Robust Security Constrained ACOPF via Conic Programming

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Motivation

- Security Constrained AC Optimal Power Flow
  - AC: accuracy is needed if close to power delivery
  - Robust security: worst contingency/contingencies is/are taken into account
- Assist the ISO to ensure a secure operation minutes prior to power delivery
- The worst contingency/contingencies is/are identified

Robust SC ACOPF

The worst

The worst happens

$$\max_{\mathbf{u}\in\mathcal{U}} \ \min_{\mathbf{y}\in\mathcal{Y}} \ f(\mathbf{x},\mathbf{u},\mathbf{y})$$

Best operator reaction

Robust SC ACOPF

$$\max_{\mathbf{u}} \quad \min_{\mathbf{y}} \quad f(\mathbf{u}, \mathbf{y})$$

$$\text{s.t.} \quad \text{s.t.} \quad \mathbf{h}^{\mathrm{O}}(\mathbf{u}, \mathbf{y}) = \mathbf{0}, \quad \mathbf{g}^{\mathrm{O}}(\mathbf{u}, \mathbf{y}) \leq \mathbf{0}, \quad \mathbf{y} \in \mathcal{Y}$$

$$\mathbf{u} \in \mathcal{U}$$

Worst contingency or contingencies

Best system operation

Robust SC ACOPF

$$
\max_{\mathbf{u}} \quad \left[\begin{array}{ll} \min_{\mathbf{y}} & f(\mathbf{u}, \mathbf{y}) \\ \text{s.t.} & \\ \mathbf{h}^{\mathrm{O}}(\mathbf{u}, \mathbf{y}) = \mathbf{0} & \\ \mathbf{g}^{\mathrm{O}}(\mathbf{u}, \mathbf{y}) \leq \mathbf{0} & \\ \mathbf{y} \in \mathcal{Y} & \end{array} \right]
$$

$$
\text{s.t.} \quad \mathbf{u} \in \mathcal{U}
$$

Inner problem: If relaxed, convex conic program

Convex Conic Program

$$\min_x 2x_1 + x_2$$

s.t.

$$\sqrt{x_1^2 + (x_2 - 1)^2} \leq 3$$

$$x_1 + x_2 \leq 2$$

Example

Conic

$$\min_x f^T x$$

s.t.

$$\|A_i x + b_i\|_2 \leq c_i^T x + d_i, \quad \forall i \in I$$

$$Fx \leq g$$

Linear

Convex Conic Program

Primal

$$\min_x f^T x$$

s.t.

$$y_i = A_i x + b_i, \quad \forall i \in I \quad (\mu_i)$$

$$t_i = c_i^T x + d_i, \quad \forall i \in I \quad (\nu_i)$$

$$\|y_i\|_2 \leq t_i, \quad \forall i \in I$$

$$Fx \leq g \quad (\lambda)$$

$$\min_x 2x_1 + x_2$$

s.t.

$$y_1 = x_1, \quad (\mu_1)$$

$$y_2 = x_2 - 1, \quad (\mu_2)$$

$$t_1 = 3, \quad (\nu_1)$$

$$\sqrt{y_1^2 + y_2^2} \leq t_1,$$

$$x_1 + x_2 \leq 2 \quad (\lambda)$$

Example

Convex Conic Program

Dual

$$\max_{\mu,\nu,\lambda} \sum_{i\in I}(-b_i^T\mu_i - d_i\nu_i) + g^T\lambda$$

s.t.

$$\sum_{i\in I}(A_i^T\mu_i + c_i\nu_i) + F\lambda = f$$

$$\|\mu_i\|_2 \le \nu_i, \quad \forall i \in I$$

$$\lambda \le 0$$

$$\max_{\mu,\nu,\lambda} \mu_2 - 3\nu_1 + 2\lambda$$

s.t.

$$\mu_1 + \lambda = 2$$

$$\mu_2 + \lambda = 1$$

$$\sqrt{\mu_1^2 + \mu_2^2} \le \nu_1$$

$$\lambda \le 0$$

Example

Convex Conic Program example

$$\min_x 2x_1 + x_2$$

s.t.

$$y_1 = x_1, \quad (\mu_1)$$

$$y_2 = x_2 - 1, \quad (\mu_2)$$

$$t_1 = 3, \quad (\nu_1)$$

$$\sqrt{y_1^2 + y_2^2} \leq t_1,$$

$$x_1 + x_2 \leq 2 \quad (\lambda)$$

```
variables z, x1, x2, y1, y2, t1;
equations of, e1, e2, e3, e4, e5;
of.. z =e= 2*x1+x2;
e1.. y1 =e= x1;
e2.. y2 =e= x2-1;
e3.. t1 =e= 3;
e4.. sqrt(power(y1,2)+power(y2,2)) =l= t1;
e5.. x1+x2 =l= 2;
model conP /all/;
solve conP using nlp minimizing z;
```

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU of	.	.	.	1.000
---- EQU e1	.	.	.	-2.000
---- EQU e2	-1.000	-1.000	-1.000	-1.000
---- EQU e3	3.000	3.000	3.000	-2.236
---- EQU e4	-INF	.	.	-2.236
---- EQU e5	-INF	-3.025	2.000	.

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR z	-INF	-5.708	+INF	.
---- VAR x1	-INF	-2.683	+INF	.
---- VAR x2	-INF	-0.342	+INF	.
---- VAR y1	-INF	-2.683	+INF	-4.890E-8
---- VAR y2	-INF	-1.342	+INF	.
---- VAR t1	-INF	3.000	+INF	.

