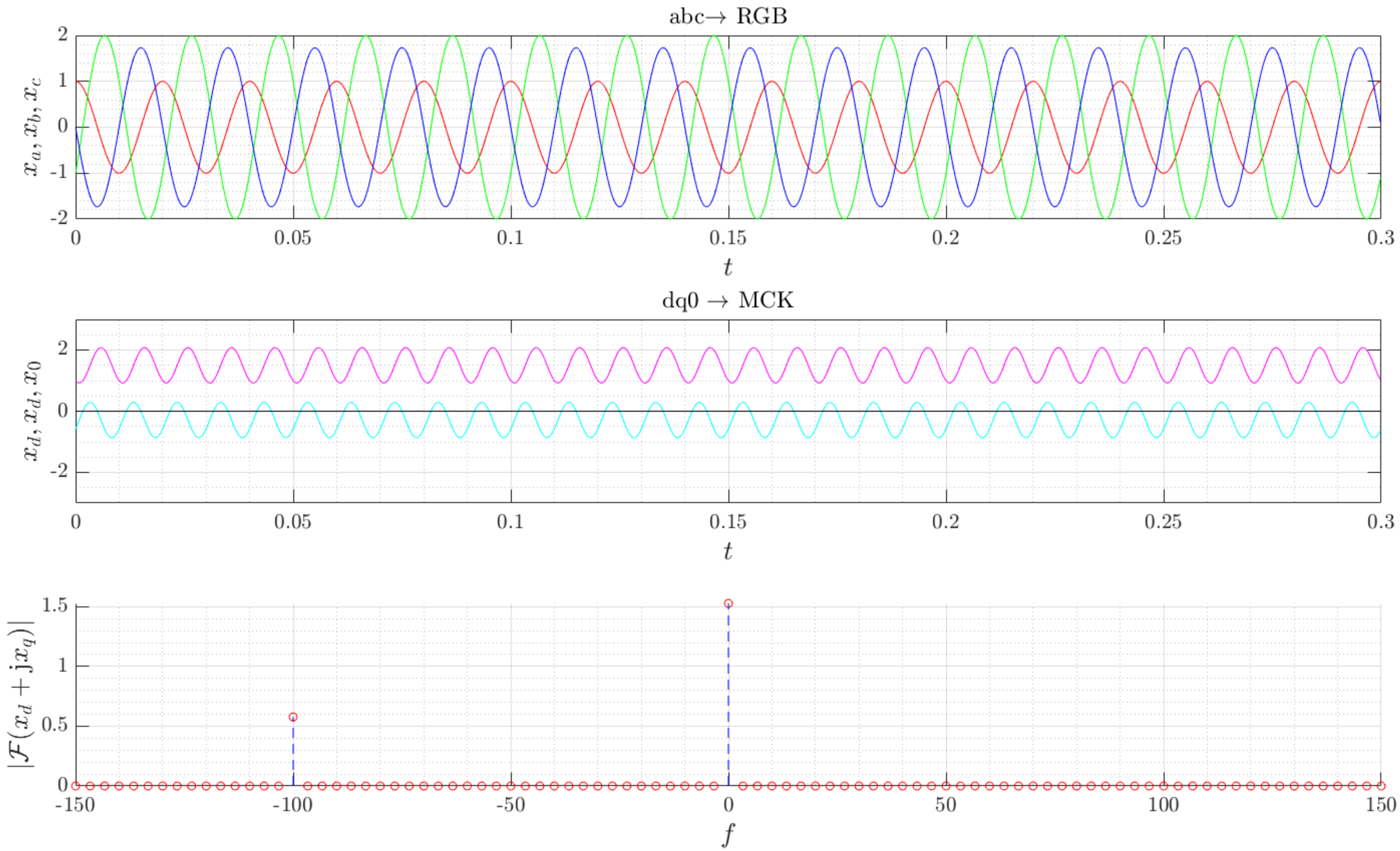
A few words on the complex Fourier transform

$$x_a(t) = \cos(\Omega t)$$
$$x_b(t) = \cos(\Omega t + \frac{2\pi}{3})$$
$$x_c(t) = -x_a(t) - x_b(t)$$

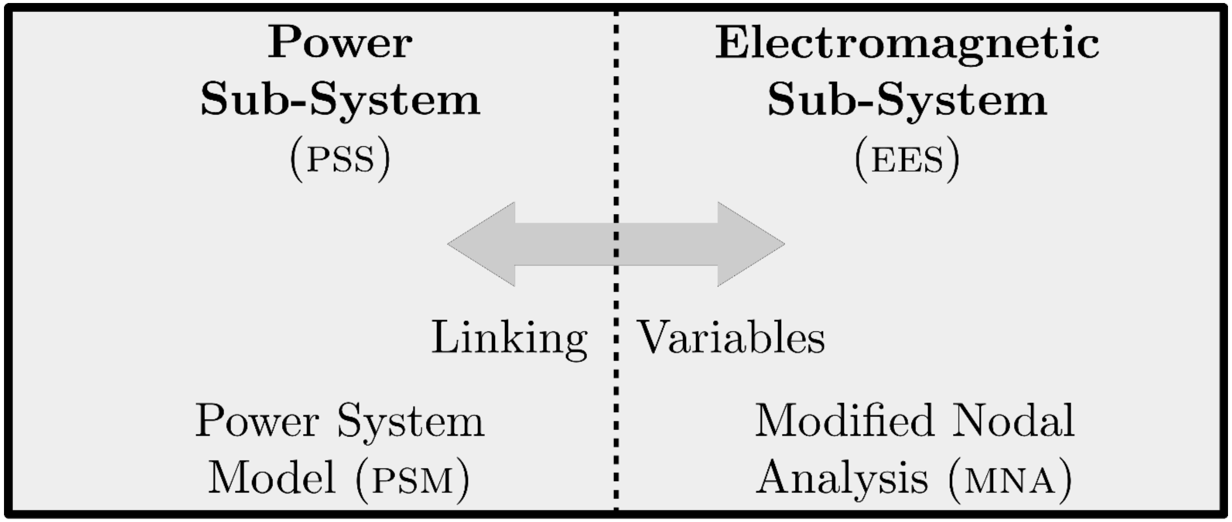


A few words on the complex Fourier transform

$$x_a(t) = \cos(\Omega t)$$
$$x_b(t) = 2\cos(\Omega t - \tfrac{2\pi}{3})$$
$$x_c(t) = -x_a(t) - x_b(t)$$



Some mathematical formalism



ELECTRICAL POWER SYSTEM

$$\frac{d\widehat{\mathbf{u}}}{dt} + \mathbf{r}\left(\widehat{\mathbf{u}}, \widehat{\mathbf{z}}\right) = 0$$

$$\mathbf{h}\left(\widehat{\mathbf{u}}, \widehat{\mathbf{z}}\right) = 0$$

$$\mathbf{r} : \mathbb{R}^{S_u+S_z} \rightarrow \mathbb{R}^{S_u}$$

$$\mathbf{h} : \mathbb{R}^{S_u+S_z} \rightarrow \mathbb{R}^{S_z}$$

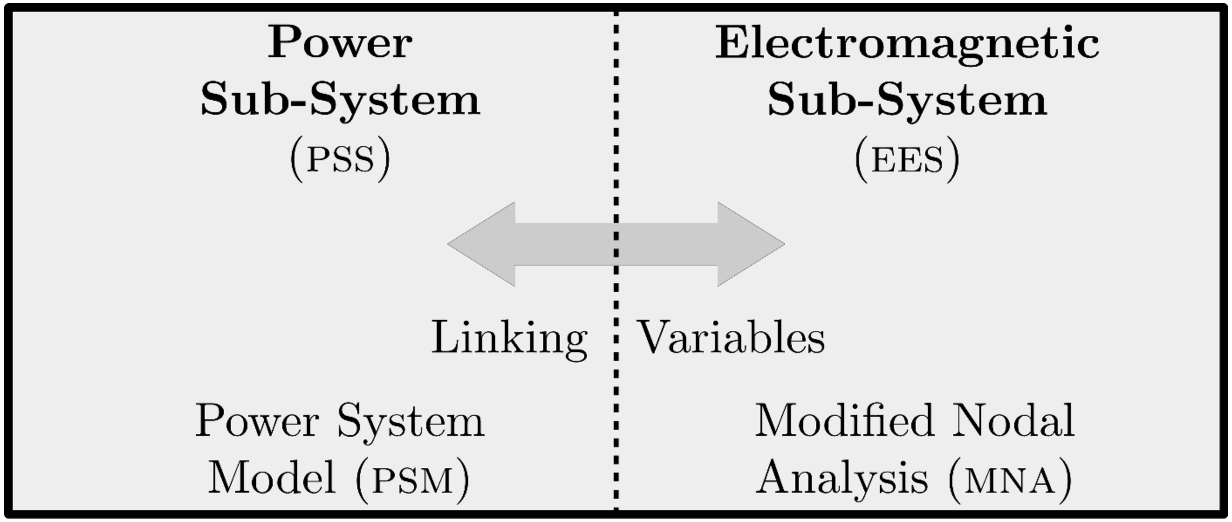
$$\widehat{\mathbf{u}} \in \mathbb{R}^{S_u}$$

$$\widehat{\mathbf{z}} \in \mathbb{R}^{S_z}$$

State variables introduced for example by generators, regulator, controllers.

Algebraic variables, such as bus voltages and currents.

Some mathematical formalism



ELECTRICAL POWER SYSTEM

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x},\mathbf{y}) = 0$$

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = 0$$

Capacitive charges and inductive fluxes.

$$\mathbf{q} : \mathbb{R}^{S_x} \rightarrow \mathbb{R}^{S_x}$$

$$\mathbf{f} : \mathbb{R}^{S_x+S_y} \rightarrow \mathbb{R}^{S_x}$$

$$\mathbf{g} : \mathbb{R}^{S_x+S_y} \rightarrow \mathbb{R}^{S_y}$$

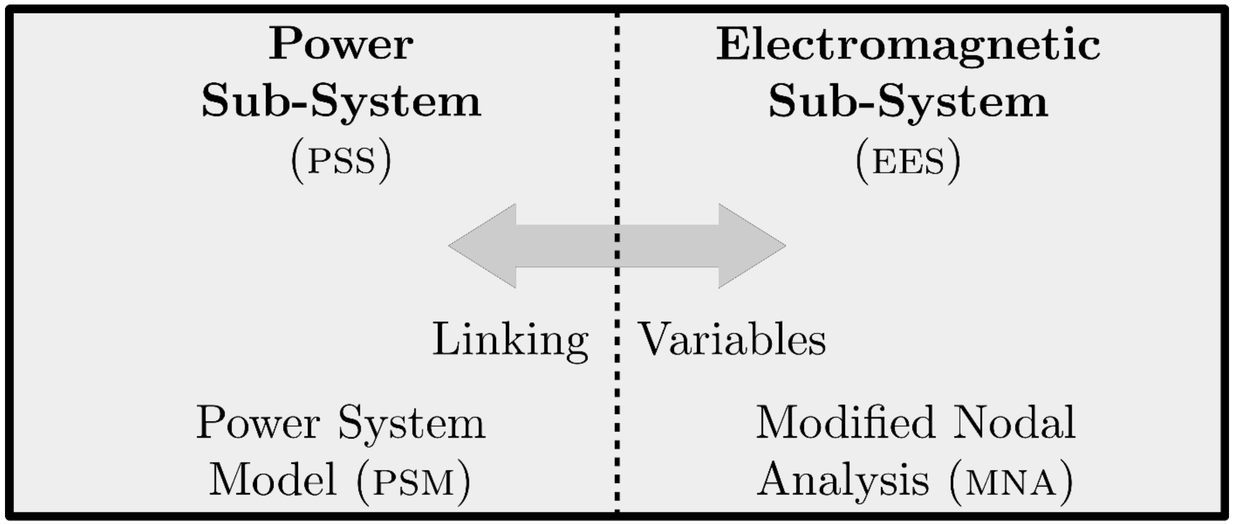
Ground-referenced node voltages and branch currents.

$$\mathbf{x} \in \mathbb{R}^{S_x}$$

Ground-referenced node voltages and branch currents.

$$\mathbf{y} \in \mathbb{R}^{S_y}$$

Some mathematical formalism

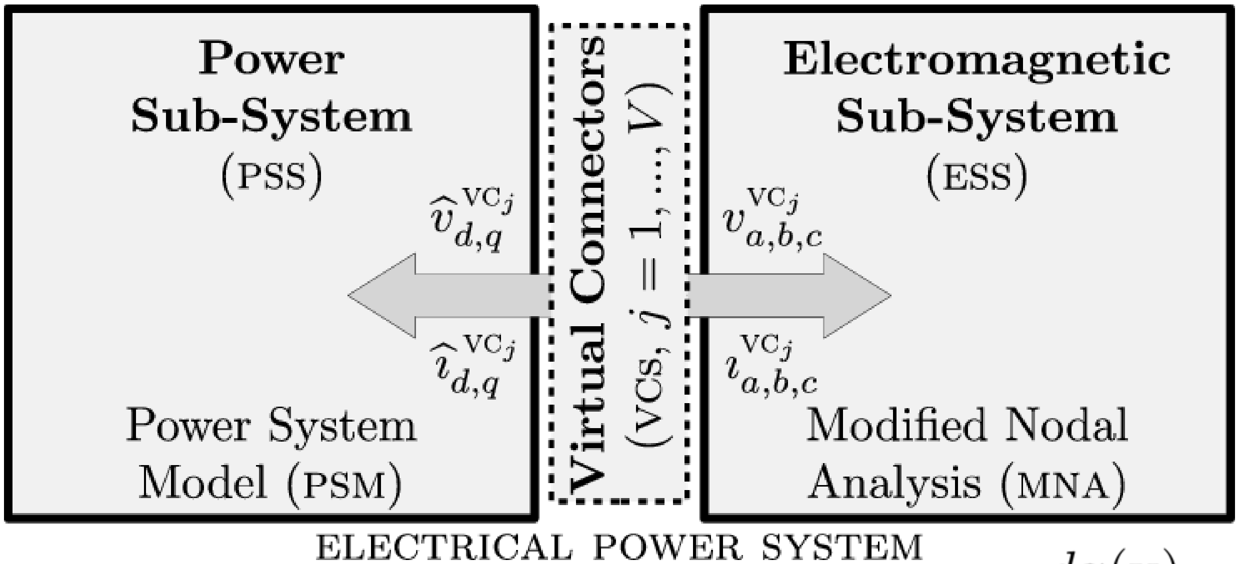


ELECTRICAL POWER SYSTEM

$$\frac{d\widehat{\mathbf{u}}}{dt} + \mathbf{r}\left(\widehat{\mathbf{u}}, \widehat{\mathbf{z}}\right) = 0$$
$$\mathbf{h}\left(\widehat{\mathbf{u}}, \widehat{\mathbf{z}}\right) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \mathbf{y}) = 0$$
$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = 0$$

