On the Impact of Auto-Correlation of Stochastic Processes on the Transient Behavior of Power Systems

Muhammad Adeen, *Member, IEEE,* and Federico Milano, *Fellow, IEEE*

***Abstract*—This letter studies the impact of auto-correlation of stochastic processes on the dynamic response of power systems. The frequency spectrum of the trajectories of the state variables of the system is utilized as a metric to evaluate this impact. The case study considers the well-known two-area system as well as with a 1479-bus dynamic model of the all-island Irish transmission system. Simulation results indicate that the auto-correlation have a direct impact on the amplitude of the dominant electro-mechanical modes of the system. Results also show that, for a wide range of the values of the auto-correlation, the impact of stochastic processes on system dynamics is local, affects differently each area of the system and, in some cases, can lead to instability and voltage collapse.**

***Index Terms*—Stochastic differential algebraic equations (SDAE), power system dynamics, Ornstein-Uhlenbeck's process, auto-correlation, electro-mechanical oscillations.**

I. Introduction

Most modern power systems include high shares of converter-interfaced renewable energy sources, which are stochastic, e.g., wind and solar. Moreover, the load consumption is not fully deterministic in the time-scale of transient stability analysis [1]–[3]. These elements introduce randomness into the systems. In dynamic studies, this randomness can be conveniently modeled by means of stochastic differential equations (SDEs). An SDE consists of two terms: the *drift* and the *diffusion* [4]. The diffusion term defines the amplitude of the noise, i.e., its standard deviation. The dynamic interaction between the drift and the diffusion terms defines the auto-correlation of the process, i.e. how the process evolves in the long term.

If the SDEs that describe the stochastic processes are combined with the conventional differential-algebraic equations that describe the transient stability model of power systems, the result is a set of stochastic differential-algebraic equations (SDAEs) [5]–[7]. This is the model considered in the letter.

A fair number of works is available on the impact of the diffusion term on the stability of power systems, e.g. [8]–[10]. However, the literature is either inadequate or silent on the topic of auto-correlation of stochastic processes. An exception is [11], where the authors exploit the property of the auto-correlation to initialize the SDAEs that model the system. This letter focuses on another feature of the auto-correlation, i.e. the dynamic coupling between the drift of stochastic processes and the electro-mechanical modes of the systems.

II. Modeling

Power systems subject to random disturbances and noise are conveniently modeled as a set of SDAEs [7], as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}) \,, \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}) \,, \\ \dot{\boldsymbol{\kappa}} &= \boldsymbol{a}(\boldsymbol{\kappa}) + \boldsymbol{b}(\boldsymbol{\kappa}) \circ \boldsymbol{\xi} \,, \end{aligned} \tag{1}$$

Equations (1) model the deterministic part of the transient behavior of a power system. $\boldsymbol{f} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^m$ are the differential equations; $\boldsymbol{g} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^l$ are the algebraic equations; $\boldsymbol{x} \in \mathbb{R}^l$ is a vector of state variables; $\boldsymbol{y} \in \mathbb{R}^m$ is a vector of algebraic variables; and $\boldsymbol{\kappa} \in \mathbb{R}^n$ represents the vector of uncorrelated stochastic processes; $\boldsymbol{\xi} \in \mathbb{R}^n$ is a vector of n-dimensional independent *Gaussian white noise*, which is the formal representation of time derivative of the Wiener process; and $\circ$ represents the Hadamard product, i.e. the element-by-element product of two vectors. Stochastic processes feature two terms: the *drift*, $\boldsymbol{a} : \mathbb{R}^n \mapsto \mathbb{R}^n$, and the *diffusion*, $\boldsymbol{b} : \mathbb{R}^n \mapsto \mathbb{R}^n$. If the drift is a vector linear functions, e.g. $\boldsymbol{a}(\boldsymbol{\kappa}) = \boldsymbol{\alpha} \circ \boldsymbol{\kappa}$, the elements of the vector $\boldsymbol{\alpha}$ are called *auto-correlation coefficients*.