Convex Conic Program example

$$\max_{\mu,\nu,\lambda} \mu_2 - 3\nu_1 + 2\lambda$$

s.t.

$$\mu_1 + \lambda = 2$$

$$\mu_2 + \lambda = 1$$

$$\sqrt{\mu_1^2 + \mu_2^2} \leq \nu_1$$

$$\lambda \leq 0$$

```
variables mu1,mu2, nu1, z;
negative variable lambda;
equations of, e1, e2, e3;
of.. z =e= mu2 - 3*nu1 + 2*lambda;
e1.. mu1 + lambda =e= 2;
e2.. mu2 + lambda =e= 1;
e3.. sqrt(power(mu1,2)+power(mu2,2)) =l= nu1;
model conD /all/;
solve conD using nlp maximizing z;
```

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU of	.	.	.	1.000
---- EQU e1	2.000	2.000	2.000	-2.683
---- EQU e2	1.000	1.000	1.000	-0.342
---- EQU e3	-INF	.	.	3.000

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR mu1	-INF	2.000	+INF	.
---- VAR mu2	-INF	1.000	+INF	.
---- VAR nu1	-INF	2.236	+INF	.
---- VAR z	-INF	-5.708	+INF	.
---- VAR lambda	-INF	.	.	5.025

Robust SC ACOPF

$$\max_{\mathbf{u}} \quad \min_{\mathbf{y}} \quad f(\mathbf{u},\mathbf{y})$$
$$\text{s.t.}$$
$$\mathbf{h}^{\mathrm{O}}(\mathbf{u},\mathbf{y}) = \mathbf{0}$$
$$\mathbf{g}^{\mathrm{O}}(\mathbf{u},\mathbf{y}) \leq \mathbf{0}$$
$$\mathbf{y} \in \mathcal{Y}$$
$$\text{s.t.}$$
$$\mathbf{u} \in \mathcal{U}$$

$$\max_{\mathbf{u}} \quad \max_{\lambda,\mu,\nu} \quad f^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu)$$
$$\text{s.t.}$$
$$\mathbf{h}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) = \mathbf{0}$$
$$\mathbf{g}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \leq \mathbf{0}$$
$$\lambda,\mu,\nu \in \mathbf{\Omega}$$
$$\text{s.t.}$$
$$\mathbf{u} \in \mathcal{U}$$

Dual of the convex conic program

Robust SC ACOPF

$$
\begin{aligned}
&\max_{\mathbf{u},\lambda,\mu,\nu} && f^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \\
&\text{s.t.} \\
&\mathbf{h}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) = \mathbf{0} \\
&\mathbf{g}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \leq \mathbf{0} \\
&\lambda,\mu,\nu \in \mathbf{\Omega} \\
&\mathbf{u} \in \mathcal{U}
\end{aligned}
$$

ACOPF rectangular coordinates

$$\min_{p,q,c,s,\theta} \sum_{j\in J} C_j^G p_j^G + \sum_{i\in I} C_i^{UP} p_i^U + \sum_{i\in I} C_i^{UQ} q_i^U \quad (1)$$

s.t.

$$\underline{p}_j^G \leq p_j^G \leq \overline{p}_j^G \quad \forall j \in J \quad (\underline{\gamma_j}, \overline{\gamma_j}) \quad (2)$$

$$\underline{q}_j^G \leq q_j^G \leq \overline{q}_j^G \quad \forall j \in J \quad (\underline{\kappa_j}, \overline{\kappa_j}) \quad (3)$$

$$\underline{V}_i^2 \leq c_i \leq \overline{V}_i^2 \quad \forall i \in I \quad (\underline{\chi_i}, \overline{\chi_i}) \quad (4)$$

$$0 \leq p_i^U \quad (\delta_i) \quad (5)$$

$$0 \leq q_i^U \quad (\eta_i) \quad (6)$$

$$\sum_{j\in J_i} p_j^G - \sum_{l|o(l)=i} (G_l c_{o(l)} + G_l c_l + B_l s_l) - \sum_{l|d(l)=i} (G_l c_{d(l)} + G_l c_l - B_l s_l) + p_i^U = p_i^D \quad \forall i \in I \quad \forall l \in L \quad (\mu_i) \quad (7)$$

$$\sum_{j\in J_i} q_j^G - \sum_{l|o(l)=i} (-B_l c_{o(l)} - B_l c_l + G_l s_l) - \sum_{l|d(l)=i} (-B_l c_{d(l)} - B_l c_l - G_l s_l) + q_i^U = q_i^D \quad \forall i \in I \quad \forall l \in L \quad (\nu_i) \quad (8)$$

$$(G_l c_{o(l)} + G_l c_l + B_l s_l)^2 + (-B_l c_{o(l)} - B_l c_l + G_l s_l)^2 \leq (f_l^{max})^2 \quad \forall l \in L \quad (9)$$

$$c_l^2 + s_l^2 = c_{o(l)} c_{d(l)} \quad \forall l \in L \quad (10)$$

Relaxed ACOPF

$$c_l^2 + s_l^2 \leq c_{o(l)} c_{d(l)} \quad \forall l \in L \qquad (11)$$

Relaxed ACOPF

$$\min_{p,q,c,s,\theta} \sum_{j\in J} C_j^G p_j^G + \sum_{i\in I} C_i^{UP} p_i^U + \sum_{i\in I} C_i^{UQ} q_i^U$$

s.t.

