Virtual Inertia and Active Power Response

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Rate of Change of Power

Power Flow Equations – I

- Let's start from the well-knonw power flow equations:

$$\bar{\boldsymbol{s}}_{\mathrm{B}}(t) = \boldsymbol{p}_{\mathrm{B}}(t) + j\boldsymbol{q}_{\mathrm{B}}(t) = \bar{\boldsymbol{v}}_{\mathrm{B}}(t) \circ \left[\bar{\mathbf{Y}}_{\mathrm{bus}}\, \bar{\boldsymbol{v}}_{\mathrm{B}}(t)\right]^{*} , \tag{1}$$

- For the sake of the derivation, it is convenient to rewrite (1) in an element-wise notation and extract the active power:

$$\begin{aligned} p_{\mathrm{B},h}(t) = v_{\mathrm{B},h}(t) \sum_{k \in \mathbb{B}} v_{\mathrm{B},k}(t)\, G_{\mathrm{bus}}^{hk} \cos\theta_{\mathrm{B},hk}(t) \\ + v_{\mathrm{B},h}(t) \sum_{k \in \mathbb{B}} v_{\mathrm{B},k}(t)\, B_{\mathrm{bus}}^{hk} \sin\theta_{\mathrm{B},hk}(t) , \end{aligned} \tag{2}$$

Power Flow Equations – II

- Let us differentiate (3) and write the active power injections as the sum of two components:

$$
\begin{aligned}
dp_{\mathrm{B},h} &= \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}} \, d\theta_{\mathrm{B},k} + \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial v_{\mathrm{B},k}} \, dv_{\mathrm{B},k} \\
&= dp'_{\mathrm{B},h} + dp''_{\mathrm{B},h} \,,
\end{aligned}
\tag{3}
$$

- In (3), $dp_{\mathrm{B},h}$ is the total variation of power at bus h, while $dp'_{\mathrm{B},h}$ is what, in the following, we will call “regulating active power”
- The other component, $dp''_{\mathrm{B},h}$, is the “passive” component of the active power.

Simplifications

- The exact expression of the regulating power is thus:

$$dp'_{\mathrm{B},h} = \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}} \, d\theta_{\mathrm{B},k} \tag{4}$$

- This expression can be conveniently simplified by assuming that, in $\frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}}$:
  - voltage magnitudes are ≈ 1;
  - line resistances are negligible; and
  - $\cos(\theta_{\mathrm{B},h} - \theta_{\mathrm{B},k}) \approx 1$.

Link between Regulating Power and Frequency

- Recovering the vector notation, equation (4) can be thus approximated (without loss of accuracy) as:

$$\boldsymbol{p}'_{\mathrm{B}}(t) = -\mathbf{B}_{\mathrm{bus}}\boldsymbol{\theta}_{\mathrm{B}}(t)\,, \tag{5}$$

- Then, differentiating (5) with respect to time gives the most important equation of this presentation, namely:

$$\boxed{\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) = -\Omega_b\mathbf{B}_{\mathrm{bus}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t) = -\hat{\mathbf{B}}_{\mathrm{bus}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)} \tag{6}$$

- In (6), $\dot{\boldsymbol{p}}'_{\mathrm{B}}(t)$ is the rate of change of (regulating) power or RoCoP, and $\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)$ are the variations of frequency at network buses.

Applications?

- So, the first question that one may ask is: Why would we even need the definition of the RoCoP?
- The answer relies on the ability to define, for each component of the grid, an expression of $\dot{p}'_{\mathrm{B},h}$.
- Fortunately, there are some very relevant cases for which one can determine $\dot{p}'_{\mathrm{B},h}$ analytically (we will discuss these cases next).
- In all other cases, we need to rely on measurements of bus frequencies to deduce $\dot{p}'_{\mathrm{B},h}$ (we will see an application at the end of this presentation).

Two (Very) Relevant Cases – I

- **Constant Admittance Loads**: for this kind of loads, one can show that:

$$\dot{p}'_{\mathrm{B},h} \equiv 0\,, \tag{7}$$

which, incidentally, implies $\dot{p}_{\mathrm{B},h} = \dot{p}''_{\mathrm{B},h}$ (fully passive device!) or, equivalently, **constant admittances do not modify the frequency**.