A. Ornstein-Uhlenbeck's Process

In the remainder of this work, we assume that $\boldsymbol{\kappa}$ in (1) are described by Ornstein-Uhlenbeck's processes (OUPs). This assumption allows simplifying the discussion of the case studies but does not impact on the generality of the conclusions. The OUP is a stochastic mean-reverting process that follows Gaussian distribution and has a bounded standard deviation. These features make the OUP adequate to model the volatility in physical processes such as stochastic load dynamics [2], [3], [12] and short-term wind fluctuations [13]–[15]. A OUP is defined as follows:

$$\dot{\kappa} = -\alpha(\kappa - \mu) + \beta\xi \tag{2}$$

where α is the auto-correlation coefficient; β is the coefficient of the diffusion term; μ is the mean value; and ξ is the white noise. κ is a real-valued process following a Gaussian probability distribution given by $\mathcal{N}(\mu, \sigma)$, where σ is the standard deviation, and $\beta = \sigma\sqrt{2\alpha}$.

In this letter, we are concerned with the impact of the auto-correlation on power system dynamics. It is thus relevant to illustrate first the effect of the auto-correlation coefficient α on the dynamic response of κ of a OUP. Figure 1 shows three realizations of (2), obtained for $\mu = 0$, $\sigma = 0.1$ and different values of α.

It is important to note that the three processes shown in Fig. 1 all have the same probability distribution in stationary condition. In fact, the probability distribution function of (2) is $P(\kappa) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\kappa-\mu}{\sigma}\right)^2}$ and does not depend on α. However, their dynamic behavior is significantly different because of the different value of α and, hence, of their auto-correlation coefficient. This can be observed in Fig. 1: the higher the value of α, the faster the variations in the stochastic processes in the unit of time.

Formally, the auto-correlation function measures the dependence of present values, of a given time series, on the past values, of the

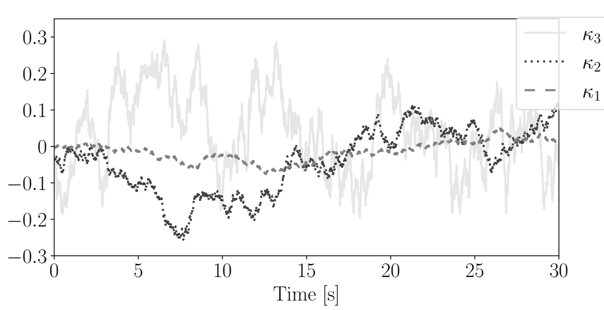

Fig. 1. Time domain profile of three Ornstein-Uhlenbeck's processes with $\mu = 0$, $\sigma = 0.1$, and $\alpha_1 = 0.01$ s^{-1}, $\alpha_2 = 0.1$ s^{-1}, and $\alpha_3 = 1$ s^{-1}.

M. Adeen and F. Milano are with School of Electrical and Electronic Engineering, University College Dublin, Belfield Campus, Dublin 4, Ireland. E-mails: muhammad.adeen@ucdconnect.ie, federico.milano@ucd.ie

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same time series, as a function of time lag:

$$R(\tau) = \frac{E[(\kappa_t - \mu)(\kappa_{t+\tau} - \mu)]}{\sigma^2} \quad (3)$$

where E is the expectation operator; κ_t is the value of the process at time t; and τ is the time lag.

Figure 2 illustrates the auto-correlation, calculated using (3), of the OUPs shown in Figure 1. The value of auto-correlation function is always equal to 1 for $\tau = 0$, by definition. For OUPs, as τ increases the correlation between current and future values decreases exponentially and decreases the faster the higher the value of α. In fact, the analytical expression of auto-correlation function of a OUP is given as $R(\tau) = e^{-\alpha\tau}$. This exponentially decaying auto-correlation is observed in several physical processes such as stochastic load dynamics [2], [12], and wind fluctuations [13], [14].