$$\underline{p}_j^G \le p_j^G \le \overline{p}_j^G \quad \forall j \in J \quad (\underline{\gamma_j}, \overline{\gamma_j})$$

$$\underline{q}_j^G \le q_j^G \le \overline{q}_j^G \quad \forall j \in J \quad (\underline{\kappa_j}, \overline{\kappa_j})$$

$$\underline{V}_i^2 \le c_i \le \overline{V}_i^2 \quad \forall i \in I \quad (\underline{\chi_i}, \overline{\chi_i})$$

$$0 \le p_i^U \quad (\delta_i)$$

$$0 \le q_i^U \quad (\eta_i)$$

$$\sum_{j\in J_i} p_j^G - \sum_{l|o(l)=i} (G_l c_{o(l)} + G_l c_l + B_l s_l) - \sum_{l|d(l)=i} (G_l c_{d(l)} + G_l c_l - B_l s_l) + p_i^U = p_i^D \quad \forall i \in I \quad \forall l \in L \quad (\mu_i)$$

$$\sum_{j\in J_i} q_j^G - \sum_{l|o(l)=i} (-B_l c_{o(l)} - B_l c_l + G_l s_l) - \sum_{l|d(l)=i} (-B_l c_{d(l)} - B_l c_l - G_l s_l) + q_i^U = q_i^D \quad \forall i \in I \quad \forall l \in L \quad (\nu_i)$$

$$(G_l c_{o(l)} + G_l c_l + B_l s_l)^2 + (-B_l c_{o(l)} - B_l c_l + G_l s_l)^2 \le (f_l^{max})^2 \quad \forall l \in L$$

$$c_l^2 + s_l^2 \le c_{o(l)} c_{d(l)} \quad \forall l \in L$$

Relaxed ACOPF

For convenience, the above formulation is recast below.

The equivalent constraints (12)-(20) below are used to replace (9) and (11).

Relaxed ACOPF

$$(G_l c_{o(l)} + G_l c_l + B_l s_l)^2 + (-B_l c_{o(l)} - B_l c_l + G_l s_l)^2 \\ \leq (f_l^{max})^2 \quad \forall l \in L \qquad (9)$$

$$c_l^2 + s_l^2 \leq c_{o(l)} c_{d(l)} \quad \forall l \in L \qquad (11)$$

Relaxed ACOPF

$$M1_l = G_l c_{o(l)} + G_l c_l + B_l s_l \quad \forall l \in L \quad (\xi_l) \tag{12}$$
$$M2_l = -B_l c_{o(l)} - B_l c_l + G_l s_l \quad \forall l \in L \quad (\psi_l) \tag{13}$$
$$M3_l = f_l^{max} \quad \forall l \in L \quad (\zeta_l) \tag{14}$$
$$M1_l^2 + M2_l^2 \leq M3_l^2 \quad \forall l \in L \tag{15}$$
$$N1_l = 2c_l \quad \forall l \in L \quad (\alpha_l) \tag{16}$$
$$N2_l = 2s_l \quad \forall l \in L \quad (\beta_l) \tag{17}$$
$$N3_l = c_{o(l)} - c_{d(l)} \quad \forall l \in L \quad (\phi_l) \tag{18}$$
$$N4_l = c_{o(l)} + c_{d(l)} \quad \forall l \in L \quad (\varphi_l) \tag{19}$$
$$N1_l^2 + N2_l^2 + N3_l^2 \leq N4_l^2 \quad \forall l \in L \tag{20}$$

Relaxed ACOPF

To formulate a corrective security constrained OPF model, the ramping-up and ramping-down variations of each generator between the normal and the contingency states need to be limited.

This can be enforced as:

$$-R_j^D \leq p_j^G - p_{j,0}^G \leq R_j^U \quad \forall j \in J \quad \left(\underline{\lambda_j}, \overline{\lambda_j}\right) \quad (21)$$

Relaxed ACOPF: Dual

$$\max_{a,\gamma,\lambda,\kappa,\chi,\mu,\nu,\xi,\psi,\zeta,\alpha,\beta,\phi,\varphi,\delta} \sum_{j\in J}(\underline{p_j^G}\underline{\gamma_j} + \overline{p_j^G}\overline{\gamma_j} + (p_{j0}^G - R_j^D)\underline{\lambda_j} + (p_{j0}^G + R_j^U)\overline{\lambda_j} + \underline{q_j^G}\underline{\kappa_j} + \overline{q_j^G}\overline{\kappa_j})$$

$$+ \sum_{i\in I}(\underline{V_i^2}\underline{\chi_i} + \overline{V_i^2}\overline{\chi_i} + p_i^D\mu_i + q_i^D\nu_i) + \sum_{l\in L} f_l^{max}\zeta_l$$

s.t.

