Rotor Speed-free Estimation of the Frequency of the Center of Inertia

Federico Milano, *IEEE Fellow*

***Abstract*— This letter proposes a formula to estimate the frequency of the center of inertia based exclusively on measures of bus frequencies, obtained, for example, from phasor measurement units; the network admittance matrix; and two parameters of synchronous generators, namely, the inertia constant and the internal reactance. The proposed formula can be utilized on-line and requires a highly reduced set of measures of bus frequencies. The letter discusses the theoretical background of the proposed expression and tests it with a 1,479-bus model of the all-island Irish transmission system.**

***Index Terms*— Frequency estimation, center of inertia, frequency divider, phasor measurement unit.**

I. Introduction

The Center of Inertia (COI) is a well-known concept utilized in transient stability analysis of power systems [1]–[3]. The frequency of the COI is the common synchronous frequency to which generators tend in stead-state conditions. In simulations, referring machine speed deviations to the frequency of the COI is also useful to avoid the drift of machine angles, which is particularly critical in long term stability analysis [4]. For these reasons, the COI is implemented in most commercial software tools for the dynamic simulation of power systems, e.g., Eurostag and PSS/E.

In practical applications, the frequency of the COI cannot be estimated because its calculation requires the availability of the measures of all synchronous machine rotor speeds. System operators do not estimate the frequency of the COI on-line but, rather, measure the frequency at some relevant (e.g., *pilot*) bus of the system. The behavior of the frequency of the pilot bus, however, does not represent the average frequency of the system as it follows the dynamics of the closest synchronous generators.

In recent years, the ability of measuring the frequency at buses has been dramatically boosted by the development of Phasor Measurement Unit (PMU) devices [5]. Actually, any device equipped with a phase-lock loop, such as power electronic converters of distributed energy resources, can accurately estimate the frequency of the voltage phasor at the point of connection [6]. Then there is a generalized trend to connect every device to the internet, which is, in turn, one of the most innovative aspects of the smart grid.

It can be thus safely assumed that bus frequency measures will be ubiquitous and easy to collect in the near future. How to meaningfully use such huge amount of information, however, is still not fully clear. This letter proposes an application for PMUs and discusses how to estimate through a simple yet accurate formula the frequency of the COI using exclusively bus frequency measures.

II. Theory

Let us consider an interconnected ac grid with n buses and m synchronous machines. The starting equations are the definition of the angular speed of the COI and the *frequency divider* (FD) formula presented in [7]. The expression of the speed of the COI is:

$$\omega_{\rm COI} = \mathbf{h}^T \boldsymbol{\omega}_{\rm G} \ , \tag{1}$$

where $\boldsymbol{\omega}_{\rm G}$ is a $m \times 1$ vector of synchronous machine rotor speeds and $\mathbf{h}$ is a $m \times 1$ vector of normalized inertia constants, i.e., the i-th element of $\mathbf{h}$ is:

$$h_i = H_i / H_{\rm T} \ , \tag{2}$$

where H_i is the inertia of the i-th machine and $H_{\rm T} = \sum_{j=1}^{m} H_j$.

F. Milano is with School of Electrical & Electronic Engineering, University College Dublin, Ireland. E-mail: federico.milano@ucd.ie

The FD formula presented in [7] is as follows:

$$\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1} = \mathbf{D}(\boldsymbol{\omega}_{\rm G} - \mathbf{1}_{m,1}) \ , \tag{3}$$

where

$$\mathbf{D} = -(\mathbf{B}_{\rm BB} + \mathbf{B}_{\rm BS})^{-1} \mathbf{B}_{\rm BG} \ , \tag{4}$$

where $\boldsymbol{\omega}_{\rm B}$ is a $n \times 1$ vector of the frequencies at system buses; $\mathbf{B}_{\rm BB}$ is the $n \times n$ network susceptance matrix, i.e., the imaginary part of the standard network admittance matrix; $\mathbf{B}_{\rm BG}$, is the susceptance $n \times m$ matrix obtained using the stator and step-up transformer impedances of the synchronous machines; and $\mathbf{B}_{\rm BS}$ is a $n \times n$ diagonal matrix that accounts for the internal susceptances of the synchronous machines at generator buses. Equation (3) can be utilized also with external networks assuming that one can measure the frequency at the point of connections and knows the equivalent Thevenin impedances and inertias of these networks. The complete set of hypothesis and the detailed mathematical derivation of the FD formula are given in [7].