An effective way to differentiate stochastic processes having same probability distribution but different auto-correlation coefficient is offered by the frequency spectrum of the time series obtained by Fourier Transform. This approach is conceptually similar to the signal probing technique, e.g., [16], [17], which utilizes a Fourier analysis of measurement data to determine the frequency, damping, and participation factors associated with the inter-area oscillatory modes of the power system. Figure 3 illustrates the frequency spectrum of the time series observed in Fig. 1. Figure 3 shows that the higher the value of α, the bigger the amplitudes of the frequencies of which a OUP is composed. Thus, the amplitudes of the frequencies, of which a OUP is composed, is directly proportional to the auto-correlation coefficient.

This letter investigates whether the noise, modeled as a set of OUPs, can trigger the electro-mechanical modes of the power system and hence, modify its dynamic response. With this aim, we first identify the electro-mechanical modes by calculating the dominant eigenvalues and their participation factors. Then, the spectrum of relevant variables of the system is analyzed to quantify the impact of the auto-correlation coefficient of stochastic processes on the overall system dynamic response.

III. Case Study

Two systems are considered, namely the well-known Kundur's two-area system and a dynamic model of the All-Island Irish Transmission System (AIITS). In all simulations, the realizations of the

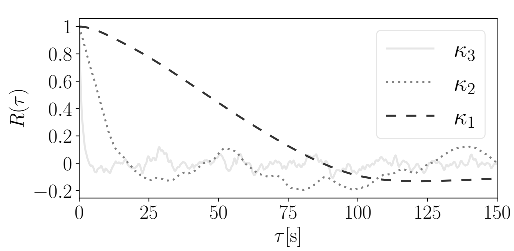

Fig. 2. Exponentially decaying auto-correlations of Ornstein-Uhlenbeck's processes for $\sigma = 0.1$, and $\alpha_1 = 0.01\ \text{s}^{-1}$, $\alpha_2 = 0.1\ \text{s}^{-1}$, and $\alpha_3 = 1\ \text{s}^{-1}$.

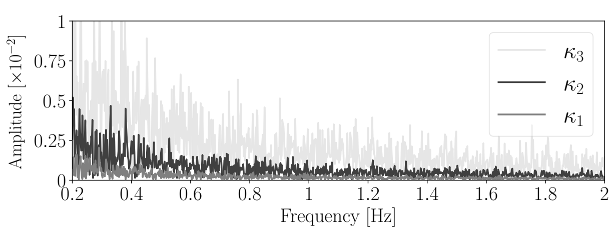

Fig. 3. Frequency spectrum of realizations of Ornstein-Uhlenbeck's processes for $\sigma = 0.1$, and $\alpha_1 = 0.01\ \text{s}^{-1}$, $\alpha_2 = 0.1\ \text{s}^{-1}$, and $\alpha_3 = 1\ \text{s}^{-1}$.

Wiener processes are integrated with the Maruyama-Euler scheme and a step size of $h = 0.01$ s [4], whereas the implicit trapezoidal method is utilized for the integration of the deterministic part with a step length $t = 0.01$ s (see also [7]). The processes are simulated for a total simulation time of 250 s, since a OUP reaches stationarity at $t_f = 2/\alpha$ [11]. All simulations are carried out using the software tool Dome [18].

Three scenarios where stochastic processes are characterized by low-, medium- and high-speed exponentially-decaying auto-correlations, respectively, are defined as follows:

- S1: $\alpha = 0.01\ \text{s}^{-1}$.
- S2: $\alpha = 0.1\ \text{s}^{-1}$.
- S3: $\alpha = 1\ \text{s}^{-1}$.

The values above are in the range of real-world stochastic processes that are found in power systems.

A. Two-Area System

The two-area system in [19] is simulated using the stochastic model described in [7]. Noise is modeled as OUPs and included in the load consumption. This model takes into account all sources of noise at the distribution level, including renewable generation and load fluctuations. The impact of the noise on the dynamic response of the system is studied considering each area independently.