- Note that, for all other types of loads, $\dot{p}'_{\mathrm{B},h} \neq 0$, which means that loads that are not pure admittances **do** modify the freqeuncy at their point of connection.

- In other words, non-constant admittance loads imply some sort of *regulation*.

Two (Very) Relevant Cases – II

- **Synchronous Machines**: In this case it is possible to show that:

$$\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) \approx -\Omega_b[\mathbf{B}_{\mathrm{BG}}\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) + \mathbf{B}_{\mathrm{G}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)] \,, \tag{8}$$

where $\mathbf{B}_{\mathrm{BG}}$ and $\mathbf{B}_{\mathrm{G}}$ and the incidence susceptance matrices formed with the internal reactances of teh machines and $\Delta\boldsymbol{\omega}_{\mathrm{G}}(t)$ are the machine rotor speeds.

- Equation (8) can be rewritten as:

$$\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) = -\hat{\mathbf{B}}_{\mathrm{BG}}\left[\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) - \Delta\boldsymbol{\omega}_{\mathrm{BG}}(t)\right] \,, \tag{9}$$

- Putting together (6) and (9), we obtain another very relevant expression:

$$\boxed{\mathbf{B}_{\mathrm{BG}}\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) = -\mathbf{B}_{\mathrm{BB}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)} \,, \tag{10}$$

where $\mathbf{B}_{\mathrm{BB}} = \mathbf{B}_{\mathrm{bus}} + \mathbf{B}_{\mathrm{G}}$.

Relevant Byproduct: Estimation of the Inertia

- A byproduct of the definition of RoCoP is the ability to estimate the *equivalent* inertia that a device connected to the grid provides to the grid:

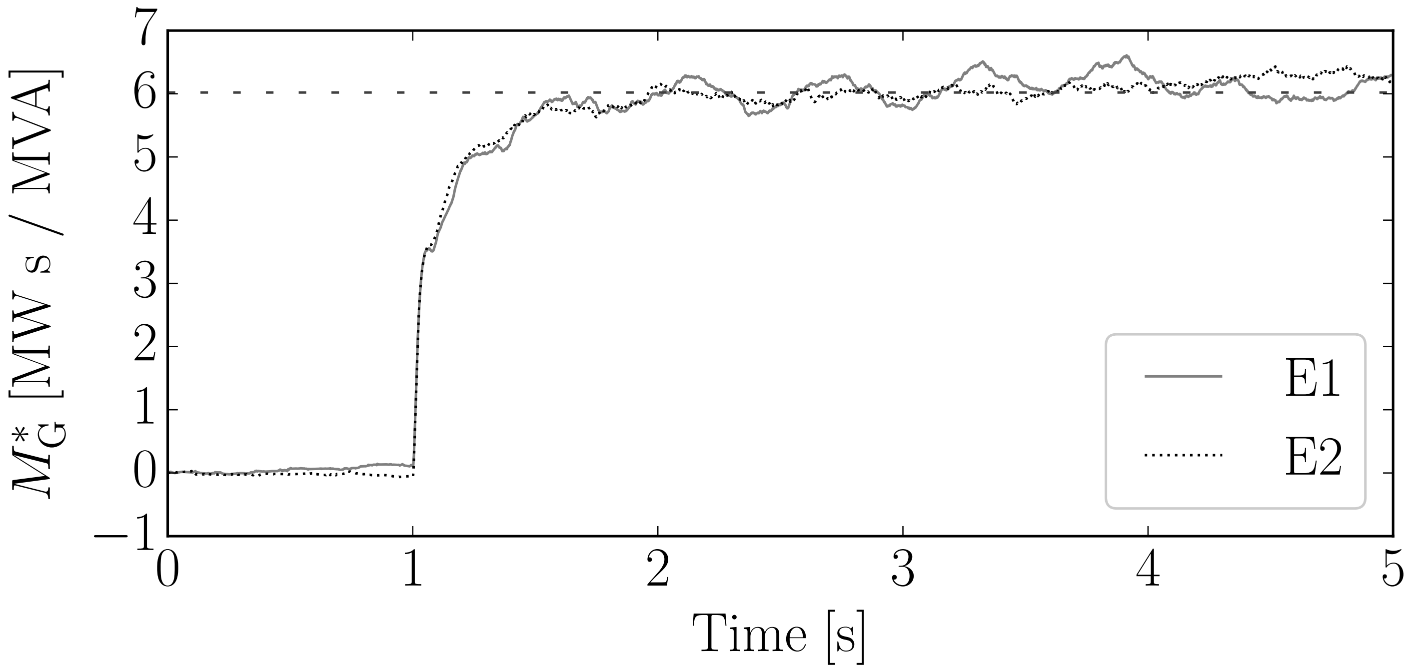
$$\boxed{M_{\blacksquare,h}(t) \approx \frac{-\dot{p}'_{\mathrm{B},h}(t)}{\frac{d^2}{dt^2}\left[\Delta\omega_{\mathrm{B},h}(t) - \hat{x}_{\blacksquare,h}\dot{p}'_{\mathrm{B},h}(t)\right]}, \text{ for } t < t^*}, \tag{11}$$