Some mathematical formalism

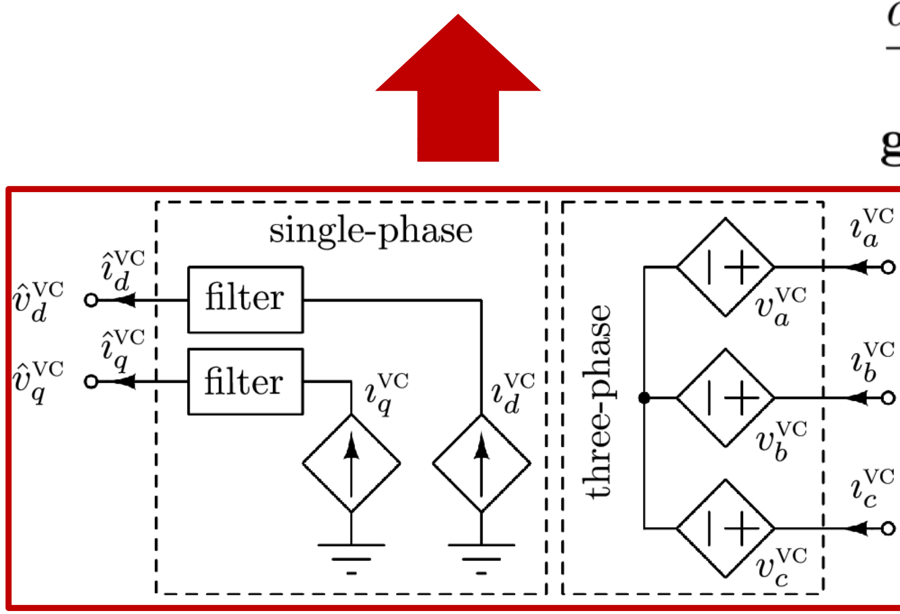


$$\frac{d\hat{\mathbf{u}}}{dt} + \mathbf{r}\left(\hat{\mathbf{u}}, \hat{\mathbf{z}}\right) = 0$$

$$\mathbf{h}\left(\hat{\mathbf{u}}, \hat{\mathbf{z}}\right) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = 0$$



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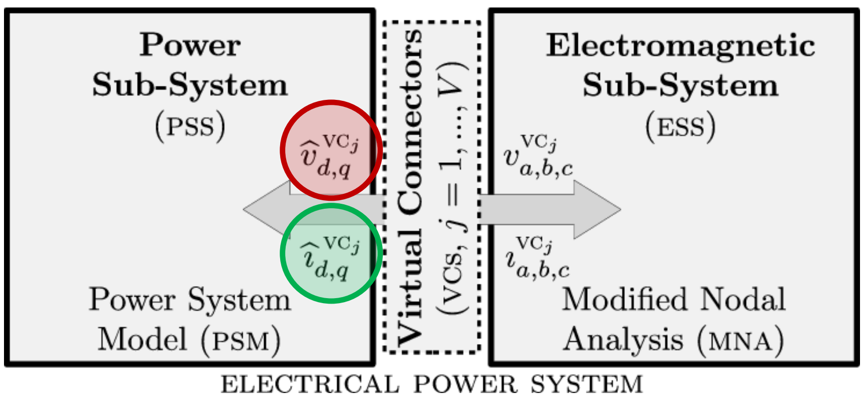
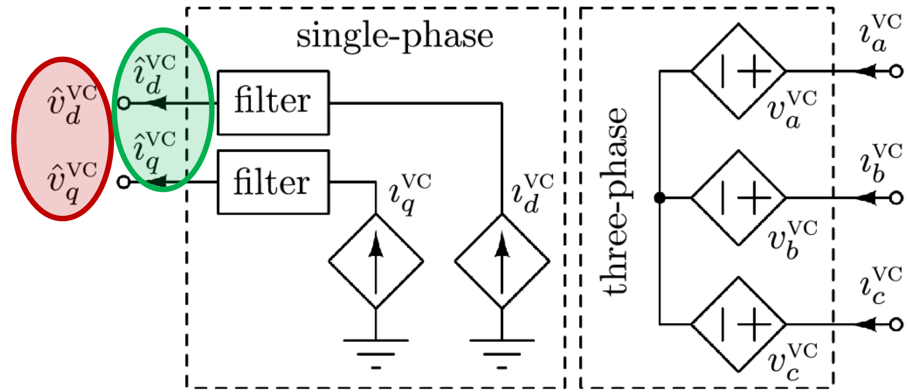
Some mathematical formalism

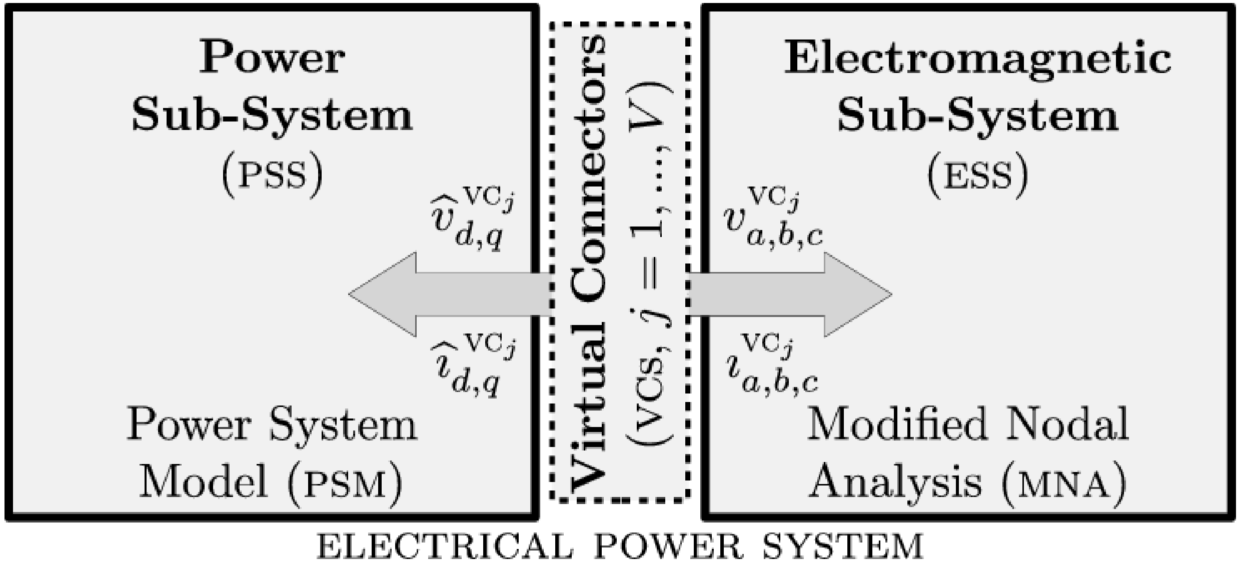




- By assuming V linking points, between the PSS and the ESS, the V subset of the B buses of the PSS plays a special role in the network since each of those is connected (also) to a VC.
- From a modeling point of view, the equivalent effect of the VCs on these buses is to make them *slack*. ***The VC forces the voltage of the bus at which it is connected and the current through it is a free variable***.

$$\widehat{\boldsymbol{v}}_{d,q}^{\,\mathrm{VC}} = \left[\widehat{v}_{d,q}^{\,\mathrm{VC}_1}, \ldots, \widehat{v}_{d,q}^{\,\mathrm{VC}_V}\right] \qquad \widehat{\boldsymbol{\imath}}_{d,q}^{\,\mathrm{VC}} = \left[\widehat{\imath}_{d,q}^{\,\mathrm{VC}_1}, \ldots, \widehat{\imath}_{d,q}^{\,\mathrm{VC}_V}\right]$$

Some mathematical formalism



$$\frac{d\widehat{\mathbf{u}}}{dt} + \mathbf{r}(\widehat{\mathbf{u}}, \overbrace{\left[\widehat{\mathbf{z}}, \widehat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}\right]}^{\widehat{\boldsymbol{\zeta}} \in \mathbb{R}^{S_z+2V}}) = 0$$

$$\mathbf{h}(\widehat{\mathbf{u}}, \widehat{\boldsymbol{\zeta}}) = 0$$

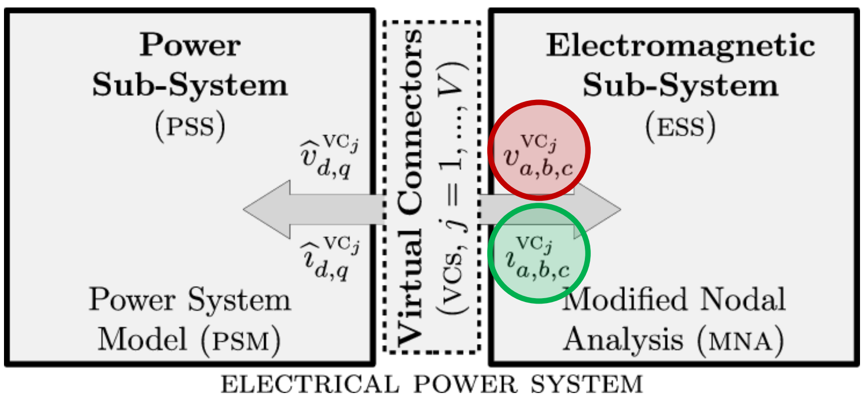
$$\mathbf{r} : \mathbb{R}^{S_u+S_z+2V} \rightarrow \mathbb{R}^{S_u}$$

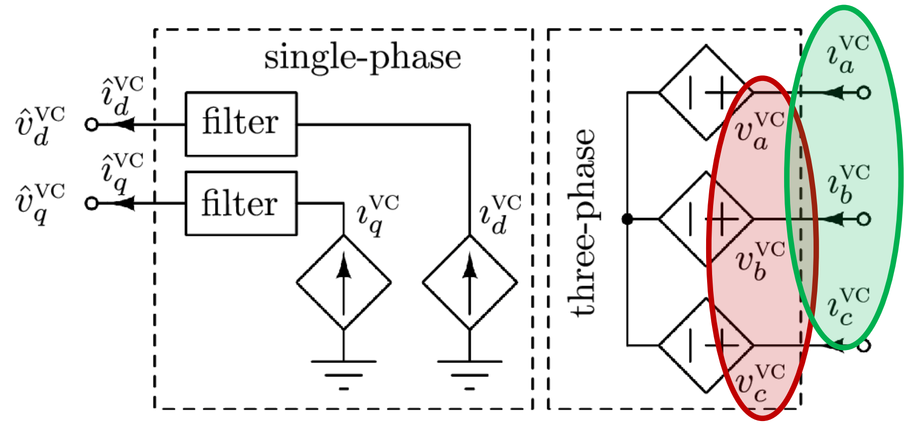
$$\mathbf{h} : \mathbb{R}^{S_u+S_z+2V} \rightarrow \mathbb{R}^{S_z+2V}$$

$$\widehat{\mathbf{u}} \in \mathbb{R}^{S_u}$$

$$\widehat{\mathbf{z}} \in \mathbb{R}^{S_z+2V}$$

Some mathematical formalism



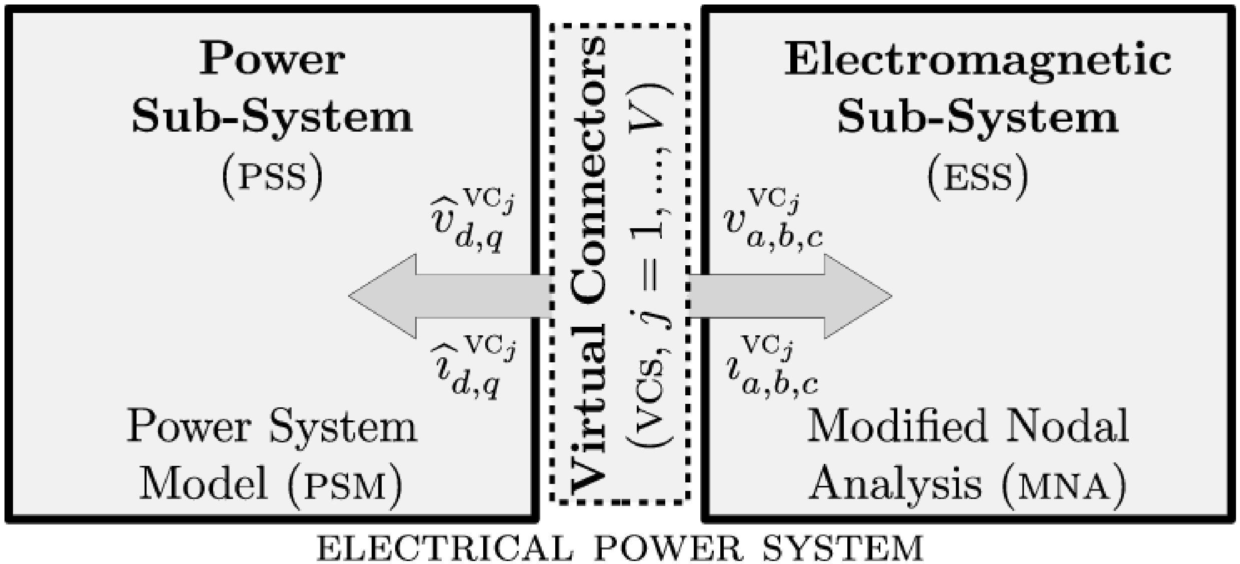


- The ESS is fed by V VCs through V triplets of voltages set by **V three-phase controlled voltage sources**.
- These components do not admit voltage basis and consequently, as in the PSS case, the algebraic-variable vector is enlarged with the V triplets of currents flowing through them.