1) Stochastic Loads only in Area 1: The dominant electro-mechanical modes of the system along with the participation factors of the machines after introducing noise in Area 1 are shown in Table I. These modes are calculated as a result of including stochastic load power variations in Area 1 through OUPs as explained in [7]. For each scenario, the OUPs used to model the noise have same frequency spectrum as shown in Fig. 3. The standard deviation is set to $\sigma = 1\%$ of the mean load value for all scenarios.

The active power injections p_g of synchronous generators G1 and G3 are shown in Fig. 4. The oscillations of these generators are higher the higher the value of α of the noise included in the load consumption. Figure 5 illustrates the frequency spectrum of p_g of the synchronous generators included in the two-area system. By comparing the frequencies of the dominant electro-mechanical modes shown in Table I with the frequency spectrum shown in Fig. 5, it is clear that an increase in α causes an increase in the amplitude of the oscillations in p_g. Note also that the frequency spectrum of p_g shows well the coupling of the oscillatory modes of the two-area system with the values of α.

The amplitude of the oscillations also depends on the participation factors of the machines. This is particularly evident for mode 1, which is the inter-area oscillatory mode and shows significant participation from all the generators. Hence, the amplitude of the inter-area oscillatory mode observed in all the generators is proportional to their participation factors. This behavior can be verified by observing the participation factors of generators in modes 2 and 3. Since mode 2 has significant participation from G1 and G2, and provided that the noise originates in Area 1, negligible oscillations are observed in mode 2 in the generators G3 and G4, located in Area 2. Whereas, mode 3 has significant participation from G3 and G4. Hence, negligible

TABLE I
ELECTRO-MECHANICAL MODES AND CORRESPONDING PARTICIPATION FACTORS OF THE TWO-AREA SYSTEM WITH STOCHASTIC LOAD IN AREA 1

Mode	Eigenvalue	Freq. [Hz]	Participation Factors			
			G1	G2	G3	G4
1	-0.063±j3.866	0.615	19.05	11.01	34.86	21.85
2	-0.300±j7.112	1.132	42.07	52.93	1.63	1.25
3	-0.300±j7.392	1.176	1.02	1.47	37.7	57.52

oscillations are observed in mode 3 from all the generators. This is due, again, to the fact that the noise is located in Area 1.

Next the impact of the auto-correlation coefficient on the stability of power system is analyzed with a Monte Carlo method. With this aim, 1000 time domain simulations are carried out. The results of these simulations are presented in Table II, which indicates that none of the trajectories were found to be unstable for the three scenarios.

TABLE II
UNSTABLE TRAJECTORIES FOR THE TWO-AREA SYSTEM

Scenario	Stoch. Procs. in Area 1 Unstable trajectories	Stoch. Procs. in Area 2 Unstable trajectories
S1	0	0
S2	0	0
S3	0	521 (52.1%)

2) Stochastic Loads only in Area 2: The dominant electro-mechanical modes of the system along with the participation factors of the machines after introducing noise in Area 2 are shown in Table III. The parameters of the stochastic loads are the same as those utilized in the example above except for the standard deviation that is set to $\sigma = 0.5\%$ of the mean load consumption.

Figure 6 illustrates p_g of synchronous generators G2 and G4 in time domain. Results show that generator G4, which belongs to Area 2, shows higher amplitude oscillations as compared to generator G1, which belongs to Area 1. The rationale of this result is given by the participation of the generators to the inter-area mode (see Table III). The frequency spectrum of p_g of all synchronous generators is shown in Fig. 7. Results are consistent with those discussed in the example above, i.e. the higher the α the higher the oscillations observed in the generators of Area 2. Note that, even though modes 2 and 3 have similar frequency, since the sources of noise are in Area 2, only the p_g of machines G3 and G4 show a relevant increase in the amplitude of the frequency of mode 3.

Finally the effect on the stability of the two-area system of auto-correlation coefficient of stochastic processes included in Area 2 is analyzed by simulating 1000 Monte Carlo simulations. The trajectories of p_g and v are observed and the results for unstable trajectories are presented in Table II. Table II shows that 52.1% of trajectories are unstable for scenario S3. For illustration purposes, a few trajectories of voltage magnitude at bus 8 for unstable cases are shown in Fig. 8. It is important to note that, for all scenarios, the standard distribution of the processes is kept the same while the auto-correlation coefficient of the processes is varied. This implies that sufficiently high values of the auto-correlation coefficient of the stochastic processes may drive a system to instability, which may end up in a voltage collapse.