The purpose of this letter is to define an expression where $\omega_{\rm COI}$ is determined through a linear combination of as few elements as possible of the vector of frequency buses $\boldsymbol{\omega}_B$. This result is obtained through algebraic manipulations of equations (1) and (3), as follows.

Observe that (1) can be rewritten as:

$$\omega_{\rm COI} - 1 = \mathbf{h}^T (\boldsymbol{\omega}_{\rm G} - \mathbf{1}_{m,1}) \ , \tag{5}$$

in fact, (1) derives directly from (2) which implies that $\mathbf{h}^T \mathbf{1}_{m,1} = \sum_{i=1}^{m} h_i = 1$. Then, using the definition of the matrix $\mathbf{D}$ given in (4), (3) can be rewritten as:

$$(\mathbf{B}_{\rm BB} + \mathbf{B}_{\rm BS})(\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1}) = -\mathbf{B}_{\rm BG}(\boldsymbol{\omega}_{\rm G} - \mathbf{1}_{m,1}) \tag{6}$$

$\mathbf{B}_{\rm BG}$ has m linearly independent columns if each power plant is modeled with a single equivalent machine. It is thus possible to define the Moore-Penrose pseudo-inverse of $\mathbf{B}_{\rm BG}$ as:

$$\mathbf{B}_{\rm BG}^{+} = (\mathbf{B}_{\rm BG}^T \mathbf{B}_{\rm BG})^{-1} \mathbf{B}_{\rm BG}^T \tag{7}$$

Matrix $\mathbf{B}_{\rm BG}^{+}$ is the left inverse of $\mathbf{B}_{\rm BG}$, i.e., $\mathbf{B}_{\rm BG}^{+} \mathbf{B}_{\rm BG} = \mathbf{I}_m$. The pseudo-inverse $\mathbf{B}_{\rm BG}^{+}$ provides the least-square solution of (6), where $\boldsymbol{\omega}_{\rm G} - \mathbf{1}_{m,1}$ is the vector of unknowns. Such a solution is unique because $\mathbf{B}_{\rm BG}$ has rank m. Multiplying (6) by $\mathbf{B}_{\rm BG}^{+}$ and equaling (6) to (5) lead to:

$$\begin{aligned} \omega_{\rm COI} - 1 &= -\mathbf{h}^T \mathbf{B}_{\rm BG}^{+} (\mathbf{B}_{\rm BB} + \mathbf{B}_{\rm BS})(\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1}) \\ &= \boldsymbol{\xi}^T (\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1}) \end{aligned} \tag{8}$$

where $\boldsymbol{\xi}^T = -\mathbf{h}^T \mathbf{B}_{\rm BG}^{+} (\mathbf{B}_{\rm BB} + \mathbf{B}_{\rm BS})$ is the sought vector of *weights* that allows calculating the frequency of the COI from the frequency of the buses $\boldsymbol{\omega}_{\rm B}$.

The following remarks are relevant.

- Equation (8) only involves sparse matrix-vector products. Moreover, the product $(\mathbf{B}_{\rm BG}^T \mathbf{B}_{\rm BG})^{-1}$ is a diagonal matrix, whose elements are the inverse of the square of the internal reactances of the synchronous machines.
- The elements of $\boldsymbol{\xi}$ can be viewed as the weights of the measurements $\boldsymbol{\omega}_{\rm B}$ for the evaluation of $\omega_{\rm COI}$. Note also that $\boldsymbol{\xi}^T \mathbf{1}_{n,1} \approx 1$, which derives from the definition of $\mathbf{h}$ and the properties of the rows of matrix $\mathbf{D}$, as discussed in [8].
- A property of $\boldsymbol{\xi}$ is that a large number of its elements is actually null or $\ll 1$. This fact has a relevant practical consequence: only a reduced number of measures of bus frequencies are needed to estimate $\omega_{\rm COI}$.