$$\boldsymbol{v}_{a,b,c}^{\mathrm{VC}} = \left[v_{a,b,c}^{\mathrm{VC}_1}, \ldots, v_{a,b,c}^{\mathrm{VC}_V}, \right]$$

$$\boldsymbol{\imath}_{a,b,c}^{\mathrm{VC}} = \left[\imath_{a,b,c}^{\mathrm{VC}_1}, \ldots, \imath_{a,b,c}^{\mathrm{VC}_V}, \right]$$

Some mathematical formalism



$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \overbrace{[\mathbf{y}, \boldsymbol{\imath}^{\mathrm{VC}}_{a,b,c}]}^{\boldsymbol{\lambda}\in\mathbb{R}^{S_y+3V}}, t) = 0$$

$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

$$\mathbf{q} : \mathbb{R}^{S_x} \rightarrow \mathbb{R}^{S_x}$$

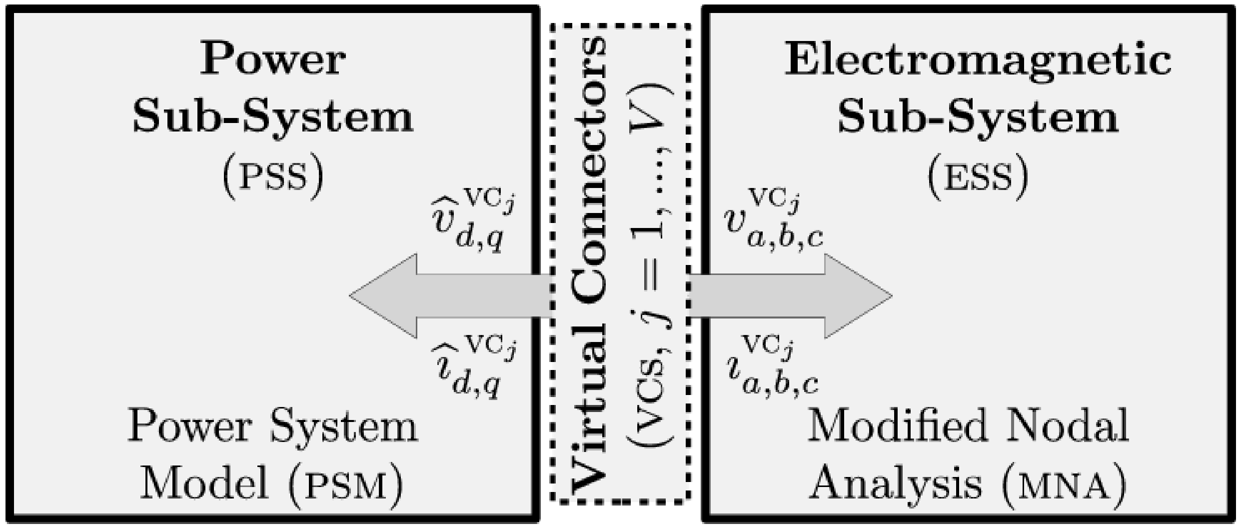
$$\mathbf{f} : \mathbb{R}^{S_x+S_y+3V} \rightarrow \mathbb{R}^{S_x}$$

$$\mathbf{g} : \mathbb{R}^{S_x+S_y+3V} \rightarrow \mathbb{R}^{S_y+3V}$$

$$\mathbf{x} \in \mathbb{R}^{S_x}$$

$$\mathbf{y} \in \mathbb{R}^{S_y+3V}$$

Some mathematical formalism



ELECTRICAL POWER SYSTEM

$$\frac{d\widehat{\mathbf{u}}}{dt} + \mathbf{r}(\widehat{\mathbf{u}}, \overbrace{\left[\widehat{\mathbf{z}}, \widehat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}\right]}^{\widehat{\boldsymbol{\zeta}} \in \mathbb{R}^{S_z + 2V}}) = 0$$

$$\mathbf{h}(\widehat{\mathbf{u}}, \widehat{\boldsymbol{\zeta}}) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \overbrace{\left[\mathbf{y}, \boldsymbol{\imath}_{a,b,c}^{\mathrm{VC}}\right]}^{\boldsymbol{\lambda} \in \mathbb{R}^{S_y + 3V}}) = 0$$

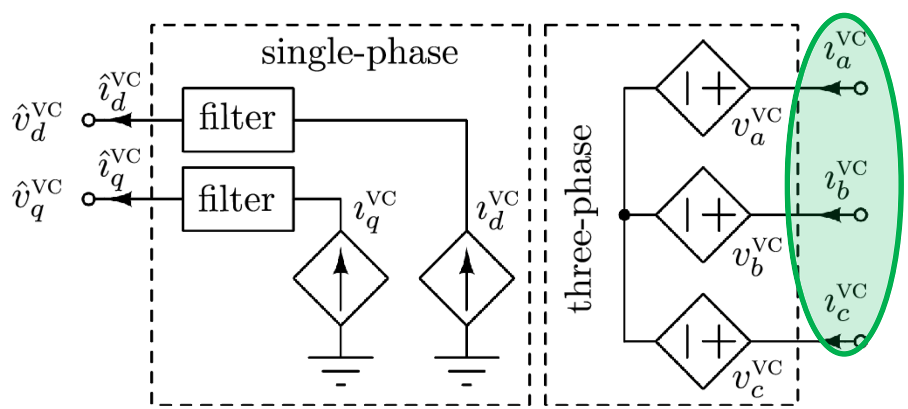
$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda},) = 0$$

The properties of Virtual Connectors

Property 1: *Being* $\imath_{a,b,c}^{\mathrm{VC}_j}$ *the currents of the* j*-th VC corresponding to* $\imath_{d,q}^{\mathrm{VC}_j}$ *in the* DQ*-frame, their zero component is null.*

$$\begin{bmatrix} \imath_d^{\mathrm{VC}_j} \\ \imath_q^{\mathrm{VC}_j} \\ 0 \end{bmatrix} = \overbrace{\frac{2}{3}\begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ 1 & 1 & 1 \end{bmatrix}}^{\boldsymbol{\Xi}} \mathbf{s}_j^{\mathrm{T}} \boldsymbol{\lambda}$$

The $\mathbf{s}_j \in \mathbb{N}^{3\times(S_y+3V)}$ vector is a selector that has only one entry equal to 1 per row. These entries select the $\imath_{a,b,c}^{\mathrm{VC}_j}$ currents of the j-th VC; the corresponding currents in the DQ-frame are $\imath_{d,q}^{\mathrm{VC}_j}$.



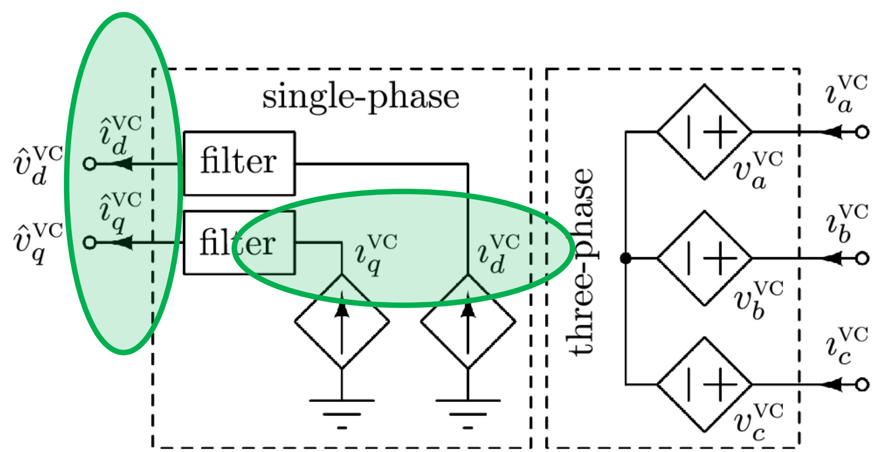
The properties of Virtual Connectors

Property 2: *Being* $\imath_{d,q}^{\mathrm{VC}_j}$ *the currents of the j-th VC in the* DQ*-frame, the PSS model is fed by*

$$\widehat{\imath}_d^{\mathrm{VC}_j} = \frac{1}{T}\int_{t_0}^{t_0+T} \imath_d^{\mathrm{VC}_j}(\tau)d\tau$$

$$\widehat{\imath}_q^{\mathrm{VC}_j} = \frac{1}{T}\int_{t_0}^{t_0+T} \imath_q^{\mathrm{VC}_j}(\tau)d\tau \ ,$$

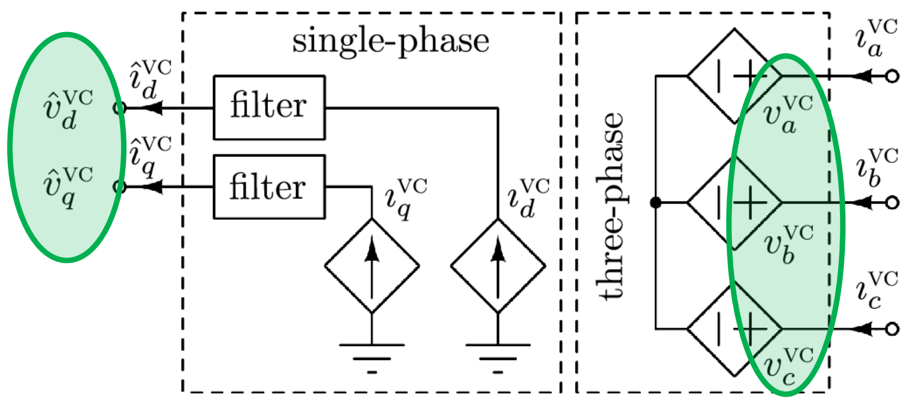
where $T = \frac{2\pi}{\Omega}$.



The properties of Virtual Connectors

Property 3: *Being* $\widehat{v}_{d,q}^{\mathrm{VC}_j}$ *the constant voltages at the bus in the PSS connected to the* j*-th VC, the* $v_{a,b,c}^{\mathrm{VC}_j}$ *voltages are derived as*

$$\begin{bmatrix} v_a^{\mathrm{VC}_j} \\ v_b^{\mathrm{VC}_j} \\ v_c^{\mathrm{VC}_j} \end{bmatrix} = \overbrace{\begin{bmatrix} \cos(\Omega\, t) & -\sin(\Omega\, t) \\ \cos\left(\Omega\, t - \frac{2\pi}{3}\right) & -\sin\left(\Omega\, t - \frac{2\pi}{3}\right) \\ \cos\left(\Omega\, t + \frac{2\pi}{3}\right) & -\sin\left(\Omega\, t + \frac{2\pi}{3}\right) \end{bmatrix}}^{\widehat{\boldsymbol{\Xi}}^{\mathrm{T}}} \begin{bmatrix} \widehat{v}_d^{\mathrm{VC}_j} \\ \widehat{v}_q^{\mathrm{VC}_j} \end{bmatrix}$$



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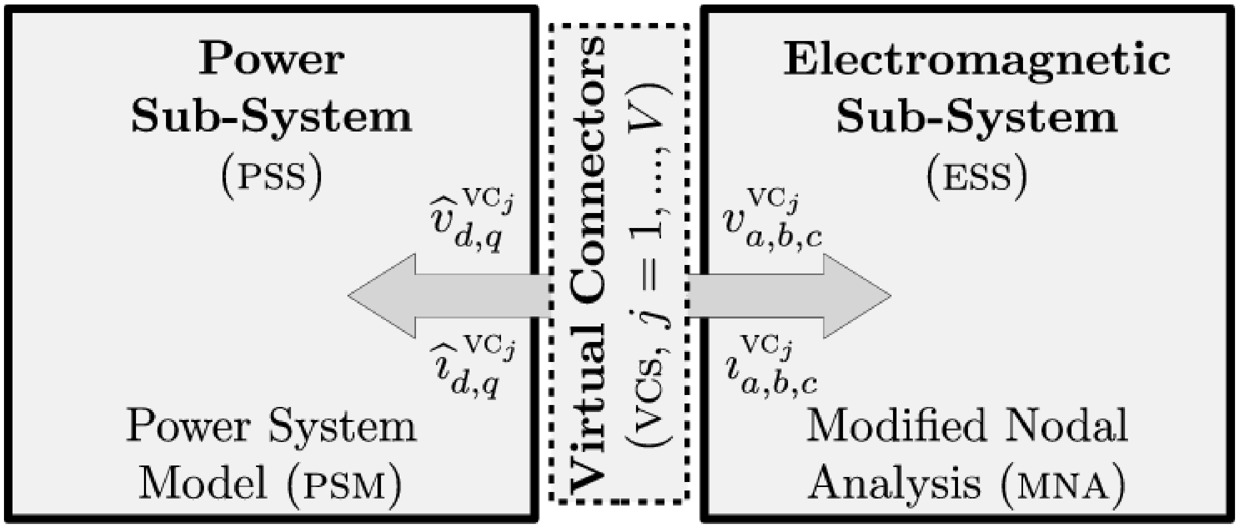
Problem formulation



ELECTRICAL POWER SYSTEM

$$\frac{d\hat{\mathbf{u}}}{dt} + \mathbf{r}(\hat{\mathbf{u}}, \overbrace{[\hat{\mathbf{z}}, \hat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}]}^{\hat{\boldsymbol{\zeta}} \in \mathbb{R}^{S_z+2V}}) = 0$$