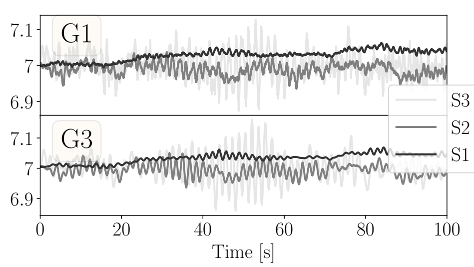

Fig. 4. Active power p_g generation of the synchronous generators G1 and G3 of the two-area system for stochastic load in $\mathrm{Area1}$.

TABLE III
ELECTRO-MECHANICAL MODES AND CORRESPONDING PARTICIPATION FACTORS OF THE TWO-AREA SYSTEM WITH STOCHASTIC LOAD IN AREA 2

Mode	Eigenvalue	Freq. [Hz]	Participation Factors			
			G1	G2	G3	G4
1	-0.139±j2.690	0.428	5.20	7.62	27.07	34.64
2	-0.292±j7.154	1.139	38.67	52.67	2.82	2.62
3	-0.331±j7.214	1.148	2.03	4.16	42.97	46.4

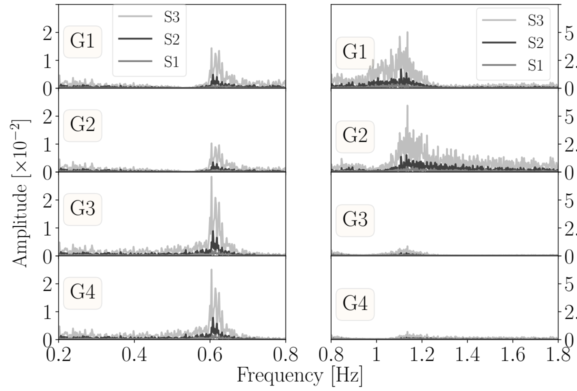

Fig. 5. Frequency spectrum of p_g of all the synchronous generators of the two-area system for stochastic load in $\mathrm{Area1}$.

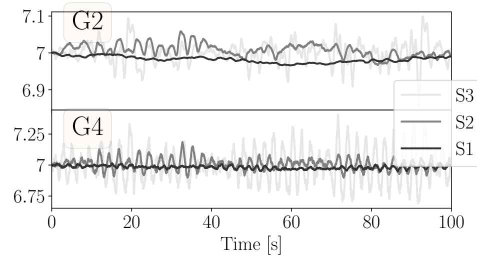

Fig. 6. Active power p_g generation of the synchronous generators G2 and G4 of the two-area system for stochastic load in $\mathrm{Area2}$.

B. All-Island Irish Transmission System

In this section, we consider a dynamic model of the AIITS, which consists of 1479 buses, 1851 lines/transformers, and 22 synchronous generators that are modeled through a VI-order model and are equipped with IEEE ST1a exciters and turbine governors to ensure a secure operation of the grid. Six conventional power plants also include a power system stabilizer. The model also includes 176 wind power plants, 34 of which are equipped with constant-speed and 142 with doubly-fed induction generators. Stochastic perturbations are included in the load consumption, voltage phasors, and wind speed. Figure 9 shows the frequency spectrum of p_g of selected synchronous generators of the AIITS. Table IV shows the dominant electro-mechanical oscillation modes of the AIITS. Results are similar to those obtained for the two-area system, i.e., the amplitude of the