$$\underline{\gamma_j} + \overline{\gamma_j} + \underline{\lambda_j} + \overline{\lambda_j} + \mu_{i|j\in J_i} = C_j^G \quad \forall j \in J$$

$$\underline{\kappa_j} + \overline{\kappa_j} + \nu_{i|j\in J_i} = 0 \quad \forall j \in J$$

$$\mu_i + \delta_i = C_i^{UP} \quad \forall i \in I$$

$$\nu_i + \eta_i = C_i^{UQ} \quad \forall i \in I$$

Relaxed ACOPF: Dual

$$\underline{\chi_i} + \overline{\chi_i} - \sum_{l \in L} G_l \mu_{o(l)} - \sum_{l \in L} G_l \mu_{d(l)} + \sum_{l \in L} B_l \nu_{o(l)} + \sum_{l \in L} B_l \nu_{d(l)} + \sum_{l \in L} G_l \xi_l - \sum_{l \in L} B_l \psi_l$$

$$+ \sum_{l|o(l)=i} \phi_l - \sum_{l|d(l)=i} \phi_l + \sum_{l|o(l)=i} \varphi_l + \sum_{l|d(l)=i} \varphi_l \leq 0 \quad \forall i \in I$$

$$- G_l \mu_{o(l)} - G_l \mu_{d(l)} + B_l \nu_{o(l)} + B_l \nu_{d(l)} + G_l \xi_l - B_l \psi_l + 2\alpha_l = 0 \quad \forall l \in L$$

$$- B_l \mu_{o(l)} + B_l \mu_{d(l)} - G_l \nu_{o(l)} + G_l \nu_{d(l)} + B_l \xi_l + G_l \psi_l + 2\beta_l = 0 \quad \forall l \in L$$

$$\xi_l^2 + \psi_l^2 \leq \zeta_l^2 \quad \forall l \in L$$

$$\alpha_l^2 + \beta_l^2 + \phi_l^2 \leq \varphi_l^2 \quad \forall l \in L$$

Robust SC ACOPF

$$\max_{\mathbf{u}} \quad \min_{\mathbf{y}} \quad f(\mathbf{u},\mathbf{y})$$
$$\text{s.t.} \quad \mathbf{h}^{\mathrm{O}}(\mathbf{u},\mathbf{y}) = \mathbf{0}$$
$$\mathbf{g}^{\mathrm{O}}(\mathbf{u},\mathbf{y}) \leq \mathbf{0}$$
$$\mathbf{y} \in \mathcal{Y}$$
$$\text{s.t.} \quad \mathbf{u} \in \mathcal{U}$$

$$\max_{a \in \Omega} \min_{p \in P(a)} \sum_{j \in J} C_j^G p_j^G + \sum_{i \in I} C_i^{UP} p_i^U + \sum_{i \in I} C_i^{UQ} q_i^U \tag{22}$$

where

$$\Omega = \Big\{ a | a_l \in \{0,1\} \quad \forall l \in L \tag{23}$$

$$\sum_{l \in L} (1 - a_l) \leq k \Big\} \tag{24}$$

$$P(a) = \Big\{ p : (2) - (5)$$

$$\sum_{j \in J_i} p_j^G - \sum_{l|o(l)=i} a_l (G_l c_{o(l)} + G_l c_l + B_l s_l) - \sum_{l|d(l)=i} a_l$$
$$(G_l c_{d(l)} + G_l c_l - B_l s_l) + p_i^U = p_i^D \;\; \forall i \in I \quad \forall l \in L \quad (\mu_i) \tag{25}$$

$$\sum_{j \in J_i} q_j^G - \sum_{l|o(l)=i} a_l (-B_l c_{o(l)} - B_l c_l + G_l s_l) - \sum_{l|d(l)=i} a_l$$
$$(-B_l c_{d(l)} - B_l c_l - G_l s_l) + q_i^U = q_i^D \quad \forall i \in I \quad \forall l \in L \quad (\nu_i) \tag{26}$$

$$M1_l = a_l (G_l c_{o(l)} + G_l c_l + B_l s_l) \quad \forall l \in L \quad (\xi_l) \tag{27}$$

$$M2_l = a_l (-B_l c_{o(l)} - B_l c_l + G_l s_l) \quad \forall l \in L \quad (\psi_l) \tag{28}$$

$$(14) - (21) \Big\}$$

Robust SC ACOPF: Dual Version

$$\max_{\mathbf{u},\lambda,\mu,\nu} \quad f^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu)$$

s.t.

$$\mathbf{h}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) = \mathbf{0}$$

$$\mathbf{g}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \leq \mathbf{0}$$

$$\lambda,\mu,\nu \in \mathbf{\Omega}$$

$$\mathbf{u} \in \mathcal{U}$$

$$\max_{a,\gamma,\lambda,\kappa,\chi,\mu,\nu,\xi,\psi,\zeta,\alpha,\beta,\phi,\varphi,\delta} \sum_{j\in J}(\underline{p_j^G}\,\underline{\gamma_j} + \overline{p_j^G}\,\overline{\gamma_j} + (p_{j0}^G - R_j^D)\underline{\lambda_j} + (p_{j0}^G + R_j^U)\overline{\lambda_j} + \underline{q_j^G}\,\underline{\kappa_j} + \overline{q_j^G}\,\overline{\kappa_j}) + \sum_{i\in I}(\underline{V_i^2}\,\underline{\chi_i} + \overline{V_i^2}\,\overline{\chi_i} + p_i^D\mu_i + q_i^D\nu_i) + \sum_{l\in L} f_l^{max}\zeta_l \qquad (29)$$

s.t.

$$a_l \in \{0,1\} \quad \forall l \in L \qquad (30)$$

$$\sum_{l\in L}(1-a_l) \leq K \qquad (31)$$

$$\underline{\gamma_j} + \overline{\gamma_j} + \underline{\lambda_j} + \overline{\lambda_j} + \mu_{i|j\in J_i} = C_j^G \quad \forall j \in J \qquad (32)$$

$$\underline{\kappa_j} + \overline{\kappa_j} + \nu_{i|j\in J_i} = 0 \quad \forall j \in J \qquad (33)$$

$$\mu_i + \delta_i = C_i^{UP} \quad \forall i \in I \qquad (34)$$

$$\nu_i + \eta_i = C_i^{UQ} \quad \forall i \in I \qquad (35)$$

Robust SC ACOPF: Dual Version

$\max_{\mathbf{u},\lambda,\mu,\nu} \quad f^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu)$

s.t.