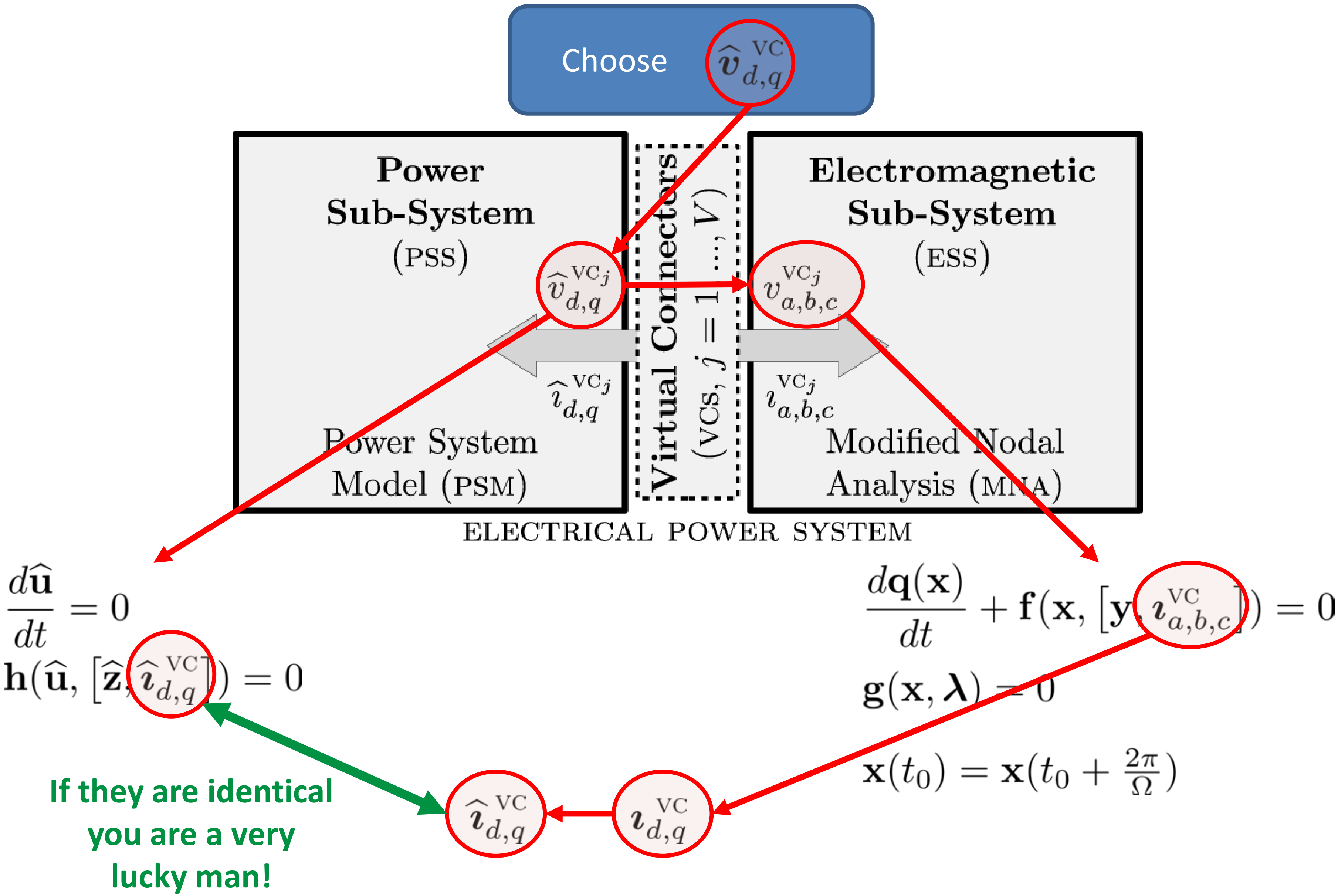
$$\mathbf{h}(\hat{\mathbf{u}}, \hat{\boldsymbol{\zeta}}) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \overbrace{[\mathbf{y}, \boldsymbol{\imath}_{a,b,c}^{\mathrm{VC}}]}^{\boldsymbol{\lambda} \in \mathbb{R}^{S_y+3V}}) = 0$$

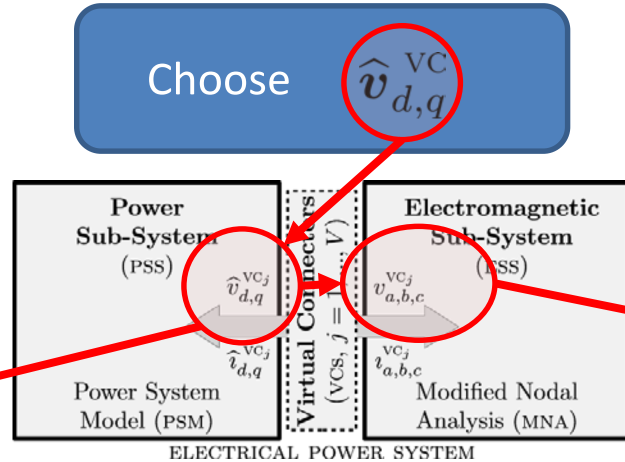
$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

Now we have to find the steady-state solution of the overall electrical system, i.e., PSS&ESS

The implementation scheme



The implementation scheme



$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, [\mathbf{y}, \imath^{\mathrm{VC}}_{a,b,c}]) = 0$$

$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

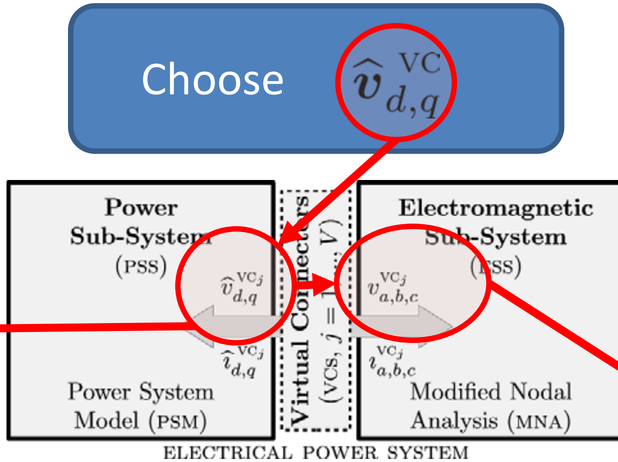
$$\mathbf{x}(t_0) = \mathbf{x}(t_0 + \tfrac{2\pi}{\Omega})$$

$$\frac{d\widehat{\mathbf{u}}}{dt} = 0$$

$$\mathbf{h}(\widehat{\mathbf{u}}, [\widehat{\mathbf{z}}, \widehat{\imath}^{\mathrm{VC}}_{d,q}]) = 0$$

- the PF solution of the PSS is a constant vector
- this **constant PF** solution, in the classical PSM, aims at representing the **constant envelope** of the actual system dynamics
- From the viewpoint of dynamical systems theory, the consideration above means that the stationary PSM solution is not an isolated equilibrium but it is embedded in a continuum of equilibria
- This is confirmed by an always null eigenvalue of the Jacobian matrix of the PSM linearized at any PF solution.

The implementation scheme



$$\frac{d\widehat{\mathbf{u}}}{dt} = 0$$
$$\mathbf{h}(\widehat{\mathbf{u}}, [\widehat{\mathbf{z}}, \widehat{\imath}^{\mathrm{VC}}_{d,q}]) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, [\mathbf{y}, \imath^{\mathrm{VC}}_{a,b,c}]) = 0$$
$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$
$$\mathbf{x}(t_0) = \mathbf{x}(t_0 + \tfrac{2\pi}{\Omega})$$

The steady-state solution of the overall electrical power system is a limit cycle with some of its components that actually oscillates (ESS) and some others that remain constant in the working period (PSS).

The solution of ESS are periodic functions in the time domain (limit cycle).

Qualitative introduction to the time-domain shooting method

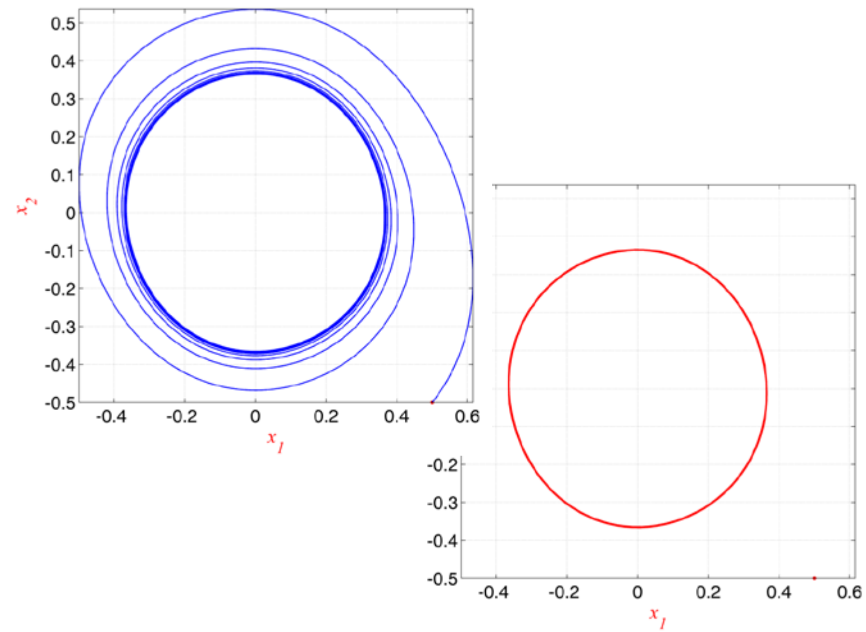
Smooth Circuit/System

$$\begin{cases} \dot{x} = f(x,t) \\ x(t_0) = x_0 \\ x(t) \in U \subset \mathbb{R}^N \\ f : \mathbb{R}^{N+1} \to \mathbb{R}^N \\ f \in C^1(\mathbb{R}^{N+1}) \end{cases}$$

Goal: efficiently find a **periodic steady state** solution of the ODE, i.e., a limit cycle (say γ)

$$\begin{cases} x_s(t) = x_s(t+T) \\ x_s(t_0) = \hat{x}_0 \in \gamma \end{cases}$$

Efficiently means that we do not want to perform a long lasting transient analysis to obtain the steady state behavior but we aim at directly find it.



Qualitative introduction to the time-domain shooting method

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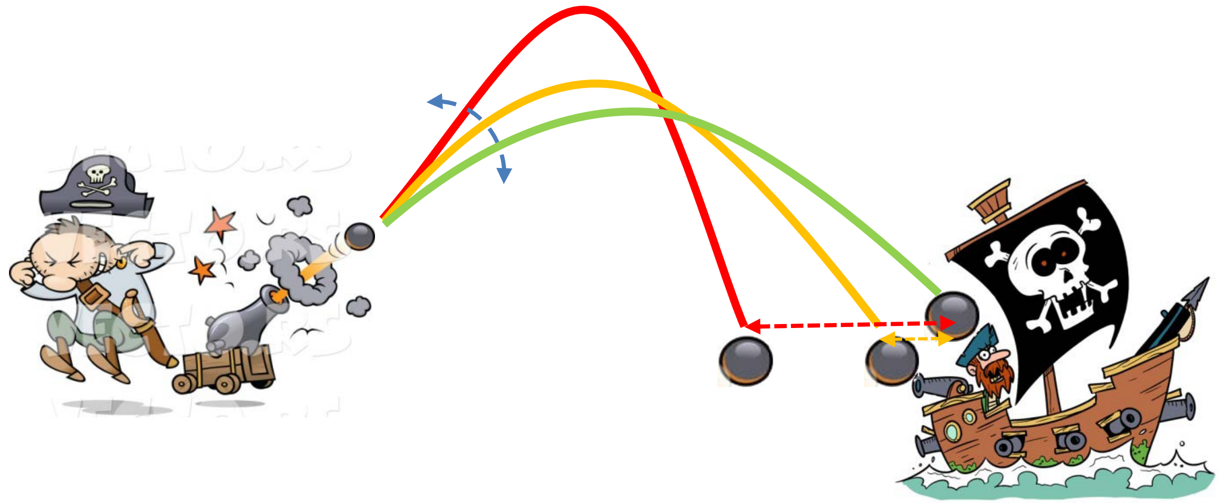


Goal: efficiently find a **periodic steady state** solution of the ODE, i.e., a limit cycle (say γ)

$$\begin{cases} x_s(t) = x_s(t+T) \\ x_s(t_0) = \hat{x}_0 \in \gamma \end{cases}$$

If a first shot misses the target, the gunner will change the tilt of the cannon, evaluate how much closer or farther he gets from his objective and finally adjust the tilt in order to (hopefully) hit the target with the next shot.

The key of the gunner's method is the perturbation of the initial guess and evaluation of the *sensitivity* of the solution (the arrival position of the cannonball) to this perturbation.

Qualitative introduction to the time-domain shooting method

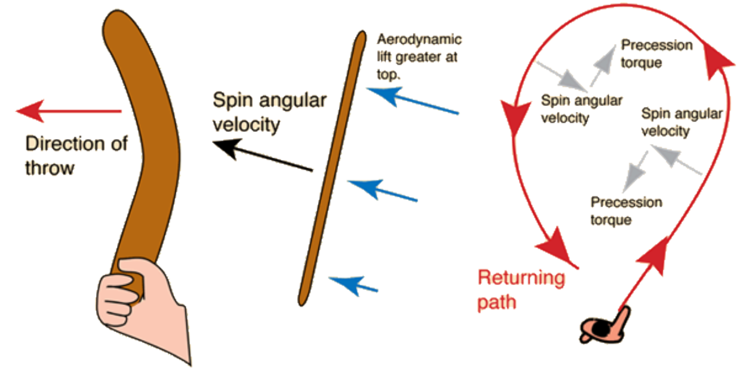
Smooth Circuit/System

$$\begin{cases} \dot{x} = f(x,t) \\ x(t_0) = x_0 \\ x(t) \in U \subset \mathbb{R}^N \\ f : \mathbb{R}^{N+1} \to \mathbb{R}^N \\ f \in C^1(\mathbb{R}^{N+1}) \end{cases}$$

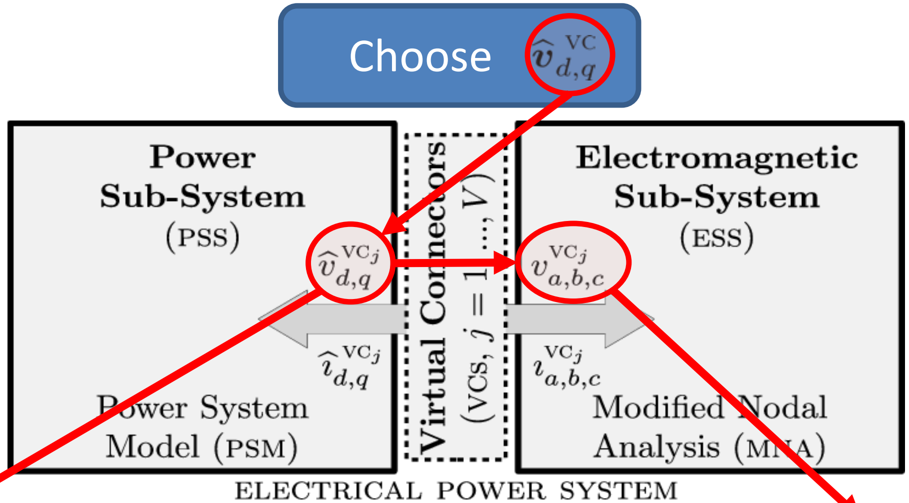
Goal: efficiently find a **periodic steady state** solution of the ODE, i.e., a limit cycle (say γ)

$$\begin{cases} x_s(t) = x_s(t+T) \\ x_s(t_0) = \hat{x}_0 \in \gamma \end{cases}$$