The proposed expression to estimate the frequency of the COI is:

$$\boxed{\omega^*_{\rm COI} = \boldsymbol{\xi}^T \boldsymbol{\omega}_{\rm B} + \boldsymbol{\alpha}} \tag{9}$$

where $\boldsymbol{\alpha} = 1 - \boldsymbol{\xi}^T \mathbf{1}_{n,1}$ is an offset, with $|\boldsymbol{\alpha}| \ll 1$. This equation is valid in time. $\boldsymbol{\xi}$ and, hence, $\boldsymbol{\alpha}$ are piece-wise constant and need to be recomputed only when a topological change occurs, e.g., a line outage, or a synchronous machine is connected to or disconnected from the grid.

The following section provides a numerical appraisal of the formula (9) based on a large real-world power system.

III. Case Study

The Irish Transmission system grid, which has been made available by EirGrid, the Irish TSO, to researchers in the author's research group, consists of 1,479 buses, 1,851 transmission lines and transformers, 245 loads, 22 conventional synchronous power plants modeled with $6^{\rm th}$ order synchronous machine models with AVRs and turbine governors, 6 PSSs, and 176 wind power plants, of which 142 are DFIGs and 34 CSWTs. This model provides a dynamic representation of the Irish electrical grid which is topologically accurate and approximates the dynamics of the actual Irish grid. All results shown in this section are obtained using Dome [9].

Expression (9) is computed using a vector of coefficients $\boldsymbol{\xi}$ where the elements $|\xi_i| < 10^{-3} = \varepsilon$ are set to 0. This threshold leads to a vector with 42 non-null elements, which is the 2.8% of the total number of buses of the system. That is: only 42 bus frequencies over 1,479 needs to be measured, e.g., by means of PMUs devices, to provide an accurate estimation of $\omega_{\rm COI}$. The accuracy of the estimation provided by (9) is illustrated in Fig. 1, which shows the frequency response following a three-phase fault that occurs at $t = 1$ s and is cleared after 50 ms.

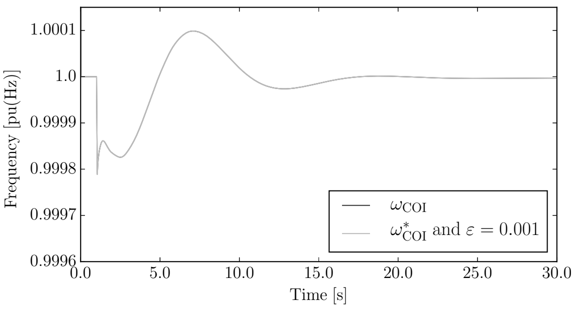

Fig. 1. Three-phase fault for the all-island Irish system – Trajectories of the actual frequency of the COI and an estimated value of $\omega^*_{\rm coi}$ obtained using (9) and $\varepsilon = 0.001$.

Observe that it would not be accurate to simply use the frequencies $\boldsymbol{\omega}_{\rm B,G}$ measured at the terminal buses where synchronous machines are connected, i.e.,

$$\tilde{\omega}_{\rm COI} = \mathbf{h}^T \boldsymbol{\omega}_{\rm B,G} \ . \tag{10}$$

Figure 2 shows the dynamic evolution of $\tilde{\omega}_{\rm COI}$ for the same contingency shown in Fig. 1. Clearly, (10) is not able to properly estimate the frequency of the COI in the first seconds after the clearing of the fault. The rationale behind this result, which might appear surprising, is that the frequencies at the terminal buses of the synchronous machines are not exactly equal to the rotor speeds because of the internal reactances of the machines themselves and the topology of the network. In other words, the term $\mathbf{B}^+_{\rm BG}(\mathbf{B}_{\rm BB}+\mathbf{B}_{\rm BS})$ in (8) is a non-trivial vector and cannot be neglected without introducing a significant estimation error.