$\mathbf{h}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) = \mathbf{0}$

$\mathbf{g}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \leq \mathbf{0}$

$\lambda,\mu,\nu \in \mathbf{\Omega}$

$\mathbf{u} \in \mathcal{U}$

$$\underline{\chi_i} + \overline{\chi_i} - \sum_{l\in L} G_l t1_l - \sum_{l\in L} G_l t2_l + \sum_{l\in L} B_l t3_l + \sum_{l\in L} B_l t4_l$$
$$+ \sum_{l\in L} G_l t5_l - \sum_{l\in L} B_l t6_l + \sum_{l|o(l)=i} \phi_l - \sum_{l|d(l)=i} \phi_l +$$
$$\sum_{l|o(l)=i} \varphi_l + \sum_{l|d(l)=i} \varphi_l \leq 0 \quad \forall i \in I \tag{36}$$

$$- G_l t1_l - G_l t2_l + B_l t3_l + B_l t4_l + G_l t5_l - B_l t6_l + 2\alpha_l$$
$$= 0 \quad \forall l \in L \tag{37}$$

$$- B_l t1_l + B_l t2_l - G_l t3_l + G_l t4_l + B_l t5_l + G_l t6_l + 2\beta_l$$
$$= 0 \quad \forall l \in L \tag{38}$$

$$t1_l = \mu_{o(l)} - h1_l \quad \forall l \in L \tag{39}$$

$$\underline{\mu_{o(l)}} a_l \leq t1_l \leq \overline{\mu_{o(l)}} a_l \quad \forall l \in L \tag{40}$$

$$\underline{\mu_{o(l)}} (1 - a_l) \leq h1_l \leq \overline{\mu_{o(l)}} (1 - a_l) \quad \forall l \in L \tag{41}$$

$$t2_l = \mu_{d(l)} - h2_l \quad \forall l \in L \tag{42}$$

$$\underline{\mu_{d(l)}} a_l \leq t2_l \leq \overline{\mu_{d(l)}} a_l \quad \forall l \in L \tag{43}$$

$$\underline{\mu_{d(l)}} (1 - a_l) \leq h2_l \leq \overline{\mu_{d(l)}} (1 - a_l) \quad \forall l \in L \tag{44}$$

$$t3_l = \nu_{o(l)} - h3_l \quad \forall l \in L \tag{45}$$

Robust SC ACOPF: Dual Version

$$\max_{\mathbf{u},\lambda,\mu,\nu} \quad f^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu)$$
s.t.
$$\mathbf{h}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) = \mathbf{0}$$
$$\mathbf{g}^{\mathrm{D}}(\mathbf{u},\lambda,\mu,\nu) \leq \mathbf{0}$$
$$\lambda,\mu,\nu \in \mathbf{\Omega}$$
$$\mathbf{u} \in \mathcal{U}$$

$$\underline{\nu_{o(l)}} a_l \leq t3_l \leq \overline{\nu_{o(l)}} a_l \quad \forall l \in L \tag{46}$$
$$\underline{\nu_{o(l)}} (1 - a_l) \leq h3_l \leq \overline{\nu_{o(l)}} (1 - a_l) \quad \forall l \in L \tag{47}$$
$$t4_l = \nu_{d(l)} - h4_l \quad \forall l \in L \tag{48}$$
$$\underline{\nu_{d(l)}} a_l \leq t4_l \leq \overline{\nu_{d(l)}} a_l \quad \forall l \in L \tag{49}$$
$$\underline{\nu_{d(l)}} (1 - a_l) \leq h4_l \leq \overline{\nu_{d(l)}} (1 - a_l) \quad \forall l \in L \tag{50}$$
$$t5_l = \xi_l - h5_l \quad \forall l \in L \tag{51}$$
$$\underline{\xi_l} a_l \leq t5_l \leq \overline{\xi_l} a_l \quad \forall l \in L \tag{52}$$
$$\underline{\xi_l} (1 - a_l) \leq h5_l \leq \overline{\xi_l} (1 - a_l) \quad \forall l \in L \tag{53}$$
$$t6_l = \psi_l - h6_l \quad \forall l \in L \tag{54}$$
$$\underline{\psi_l} a_l \leq t6_l \leq \overline{\psi_l} a_l \quad \forall l \in L \tag{55}$$
$$\underline{\psi_l} (1 - a_l) \leq h6_l \leq \overline{\psi_l} (1 - a_l) \quad \forall l \in L \tag{56}$$
$$\xi_l^2 + \psi_l^2 \leq \zeta_l^2 \quad \forall l \in L \tag{57}$$
$$\alpha_l^2 + \beta_l^2 + \phi_l^2 \leq \varphi_l^2 \quad \forall l \in L \tag{58}$$

Nonlinearities!

Non-linearities originate from the product of binary and continuous variables.