- A similar expression can be obtained to estimated the gain of the droop control of devices regulating the frequency.

Examples

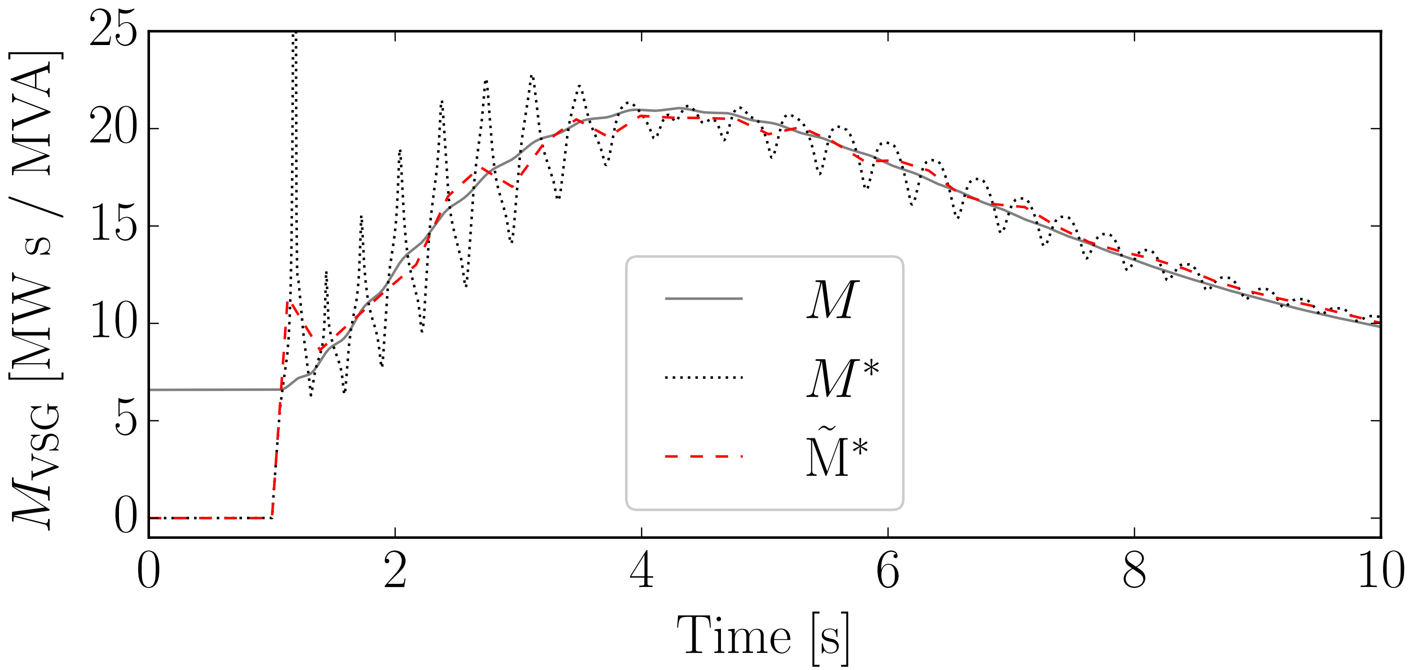
Estimation of the Inertia of a Synchronous Machine

- Transient behavior of the estimator for a synchronous machine.