We are not gunners … we play with a boomerang since we are looking for a periodic trajectory (the initial point must coincide we the final one)



The implementation scheme and the solution approach



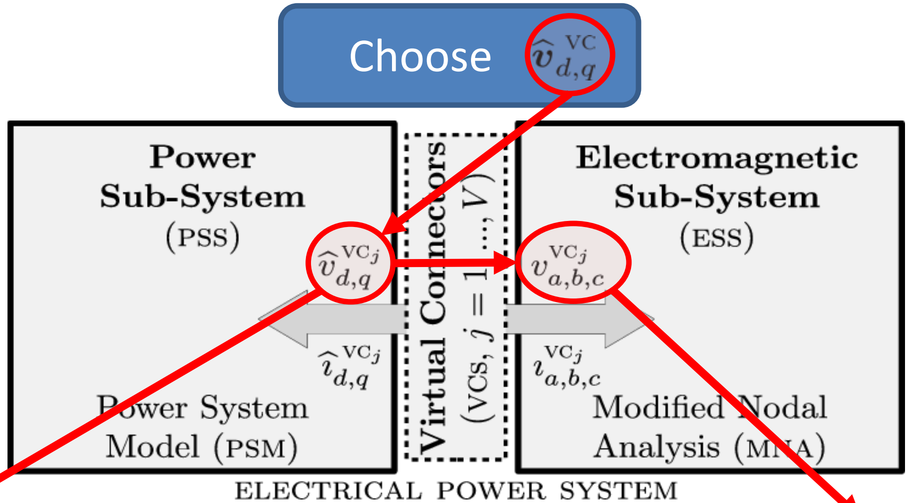
$$\frac{d\widehat{\mathbf{u}}}{dt} = 0$$
$$\mathbf{h}(\widehat{\mathbf{u}}, [\widehat{\mathbf{z}}, \widehat{\boldsymbol{\imath}}^{\mathrm{VC}}_{d,q}]) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, [\mathbf{y}, \boldsymbol{\imath}^{\mathrm{VC}}_{a,b,c}]) = 0$$
$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$
$$\mathbf{x}(t_0) = \mathbf{x}(t_0 + \tfrac{2\pi}{\Omega})$$

$\widehat{\boldsymbol{\imath}}^{\mathrm{VC}}_{d,q}$ ← $\boldsymbol{\imath}^{\mathrm{VC}}_{d,q}$

If they are identical you are a very lucky man!

The implementation scheme and the solution approach



$$\frac{d\hat{\mathbf{u}}}{dt} = 0$$

$$\mathbf{h}(\hat{\mathbf{u}}, [\hat{\mathbf{z}}, \hat{\boldsymbol{\imath}}^{VC}_{d,q}]) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, [\mathbf{y}, \boldsymbol{\imath}^{VC}_{a,b,c}]) = 0$$

$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

$$\mathbf{x}(t_0) = \mathbf{x}(t_0 + \tfrac{2\pi}{\Omega})$$

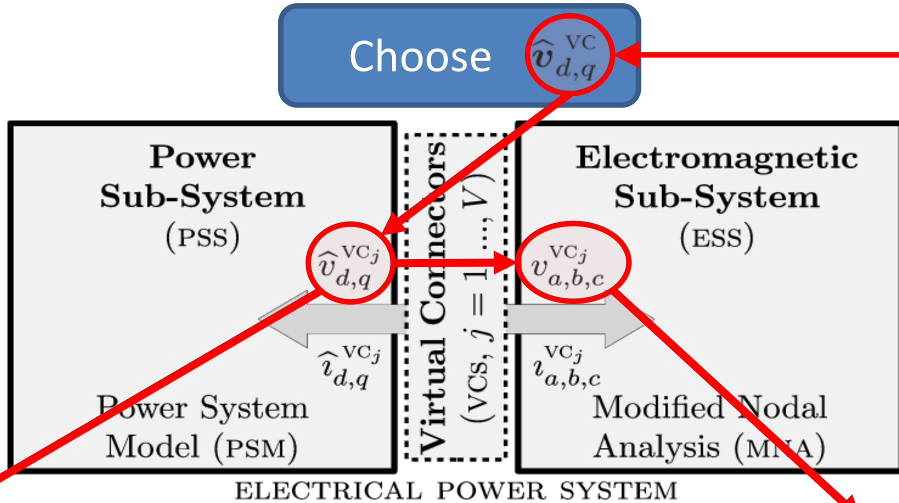
$$\boldsymbol{\imath}^{VC}_{d,q}$$

The PF solution of the PSS and the periodic steady state solution of the EES are thus derived independently and in parallel. The residue nonlinear function of the j-th VC

$$\rho^{VC_j}_{d,q}\left(\hat{\boldsymbol{v}}^{VC}_{d,q}\right) = \hat{\imath}^{VC_j}_{d,q} - \frac{1}{T}\int_{t_0}^{t_0+T} \imath^{VC_j}_{d,q}(\tau)\,d\tau$$

is computed for $j = 1, ..., V$.

The implementation scheme and the solution approach



$$\frac{d\widehat{\mathbf{u}}}{dt} = 0$$
$$\mathbf{h}(\widehat{\mathbf{u}}, [\widehat{\mathbf{z}}, \widehat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}]) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, [\mathbf{y}, \boldsymbol{\imath}_{a,b,c}^{\mathrm{VC}}]) = 0$$
$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$
$$\mathbf{x}(t_0) = \mathbf{x}(t_0 + \tfrac{2\pi}{\Omega})$$

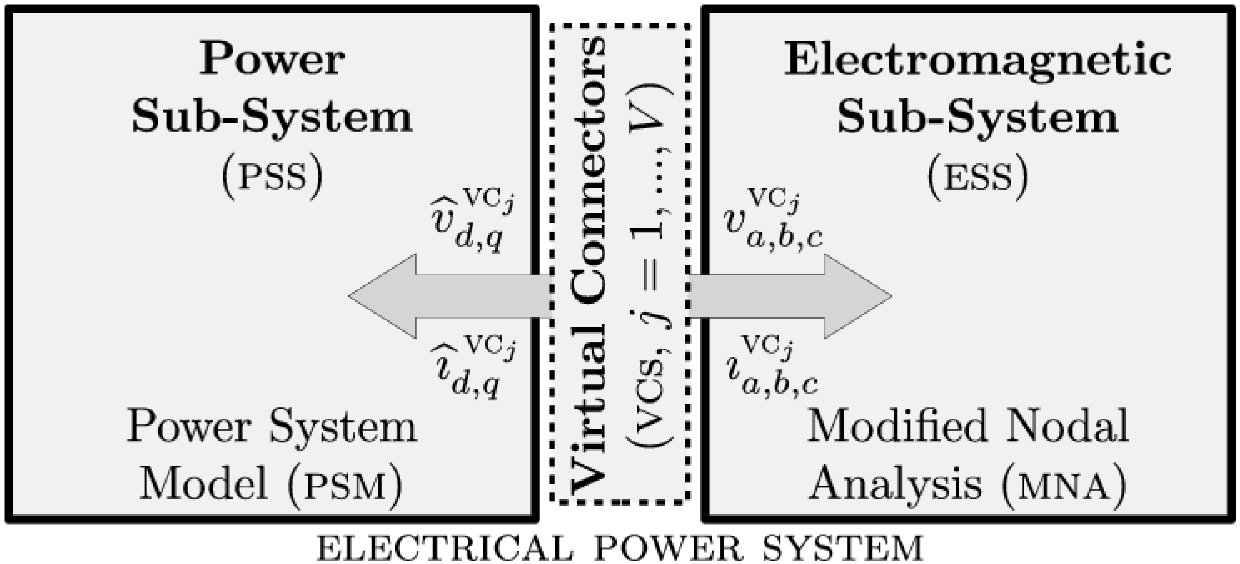
$\widehat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}$ $\quad$ $\boldsymbol{\imath}_{d,q}^{\mathrm{VC}}$

We use the iterative Newton method to compute the zeros of the residue functions that leads to the recursion expression

$$\widehat{\boldsymbol{v}}_{d,q}^{\mathrm{VC}^k} - \frac{\partial \boldsymbol{\rho}_{d,q}^{\mathrm{VC}^k}}{\partial \widehat{\boldsymbol{v}}_{d,q}^{\mathrm{VC}^k}} \boldsymbol{\rho}_{d,q}^{\mathrm{VC}^k}\left(\widehat{\boldsymbol{v}}_{d,q}^{\mathrm{VC}^k}\right) = \widehat{\boldsymbol{v}}_{d,q}^{\mathrm{VC}^{k+1}}$$

where k is the iteration of the Newton method.

The stability of the solution



$$\frac{d\widehat{\mathbf{u}}}{dt} + \mathbf{r}(\widehat{\mathbf{u}}, \overbrace{[\widehat{\mathbf{z}}, \widehat{\boldsymbol{\imath}}_{d,q}^{\mathrm{VC}}]}^{\widehat{\boldsymbol{\zeta}} \in \mathbb{R}^{S_z + 2V}}) = 0$$

$$\mathbf{h}(\widehat{\mathbf{u}}, \widehat{\boldsymbol{\zeta}}) = 0$$

$$\frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \overbrace{[\mathbf{y}, \boldsymbol{\imath}_{a,b,c}^{\mathrm{VC}}]}^{\boldsymbol{\lambda} \in \mathbb{R}^{S_y + 3V}}) = 0$$

$$\mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

A by product of this solution approach is the sensitivity matrix of the overall system with respect to its initial conditions: the monodromy matrix

The stability of the solution

$$\frac{d\hat{\mathbf{u}}}{dt} + \mathbf{r}(\hat{\mathbf{u}}, \overbrace{[\hat{\mathbf{z}}, \hat{\boldsymbol{\imath}}^{\mathrm{VC}}_{d,q}]}^{\hat{\boldsymbol{\zeta}} \in \mathbb{R}^{S_z+2V}}) = 0 \qquad \frac{d\mathbf{q}(\mathbf{x})}{dt} + \mathbf{f}(\mathbf{x}, \overbrace{[\mathbf{y}, \boldsymbol{\imath}^{\mathrm{VC}}_{a,b,c}]}^{\boldsymbol{\lambda} \in \mathbb{R}^{S_y+3V}}) = 0$$

$$\mathbf{h}(\hat{\mathbf{u}}, \hat{\boldsymbol{\zeta}}) = 0 \qquad \mathbf{g}(\mathbf{x}, \boldsymbol{\lambda}) = 0$$

A by product of this solution approach is the sensitivity matrix of the overall system with respect to its initial conditions: the monodromy matrix

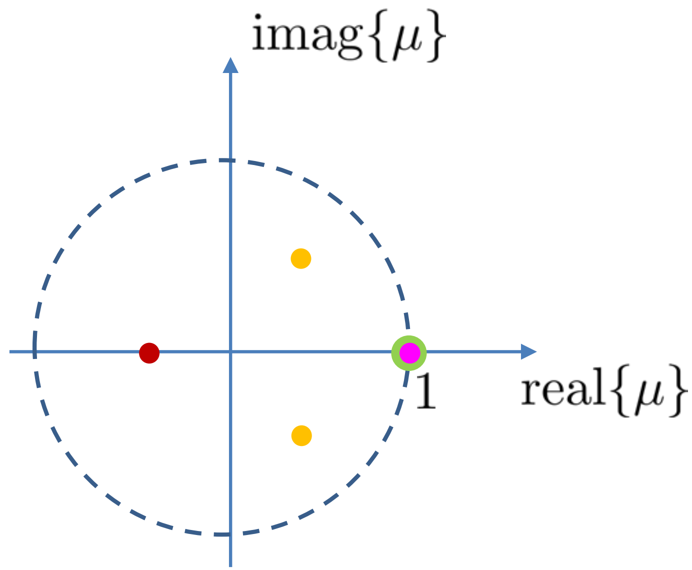
The eigenvalues of the monodromy matrix are known as the Floquet multipliers