However, these products are easily linearized using additional continuous variables

Once the worst contingency/contingencies is/are found…

Corrective Pre-dispatch

$$\min_{p,q,c,s,\theta} \sum_{j\in J} C_j^G p_j^G + \sum_{i\in I} C_i^{UP} p_i^U + \sum_{i\in I} C_i^{UQ} q_i^U \quad (59)$$

s.t.

$$(2)-(6) \quad (60)$$

$$\sum_{j\in J_i} p_j^G - \sum_{l|o(l)=i} A_l(G_l c_{o(l)} + G_l c_l + B_l s_l) - \sum_{l|d(l)=i} A_l \cdot (G_l c_{d(l)} + G_l c_l - B_l s_l) + p_i^U = p_i^D \quad \forall i \in I \quad \forall l \in L \quad (61)$$

$$\sum_{j\in J_i} q_j^G - \sum_{l|o(l)=i} A_l(-B_l c_{o(l)} - B_l c_l + G_l s_l) - \sum_{l|d(l)=i} A_l \cdot (-B_l c_{d(l)} - B_l c_l - G_l s_l) + q_i^U = q_i^D \quad \forall i \in I \quad \forall l \in L \quad (62)$$

$$M1_l = A_l(G_l c_{o(l)} + G_l c_l + B_l s_l) \quad \forall l \in L \quad (63)$$

$$M2_l = A_l(-B_l c_{o(l)} - B_l c_l + G_l s_l) \quad \forall l \in L \quad (64)$$

$$(14)-(20) \quad (65)$$

$$-R_j^D \le p_j^G - p_{j,0}^G \le R_j^U \quad \forall j \in J \quad (66)$$

Preventive Pre-dispatch

$$\min_{p,q,c,s,\theta} \sum_{j \in J} C_j^G p_j^G + \sum_{i \in I} C_i^{UP} p_i^U + \sum_{i \in I} C_i^{UQ} q_i^U \tag{67}$$

s.t.

$$(2) - (6) \tag{68}$$

$$\sum_{j \in J_i} p_j^G - \sum_{l|o(l)=i} (G_l c_{o(l)} + G_l c_l + B_l s_l) - \sum_{l|d(l)=i} (G_l c_{d(l)} + G_l c_l - B_l s_l) + p_i^U = p_i^D \quad \forall i \in I \quad \forall l \in L \tag{69}$$

$$\sum_{j \in J_i} q_j^G - \sum_{l|o(l)=i} (-B_l c_{o(l)} - B_l c_l + G_l s_l) - \sum_{l|d(l)=i} (-B_l c_{d(l)} - B_l c_l - G_l s_l) + q_i^U = q_i^D \quad \forall i \in I \quad \forall l \in L \tag{70}$$

Preventive Pre-dispatch

$$\sum_{j \in J_i} p_j^G - \sum_{l|o(l)=i} A_l(G_l c_{o(l)}^{(1)} + G_l c_l^{(1)} + B_l s_l^{(1)}) -$$
$$\sum_{l|d(l)=i} A_l(G_l c_{d(l)}^{(1)} + G_l c_l^{(1)} - B_l s_l^{(1)}) + p_i^U = p_i^D$$
$$\forall i \in I \quad \forall l \in L \tag{71}$$

$$\sum_{j \in J_i} q_j^G - \sum_{l|o(l)=i} A_l(-B_l c_{o(l)}^{(1)} - B_l c_l^{(1)} + G_l s_l^{(1)}) -$$
$$\sum_{l|d(l)=i} A_l(-B_l c_{d(l)}^{(1)} - B_l c_l^{(1)} - G_l s_l^{(1)}) + q_i^U = q_i^D$$
$$\forall i \in I \quad \forall l \in L \tag{72}$$

$$M1_l = A_l(G_l c_{o(l)} + G_l c_l + B_l s_l) \quad \forall l \in L \tag{73}$$

$$M2_l = A_l(-B_l c_{o(l)} - B_l c_l + G_l s_l) \quad \forall l \in L \tag{74}$$

Preventive Pre-dispatch

$$(14) - (20) \tag{75}$$

$$M4_l = G_l c_{o(l)}^{(1)} + G_l c_l^{(1)} + B_l s_l^{(1)} \quad \forall l \in L \tag{76}$$

$$M5_l = -B_l c_{o(l)}^{(1)} - B_l c_l^{(1)} + G_l s_l^{(1)} \quad \forall l \in L \tag{77}$$

$$M6_l = f_l^{max} \quad \forall l \in L \tag{78}$$

$$M4_l^2 + M5_l^2 \leq M6_l^2 \quad \forall l \in L \tag{79}$$

$$N5_l = 2c_l^{(1)} \quad \forall l \in L \tag{80}$$

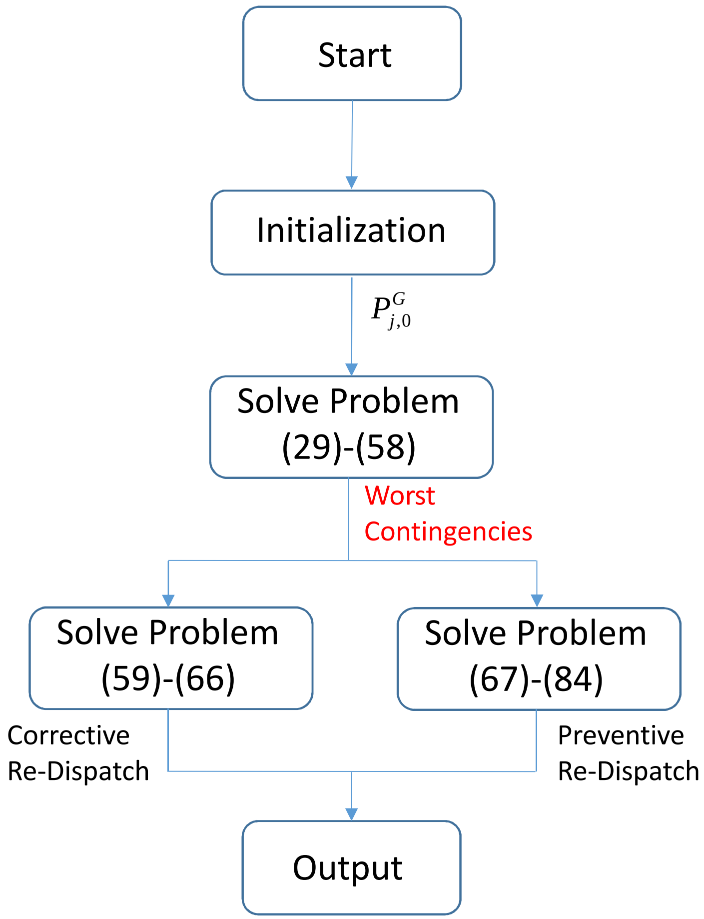
$$N6_l = 2s_l^{(1)} \quad \forall l \in L \tag{81}$$