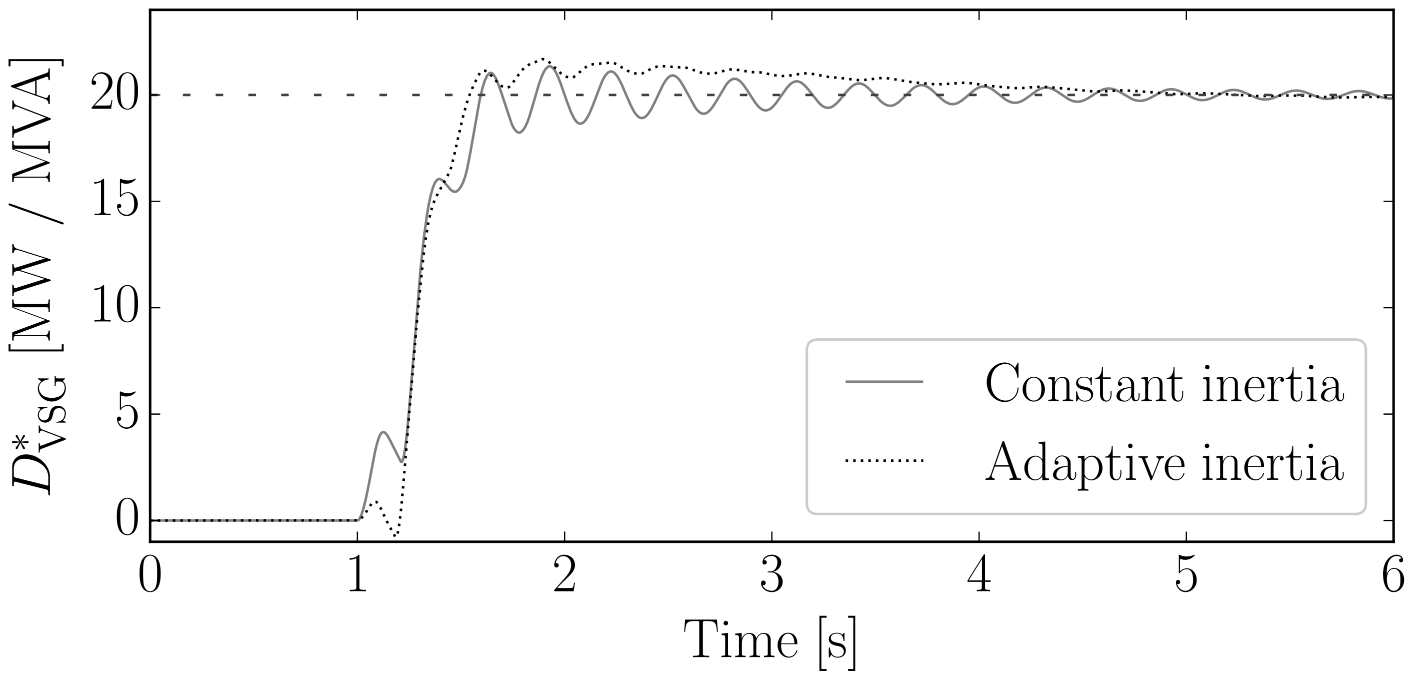
Estimation of the Inertia of a Virtual Synchronous Machine

- Transient behavior of the estimator for a virtual synchronous machine with variable inertia.