$\mu_{1,1} = \mu_{1,2} = 1$

μ_j and $\mu_j^* \in \mathbb{C}$

$\mu_r \in \mathbb{R}$

If they are within the unit circle the system is stable



The INstability of the solution

$\mu_{1,1} = \mu_{1,2} = 1$

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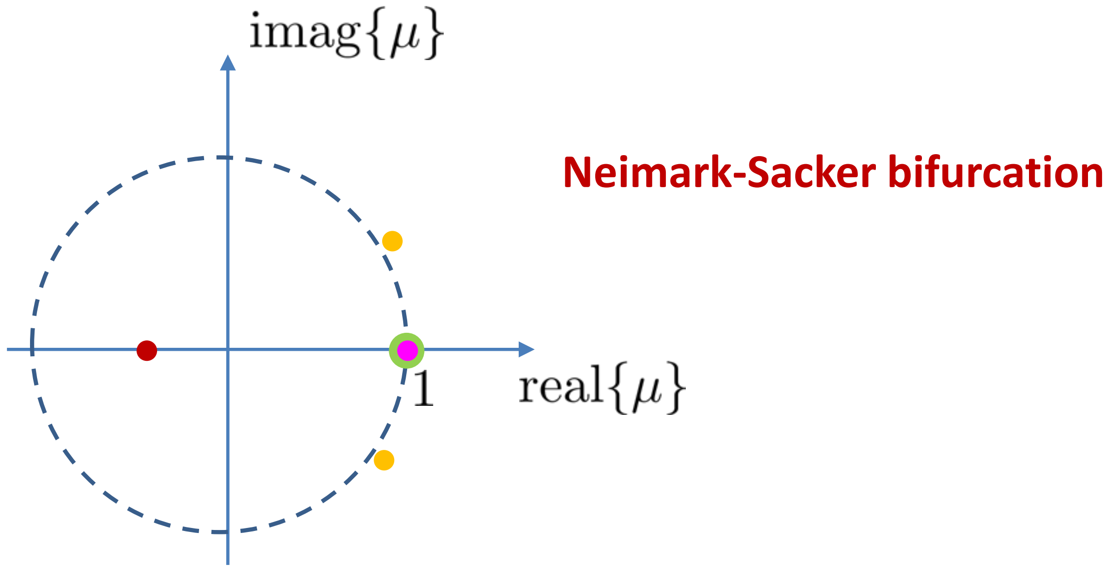
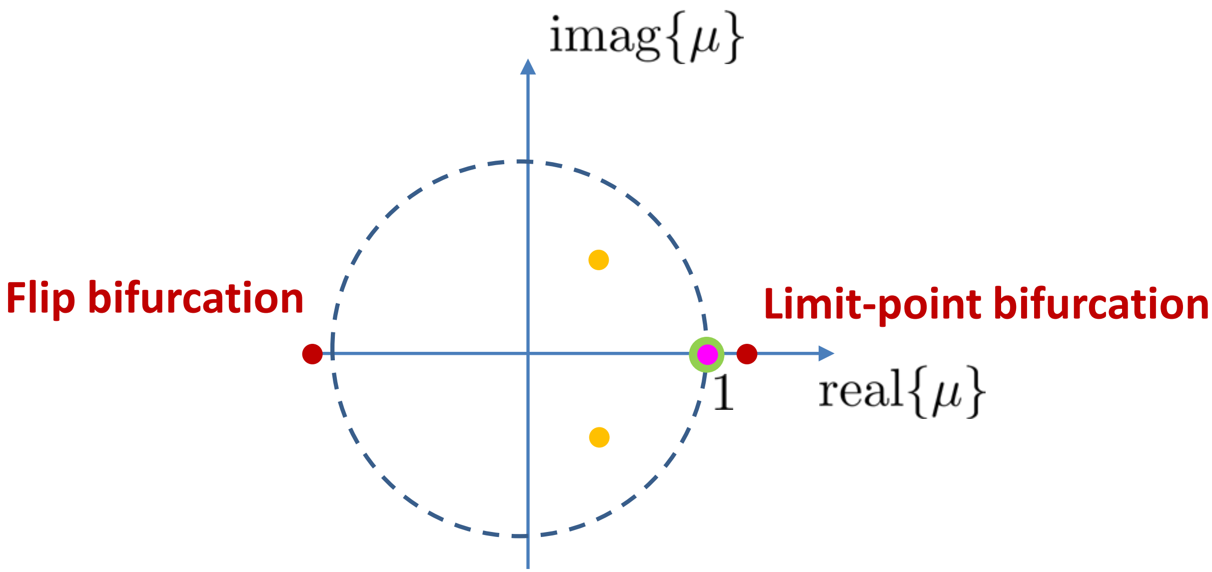
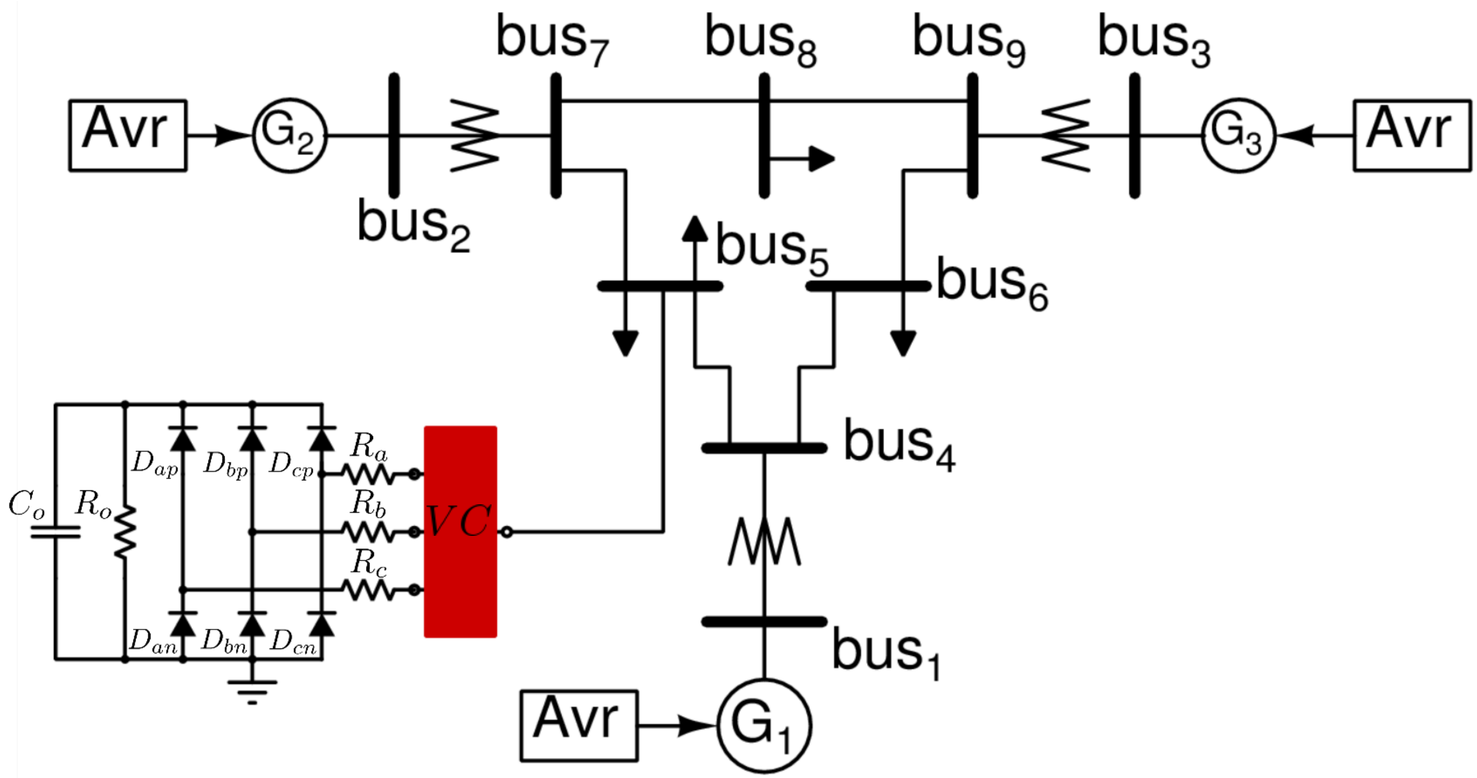




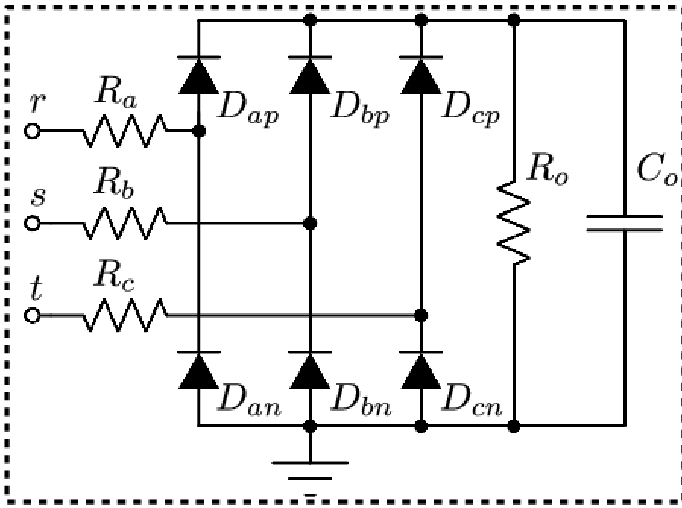
Numerical results

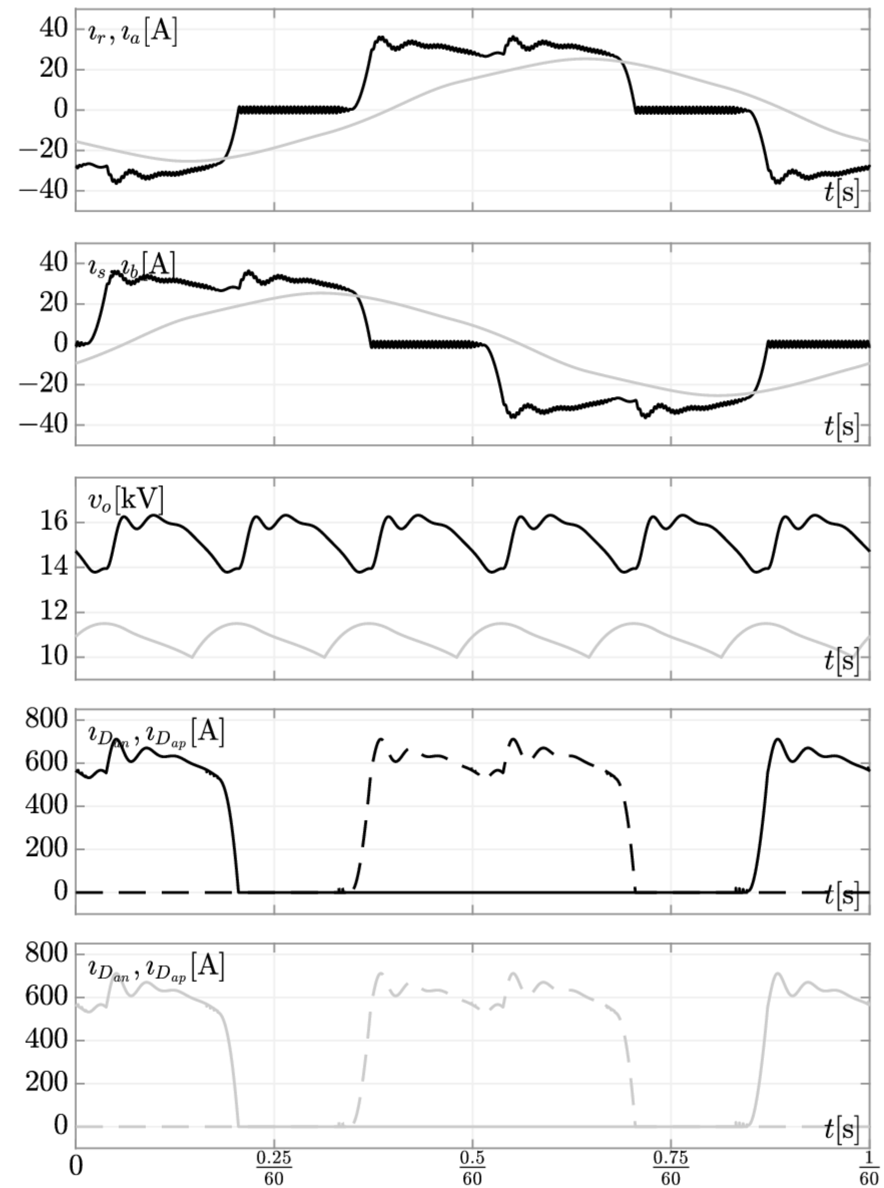


The application of the proposed method shows that the power flowing through the VC is $P = 8.47\,\mathrm{MW}$, $Q = 4.03\,\mathrm{MVAR}$. The active power of the slack generator increases from $71.641\,\mathrm{MW}$ (no rectifier connected) to $80.118\,\mathrm{MW}$.

Numerical results

Focus on black traces

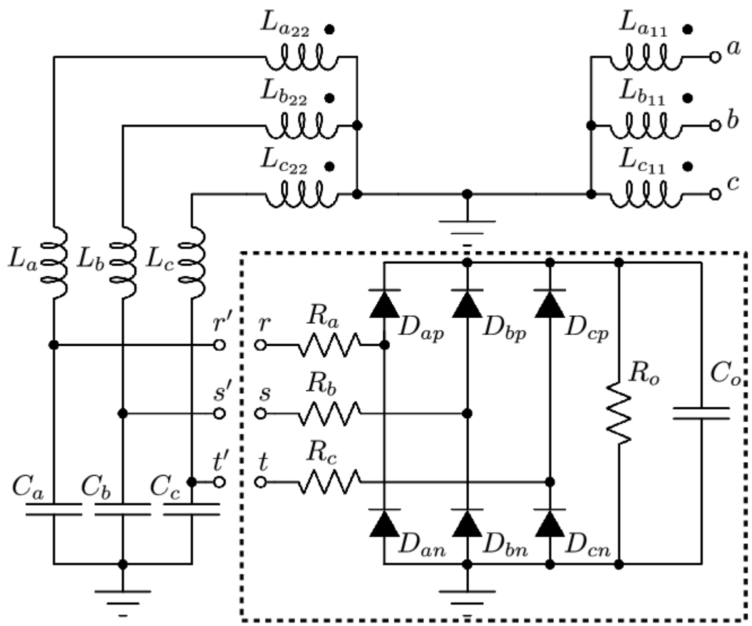




$$\widehat{\mathcal{E}}_{\text{hd}} = 0.483$$

$$\widehat{\mathcal{E}}_{\text{hd}} = \frac{|\widetilde{I}_0|^2}{\sum_{k=-K}^{K} |\widetilde{I}_k|^2}$$