$$N7_l = c_{o(l)}^{(1)} - c_{d(l)}^{(1)} \quad \forall l \in L \tag{82}$$

$$N8_l = c_{o(l)}^{(1)} + c_{d(l)}^{(1)} \quad \forall l \in L \tag{83}$$

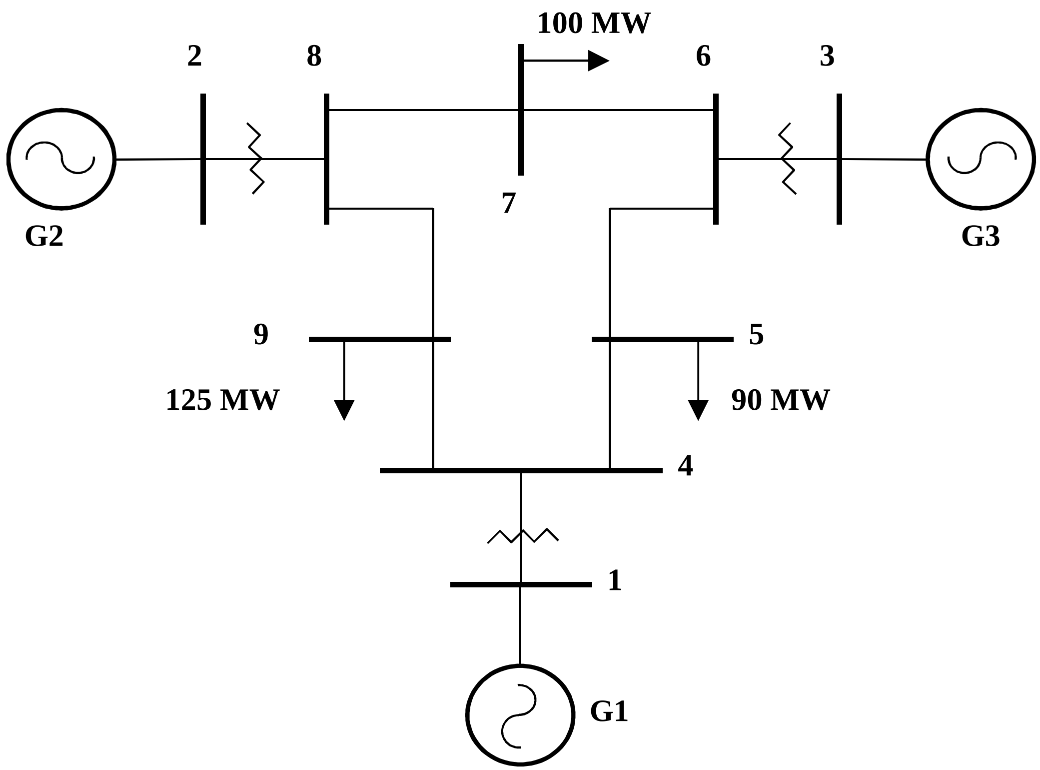
$$N5_l^2 + N6_l^2 + N7_l^2 \leq N8_l^2 \quad \forall l \in L \tag{84}$$

How to proceed?



Example

Data



Example

Data

Table 1: Data for generators and lines

Generator	Cost ($/MWh)	Capacity (MW)	Line	Line Capacity (MVA)
G1	5	250	1-4	375
G2	1.2	300	4-5	375
G3	1	270	5-6	225
			3-6	450
			6-7	225
			7-8	375
			8-2	375
			8-9	375
			9-4	375