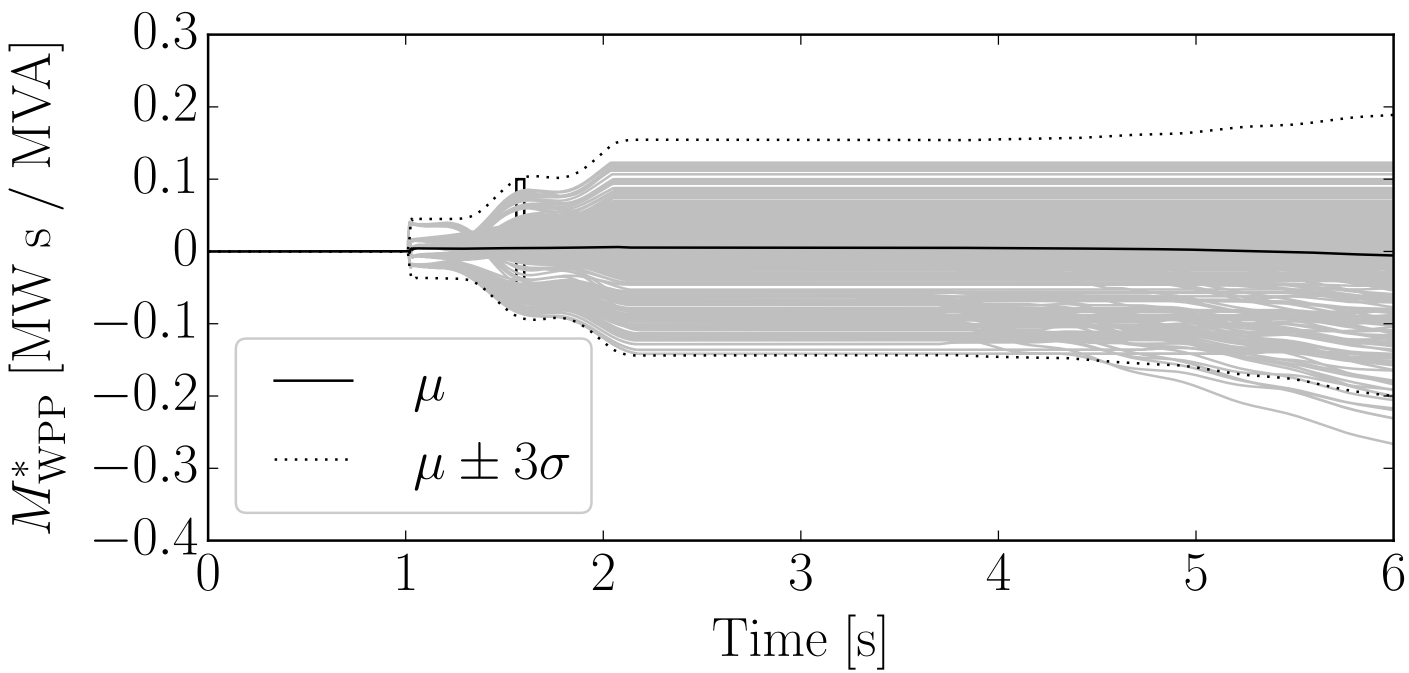
Estimation of the Droop of a Virtual Synchronous Machine

- Transient behavior of the estimator for a virtual synchronous machine with constant droop.



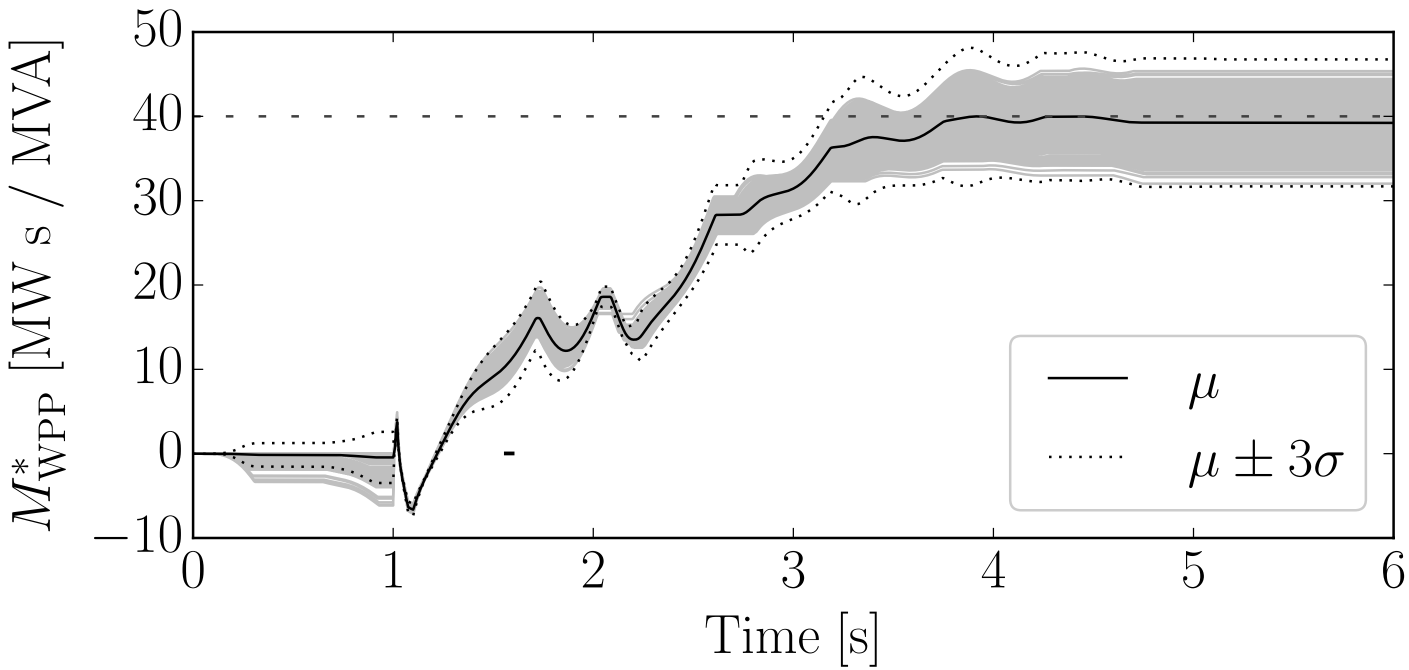
Estimation of the Inertia of a Wind Power Plant