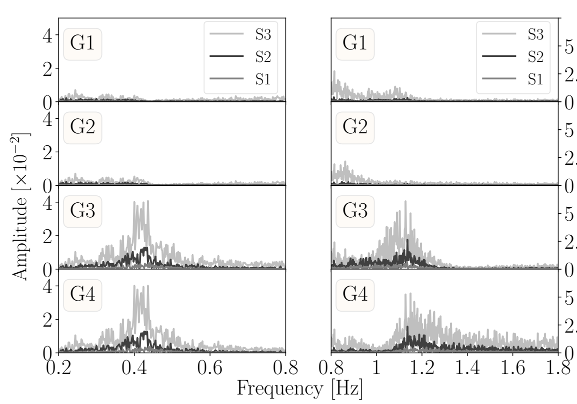

Fig. 7. Frequency spectrum of p_g of all the synchronous generators of the two-area system for stochastic load in $\mathrm{Area2}$.

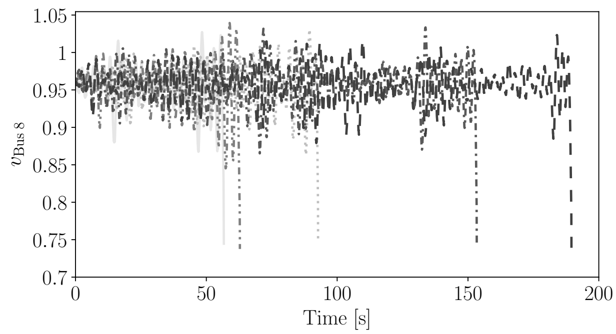

Fig. 8. Voltage at bus 8 of the two-area system for stochastic load in Area 2 for scenario S3.

frequency of the electro-mechanical oscillation modes is increased locally by increasing α.

TABLE IV
ELECTRO-MECHANICAL MODES OF THE AIITS AND CORRESPONDING PARTICIPATION FACTORS

Mode	Eigenvalue	Freq. [Hz]	Participation Factors			
			G1	G2	G3	G4
1	-0.392±j4.689	0.746	54.50	29.73	–	–
2	-0.677±j6.322	1.006	–	–	84.75	–
3	-1.150±j6.368	1.0135	–	–	–	91.26

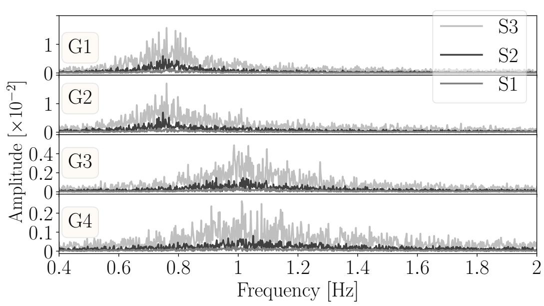

Fig. 9. Frequency spectrum of p_g of selected synchronous generators of the AIITS.

IV. CONCLUSIONS

The letter analyzes the impact of the auto-correlation of the stochastic processes on power system electro-mechanical modes. Simulations are carried out considering the Kundur's two-area system and a detailed dynamic model of All-Island Irish Transmission System.

Simulation results lead to the following relevant remarks.

- The higher the auto-correlation coefficient of the stochastic processes, the higher the amplitude of the frequency of dominant electro-mechanical modes. Thus it is important not only to know the stationary probability distribution and standard deviation of the stochastic processes of the system but also their "dynamic" behavior, which is defined by the auto-correlation of these processes.
- The presence of stochastic processes in an area of the system has a reduced effect on the local modes of other areas. This is due to the fact that the noise originated in an area can only propagate to other areas through inter-area modes.
- Stochastic processes exhibiting higher values of auto-correlation coefficient present in one area may cause instability in the power system than those present in another area with the same statistical properties. Hence, it is crucial to know the auto-correlation coefficient of the stochastic processes along with their distribution.

The main recommendation that can be drawn from this work is that system operators should perform time-domain simulations for power systems subject to stochastic disturbances with the proper values of auto-correlation coefficients along with the standard deviation of the stochastic processes. Using correct values of the auto-correlation coefficients can prevent overlooking some potential instabilities that may arise due to fast-varying stochastic processes.

Future work will focus on the design of local controllers capable to reduce the coupling of noise on system dynamics.

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