Example

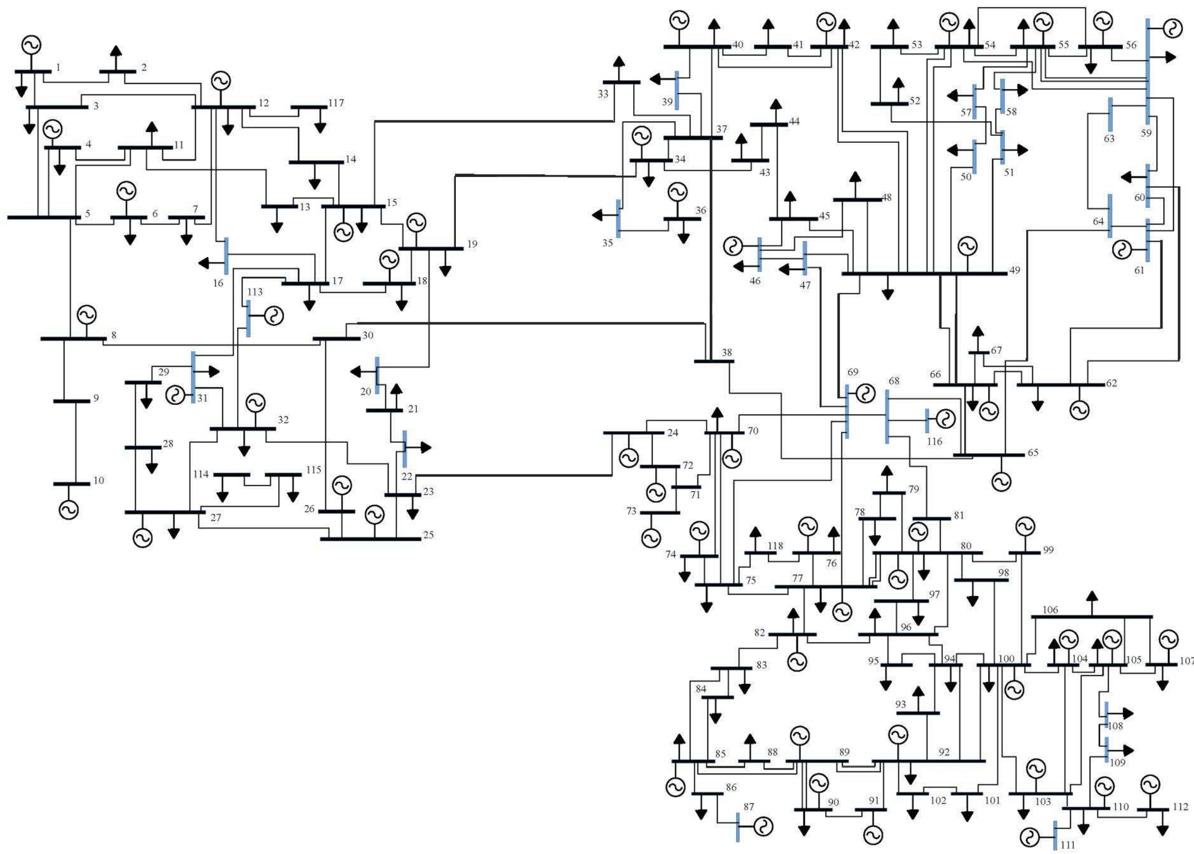
Results

Table 2: WSCC 9-Bus System Results

Normal Dispatch			
Cost	P_{G1}	P_{G2}	P_{G3}
374.34	10	45.28	270
Corrective Dispatch			
Cost	P_{G1}	P_{G2}	P_{G3}
399.88	10	107.34	221.07
Preventive Dispatch			
Cost	P_{G1}	P_{G2}	P_{G3}
402.22	10	106.13	224.86

Case Study

Data



Case Study

Results

Table 3: IEEE 118-Bus System Single-Contingency Results

Generator	Normal Dispatch (MW)	Corrective Re-Dispatch (MW)	Preventive Re-Dispatch (MW)
$G5$	211	211	210
$G6$	370	370	370
$G11$	163	158	149
$G12$	179	182	167
$G14$	199	199	198
$G20$	90	90	91
$G21$	319	319	324
$G22$	296	296	296
$G25$	305	305	305
$G26$	142	143	143
$G28$	126	133	130
$G29$	139	131	135
$G30$	410	381	384
$G37$	479	475	451
$G39$	28	27	27
$G40$	354	355	354
$G45$	267	265	264
$G46$	148	148	148
$G51$	79	79	79
Cost	\$86,063	\$86,753	\$87,654

Case Study

Results

Table 4: IEEE 118-Bus System Double-Contingency Results

Generator	Normal Dispatch (MW)	Corrective Re-Dispatch (MW)	Preventive Re-Dispatch (MW)
$G5$	211	211	211
$G6$	370	370	370
$G11$	163	163	159
$G12$	179	178	185
$G14$	199	199	200
$G20$	90	90	90
$G21$	319	319	319
$G22$	296	296	296
$G25$	305	305	305
$G26$	142	143	143
$G28$	126	133	129
$G29$	139	130	137
$G30$	410	402	416
$G37$	479	378	341
$G39$	28	28	28
$G40$	354	353	359
$G45$	267	266	274
$G46$	148	148	151
$G51$	79	79	79

Case Study

Results

Table 4: IEEE 118-Bus System Double-Contingency Results

Generator	Normal Dispatch (MW)	Corrective Re-Dispatch (MW)	Preventive Re-Dispatch (MW)
Total Generation	4,303	4,191	4,193
Unserved Real Power	0	110	110
Generation Cost	\$86,063	\$83,818	\$83,853
Unserved Real Power Cost	-	$\$1.1 \times 10^5$	$\$1.1 \times 10^5$
Total Cost	\$86,063	$\$1.94 \times 10^5$	$\$1.94 \times 10^5$

Summary & Conclusions

A robust AC SCOPF bi-level conic formulation is presented to retain AC constraints.

Binary variables are used to represent the impact of contingencies.

A bi-level max-min optimization model is developed to find the worst contingencies.

Conic duality is used to convert the bi-level problem into a solvable single-level problem.

Summary & Conclusions

The solution of this model identifies the worst contingencies, which are used to determine either a corrective or a preventive generation dispatch.

This novel technique requires a reasonable computation time.

The ACOPF relaxation is only used to identify the worst contingencies.

Thank you!