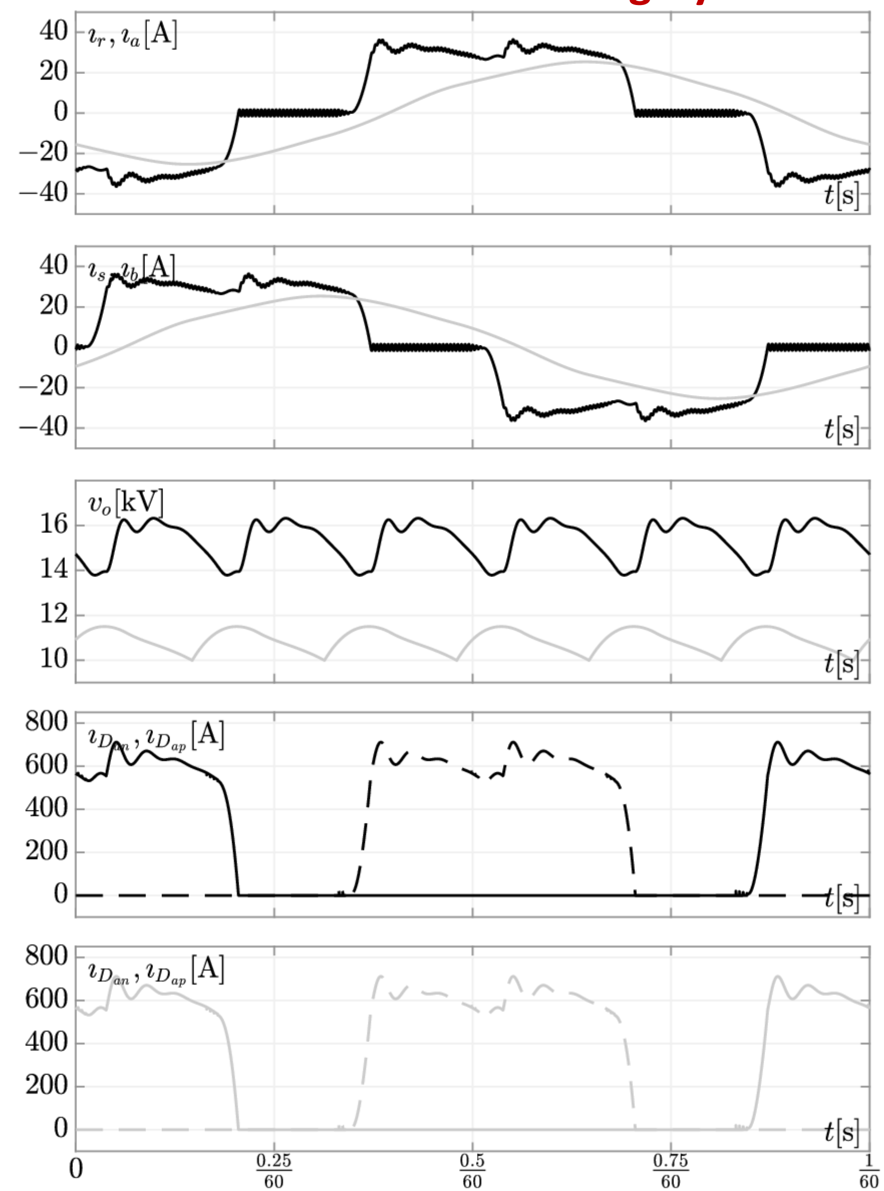
Numerical results



$$\mathcal{E}_{\rm hd} = 0.891$$

The power flowing through the VC is now $P = 4.7\text{MW}$ and $Q = 5.2\,\text{kVAR}$, whereas the active power of the slack generator lowers to $76.542\,\text{MW}$.

Focus on gray traces



Numerical results

F. Bizzarri and A. Brambilla, "Generalized Power Flow Analysis of Electrical Power Systems Modeled as Mixed Single-Phase/Three-Phase Sub-Systems," in IEEE Transactions on Power Systems.

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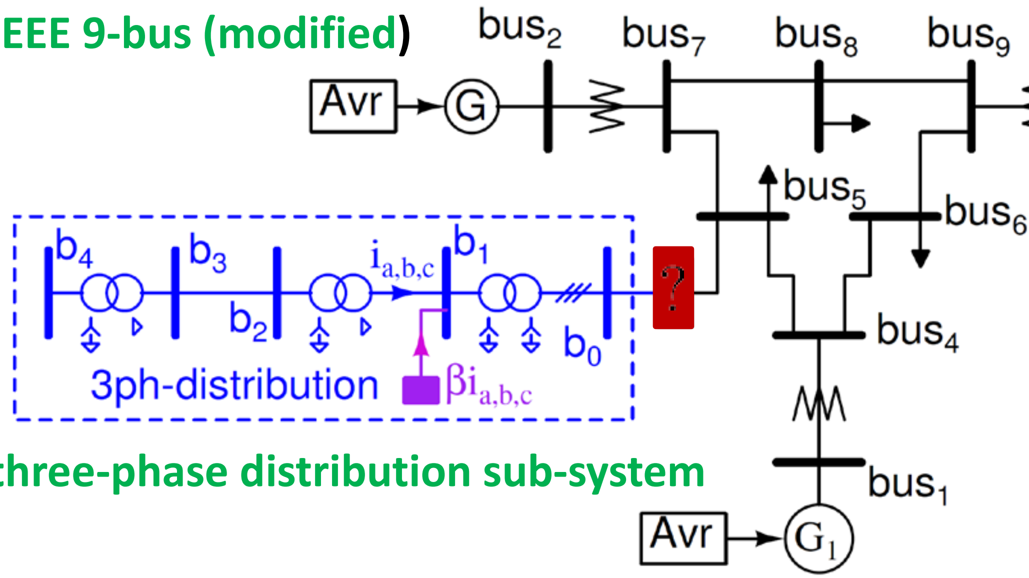
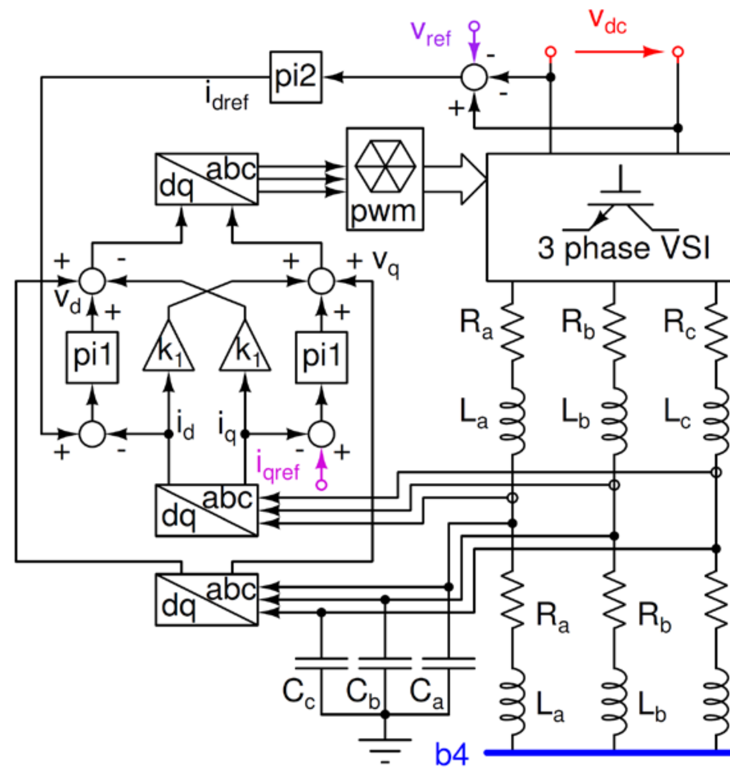
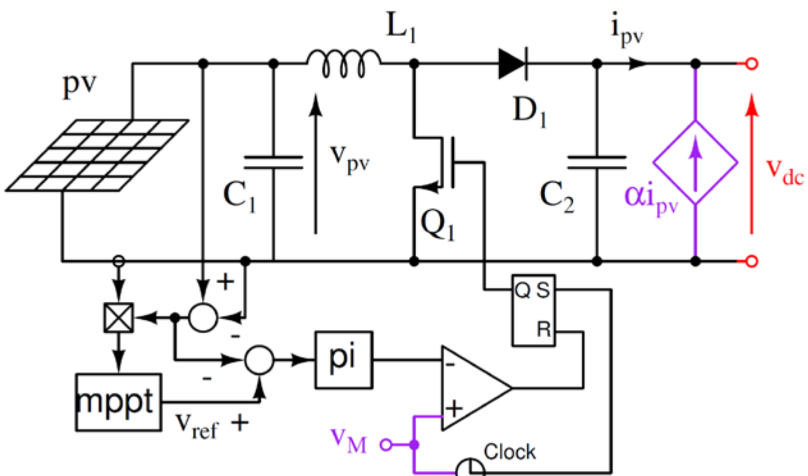


three-phase distribution sub-system

three-phase LCL VSC

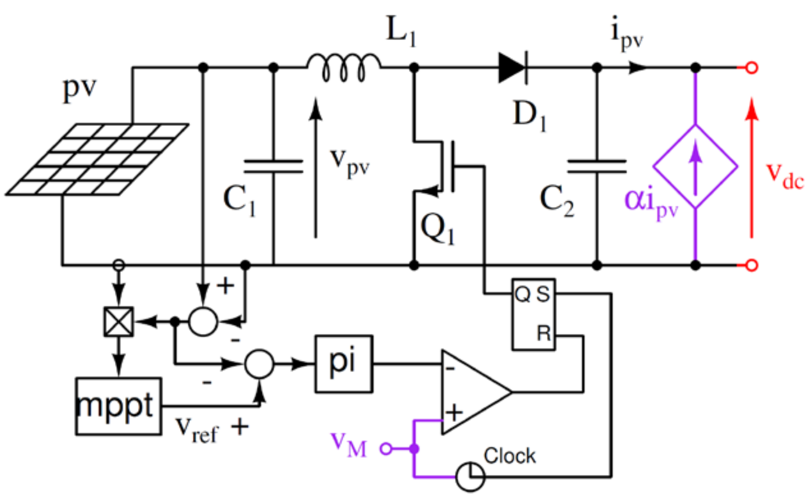




DC-DC converter with MPPT connected to a PV plant

Numerical results

The PF solution depends on the efficiency of the converters, on the behaviour of the MPPT, on the solar radiation and working temperature of the PV array of solar panels and on the characteristics of the distribution system. We compute the PF solution at different levels of the S solar irradiance ($200\,\mathrm{Wm}^{-2}$, $1\,\mathrm{kWm}^{-2}$ and $1.15\mathrm{kWm}^{-2}$).

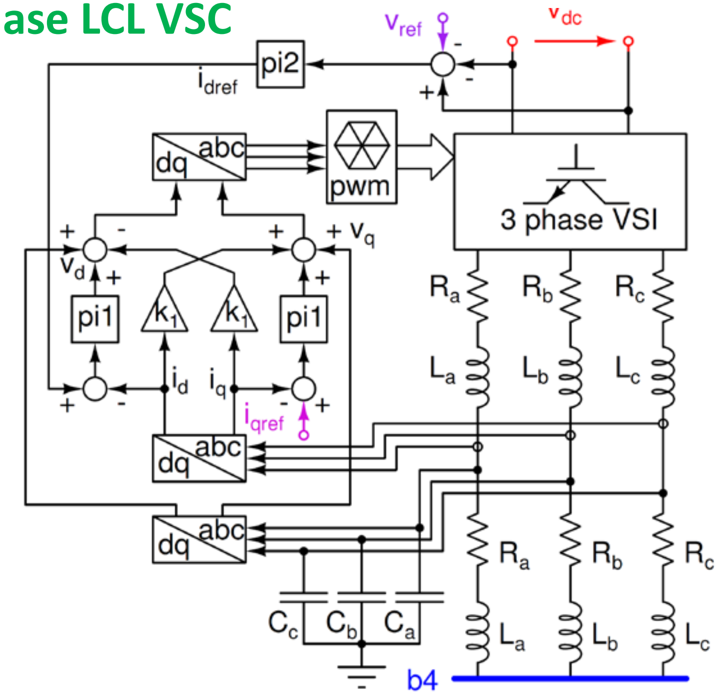


DC-DC converter with MPPT connected to a PV plant

POLITECNICO MILANO 1863

Numerical results

three-phase LCL VSC



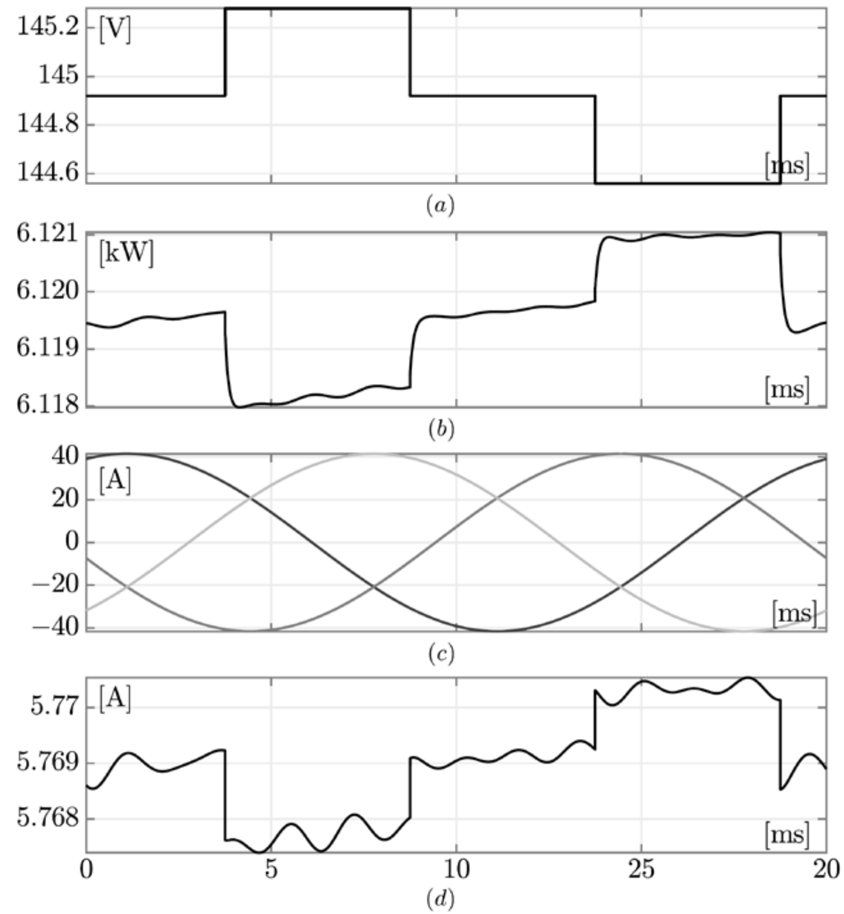
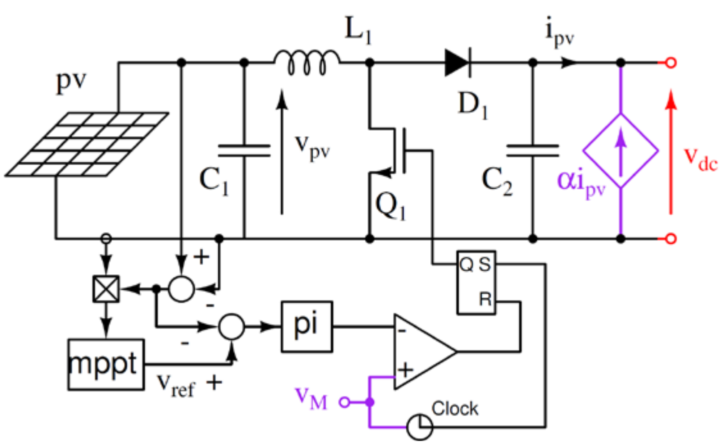


Figure 6. $S = 1000\,\mathrm{W\,m^{-2}}$ (a) v_{ref} voltage determined by the MPPT, y-axis: voltage [V]. (b) The instantaneous power at the output of the DC/DC converter, y-axis: power [kW]. (c) The $i_{a,b,c}$ three-phase currents of the VC in Fig. 3, (d) The i_{dref} driving signal of the LCL converter. y-axis: current [A]. x-axes: time [ms].