While using only $\mathbf{h}$ leads to inconsistent results, one may wonder whether it is possible to obtain accurate results with a reduced number of measures by increasing the threshold ε.

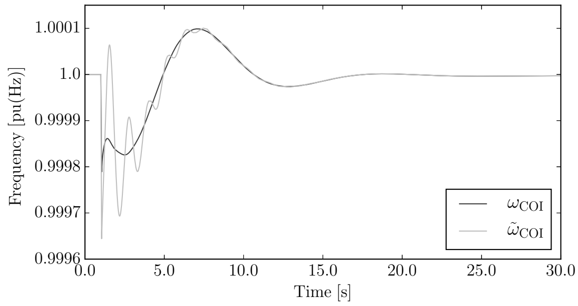

Fig. 2. Three-phase fault for the all-island Irish system – Trajectories of the actual frequency of the COI and an approximated value ($\tilde{\omega}_{\rm COI}$) obtained using (10).

Figure 3 shows the behavior of (9) when $\boldsymbol{\xi}$ is approximated considering a threshold $\varepsilon = 0.05$. With such a threshold, there are exactly 22 non-zero elements left in $\boldsymbol{\xi}$, which correspond to the frequencies at generator buses (1.5% of the total number of buses). While the results is more accurate than that obtained with (10), the transient in the first seconds after the fault clearing is not captured well.

Finally, it is relevant to note that the proposed expression (9) allows estimating the error when some signals are neglected, missing or erroneous. As previously discussed, in fact, the elements of $\boldsymbol{\xi}$ are the weights of bus frequencies to estimate the frequency of the COI. This property can be utilized to identify the most important nodes – or areas – of the system with respect to frequency dynamics. Future work will focus on the impact of noise and measurement data quality on the estimation of $\omega_{\rm COI}$.

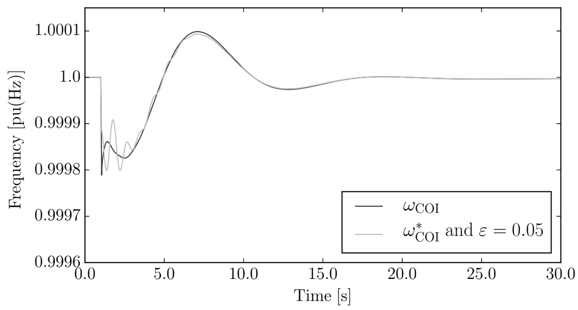

Fig. 3. Three-phase fault for the all-island Irish system – Trajectories of the actual frequency of the COI and an estimated value of $\omega^*_{\rm coi}$ obtained using (9) and $\varepsilon = 0.05$.

References

[1] C. J. Tavora and O. J. M. Smith, "Characterization of equilibrium and stability in power systems," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-91, no. 3, pp. 1127–1130, May 1972.
[2] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Upper Saddle River, NJ: Prentice Hall, 1998.
[3] F. Milano, *Power System Modelling and Scripting*. London: Springer, 2010.
[4] D. Fabozzi and T. V. Cutsem, "On angle references in long-term time-domain simulations," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 483–484, Feb. 2011.
[5] H. Bevrani, M. Watanabe, and Y. Mitani, *Oscillation Dynamics Analysis Based on Phasor Measurements*. Wiley-IEEE Press, 2014, pp. 288–.
[6] R. Teodorescu, M. Liserre, and P. Rodríguez, *Grid Converters for Photovoltaic and Wind Power Systems*. Chichester, UK: Wiley, 2011.
[7] F. Milano and Á. Ortega, "Frequency divider," *IEEE Trans. on Power Systems*, vol. 32, no. 2, pp. 1493–1501, Mar. 2017.
[8] I. K. Dassios, P. Cuffe, and A. Keane, "Visualizing voltage relationships using the unity row summation and real valued properties of the F_{LG} matrix," *Electric Power Systems Research*, vol. 140, pp. 611 – 618, 2016.
[9] F. Milano, "A Python-based software tool for power system analysis," in *Procs. of the IEEE PES General Meeting*, Vancouver, BC, July 2013.

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