- Transient behavior of the estimator for a wind power plant.



Estimation of the Inertia of a Wind Power Plant + Energy Storage

- Transient behavior of the estimator for a wind power plant coupled with an energy storage system that regulates the frequency.



Conclusions and References

Conclusing Remarks

- A general expression to estimate frequency variations during the transient of electric power systems has been deduced.
- The proposed expression is derived based on standard assumptions of power system models for transient stability analysis and can be readily implemented in power system software tools for transient stability analysis.
- The formula allows estimating whether a device is providing freqeuncy regulation or not.
- A by product of the formula is the ability to estimate the inertia and the droop of the devices.

References

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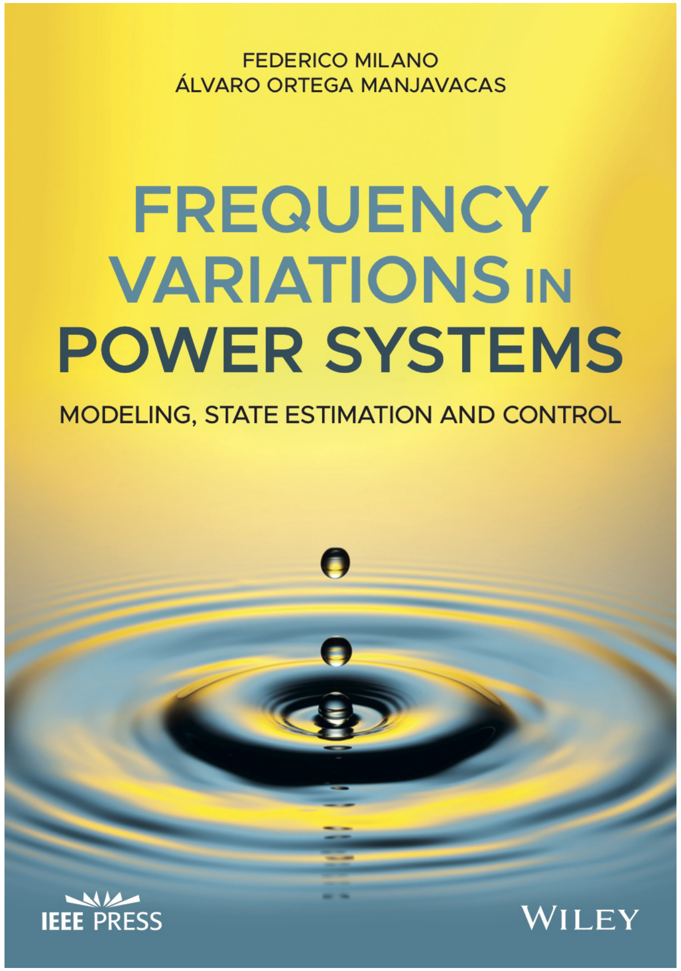
Monograph just published by Wiley on July 2020!



Thanks much for your attention!