DC-DC converter with MPPT connected to a PV plant

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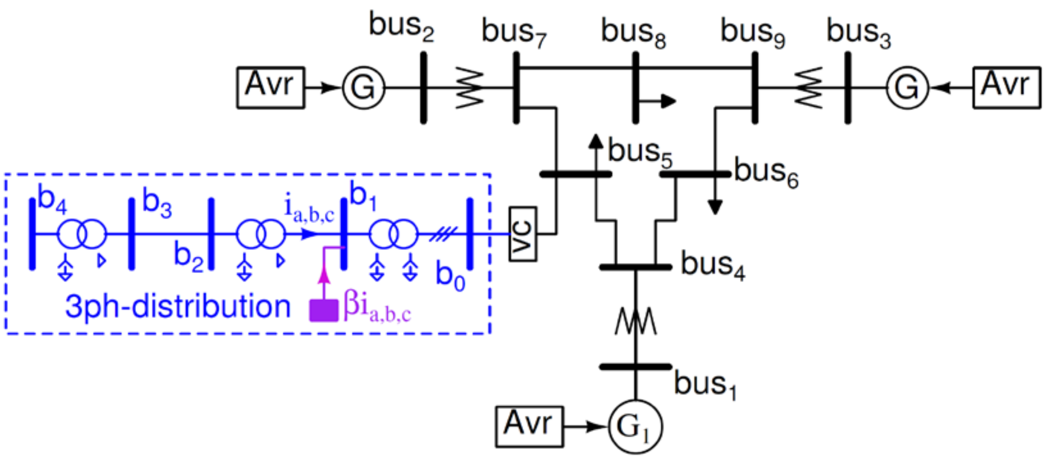
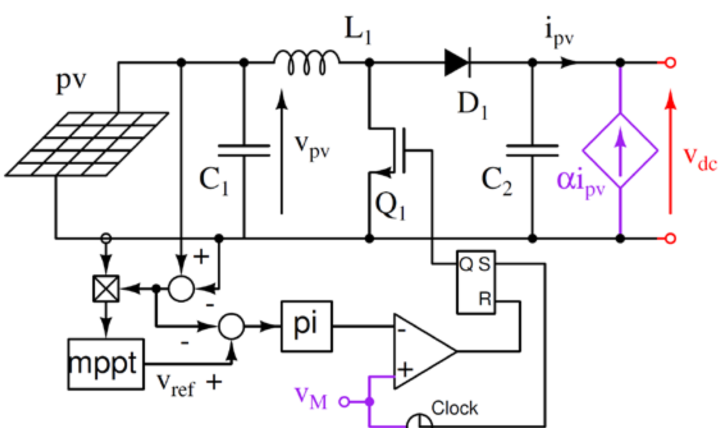
Numerical results

IEEE 9-bus (modified)



three-phase distribution sub-system



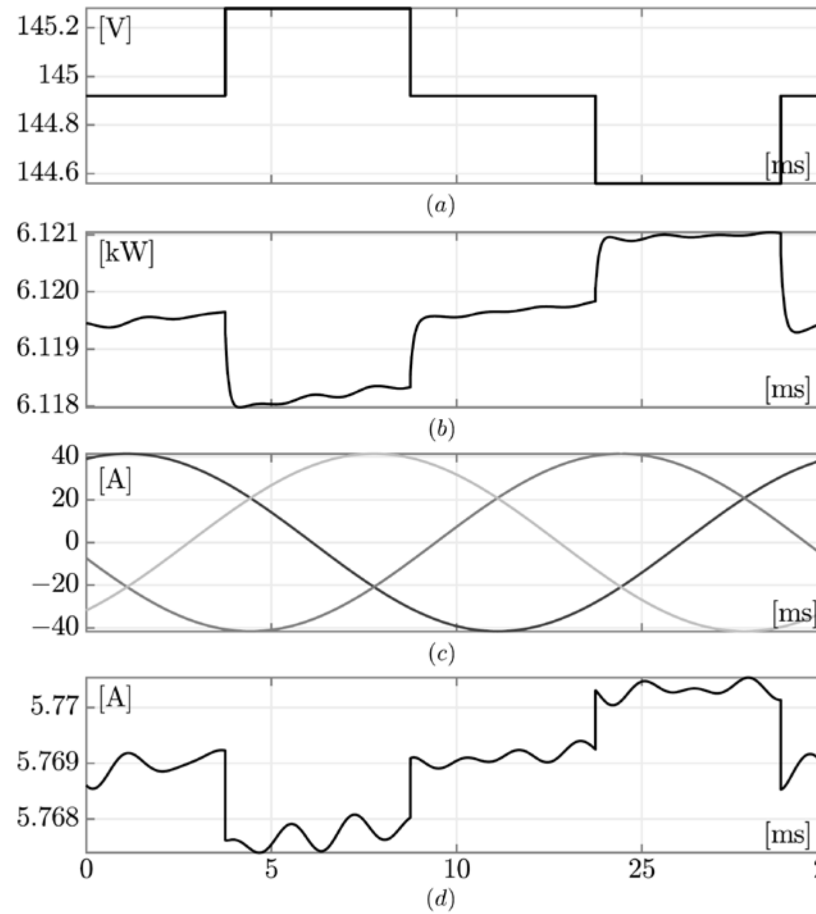


Figure 6. $S = 1000\ \mathrm{Wm}^{-2}$ (a) v_{ref} voltage determined by the MPPT, y-axis: voltage [V]. (b) The instantaneous power at the output of the DC/DC converter, y-axis: power [kW]. (c) The $i_{a,b,c}$ three-phase currents of the VC in Fig. 3, (d) The i_{dref} driving signal of the LCL converter. y-axis: current [A]. x-axes: time [ms].

DC-DC converter with MPPT connected to a PV plant

Numerical results

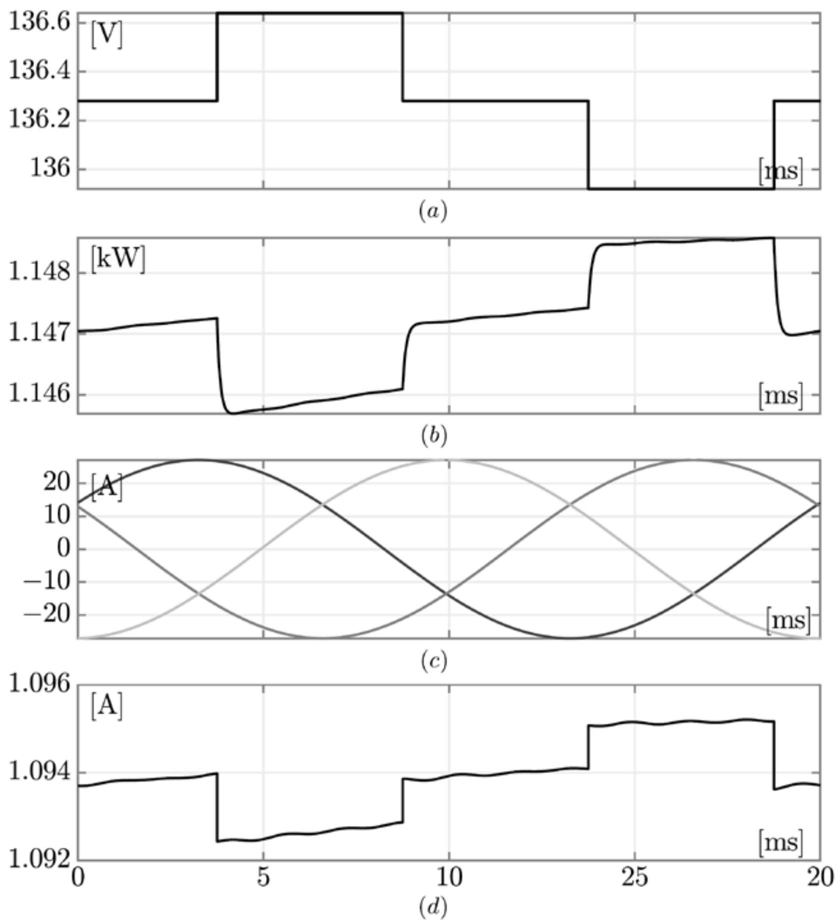


Figure 7. $S = 200\,\mathrm{Wm}^{-2}$. (a) v_{ref} voltage determined by the MPPT, y-axis: voltage [V]. (b) The instantaneous power at the output of the DC/DC converter, y-axis: power [kW]. (c) The $i_{a,b,c}$ three-phase currents of the VC in Fig. 3, (d) The i_{dref} driving signal of the LCL converter. y-axis: current [A]. x-axes: time [ms].

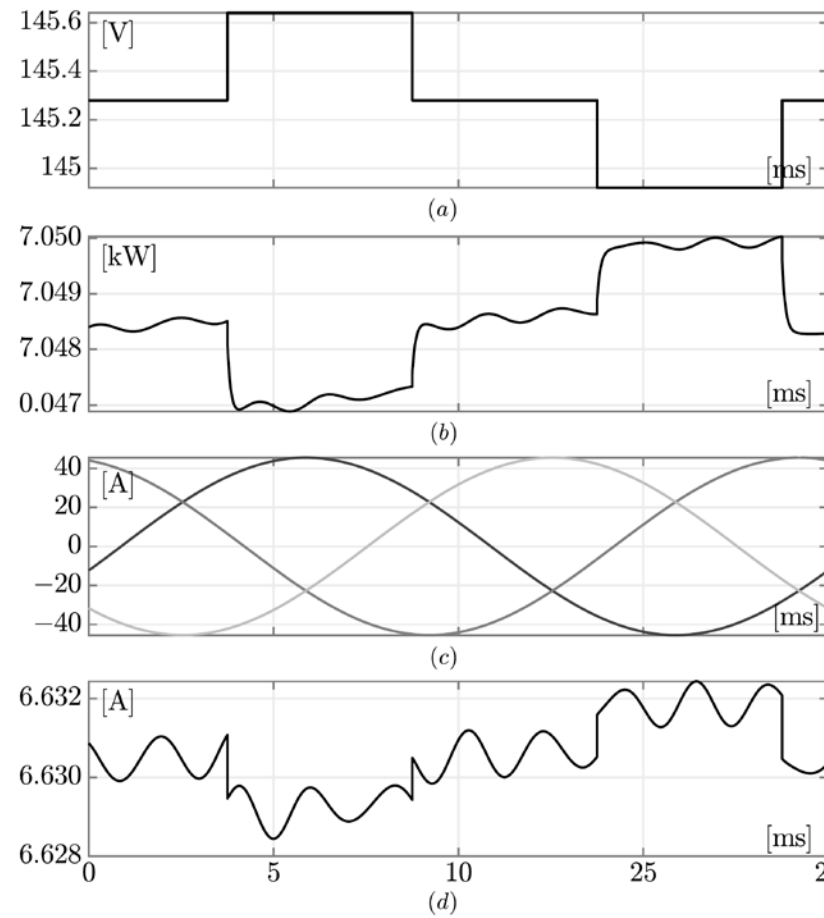


Figure 8. $S = 1150\,\mathrm{Wm}^{-2}$ (a) v_{ref} voltage determined by the MPPT, y-axis: voltage [V]. (b) The instantaneous power at the output of the DC/DC converter, y-axis: power [kW]. (c) The $i_{a,b,c}$ three-phase currents of the VC in Fig. 3, (d) The i_{dref} driving signal of the LCL converter. y-axis: current [A]. x-axes: time [ms].

Numerical results

Table 1: PSS solutions with different values of the S irradiance.

S	200	1000	1150	No ESS
P_{G_1}	69.884	62.381	60.986	71.641
Q_{G_1}	22.338	21.778	21.693	27.046
Q_{G_2}	3.819	3.480	3.425	6.653
Q_{G_3}	−12.220	−12.328	−12.343	−10.860
$v_{d,q}^{\mathrm{bus}_5}$	(230.06, −15.54)	(230.40, −13.37)	(230.46, −12.96)	(228.44, −15.39)
$\mathcal{E}_{\mathrm{hd}}$	0.80%	0.74%	0.63%	–

Irradiance is expressed in Wm^{-2}; active and reactive powers are in MW and MVAR, respectively; (d,q) voltages are in kV.

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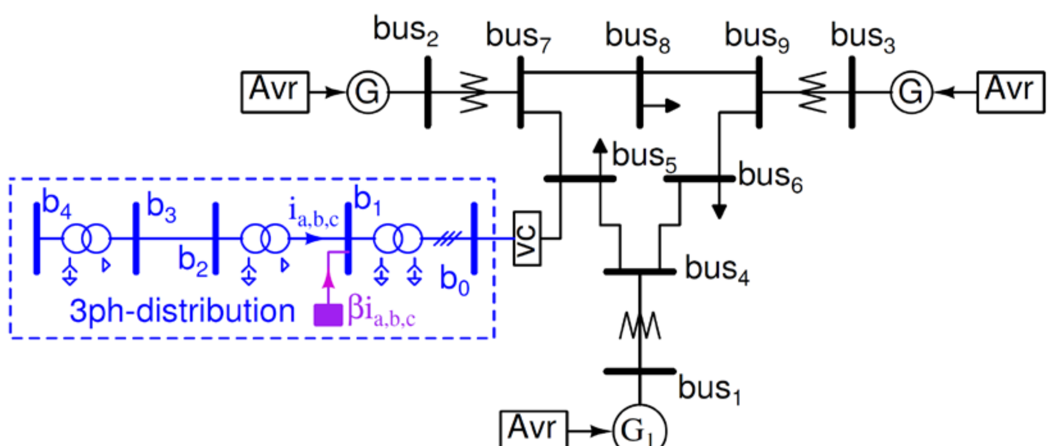

three-phase distribution sub-system