Estimation of Voltage Dependent Load Models through Power and Frequency Measurements

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***Abstract*— This letter proposes an optimization-free technique to estimate the parameters of voltage dependent loads through the measurements of only the load active and reactive power consumption and frequency deviations at the load bus. The technique is shown to be a relevant consequence of the dependency of power flow equations on the bus voltage phase angles. The WSCC 9-bus and the IEEE 14-bus systems serve to illustrate the proposed technique.**

***Index Terms*— Voltage dependent load, state estimation, frequency measurement, frequency divider, phasor measurement unit (PMU).**

I. Introduction

Measurement-based approaches to estimate the behavior of loads have flourished in recent years due to the advances and ubiquity of Phasor Measurement Unit (PMU) technology. Relevant techniques have been proposed to estimate the parameters of different load models commonly used in dynamic and stability analyses of power systems such as the exponential Voltage Dependent Load (VDL), both with and without exponential recovery [1], voltage and frequency dependent loads [2], and ZIP loads [3], [4]. The main features that the approaches proposed in the references above have in common is that measurements are taken either at the load bus, only, or at every bus of the system to ensure system observability, and they are generally based on often intricate optimization techniques.

In this letter we propose an optimization-free technique to estimate the parameters of the models of VDLs based on instantaneous power and frequency measurements. The most noteworthy features of this technique are (i) its simplicity of formulation, implementation and adjustment for different systems and locations, and (ii) that only the measurements of the frequencies at the point of connection of the load and at neighboring buses, and the measurement of the power consumption of the load, are required. No confidential information of the load itself is required.

II. Background

Our starting point is the set of equations that defines the net active power injections at each bus h of the grid:

$$p_h(t) = v_h(t) \sum_{k\in\mathbb{B}} v_k(t)\big(G_{hk}\cos\theta_{hk}(t) + B_{hk}\sin\theta_{hk}(t)\big)\,, \quad (1)$$

where $\mathbb{B}$ is the set of network buses; G_{hk} and B_{hk} are the real and imaginary parts of the element (h,k) of the network admittance matrix, i.e. $\bar{Y}_{hk} = G_{hk} + jB_{hk}$; v_h and v_k denote the voltage magnitudes at buses h and k, respectively; and $\theta_{hk} = \theta_h - \theta_k$, where θ_h and θ_k are the voltage phase angles at buses h and k, respectively.

For the purpose of our discussion, we consider only active power variations and rewrite the active power injections as the sum of two components:

$$dp_h = \sum_{k\in\mathbb{B}} \frac{\partial p_h}{\partial\theta_k}\, d\theta_k + \sum_{k\in\mathbb{B}} \frac{\partial p_h}{\partial v_k}\, dv_k = dp'_h + dp''_h\,. \quad (2)$$

In the remainder of this letter, we focus on the first component, p'_h, that is, the component that mostly depends on the voltage phase angle variations at network buses. With this aim, we simplify the first term of the right-hand side of (2) with the usual assumptions of the DC power flow analysis [5], namely, $G_{hk}\approx 0$, $v_h v_k \approx 1$ pu(kV) and $\sin(\theta_{hk})\approx\theta_{hk}$. With these approximations, we obtain that

$$\frac{\partial p_h}{\partial\theta_k} \approx B_{hk}\,, \quad (3)$$

and, hence, the component p'_h in (2) can be approximated as:

$$p'_h(t) \approx \sum_{k\in\mathbb{B}} B_{hk}\,\theta_{hk}(t)\,, \quad (4)$$

from where one can deduce that the DC power flow equations are an approximation of only the component p'_h of the active power as defined in this letter.

Despite the fact that in recent years PMUs have enabled the measurement of the phase angles of voltages and current, the precise determination of θ_{hk} in (4) is still a difficult task. For this reason, p'_h cannot be easily measured in practice. However, the differentiation of (4) with respect to time leads to:

$$\dot{p}'_h(t) = \Omega_o \sum_{k\in\mathbb{B}} B_{hk}\,\big(\omega_h(t) - \omega_k(t)\big)\,, \quad (5)$$

where ω_h and ω_k are the frequency deviations in per unit with respect to the reference synchronous speed at the buses and Ω_o is the reference synchronous angular speed in rad/s. Multiplying by Ω_o is necessary to take into account the fact that the phase angles in (4) are in radians, while the angular frequencies in (5) and in all other equations in the remainder of this letter are expressed in per unit. The precise measurement of the bus frequencies ω_h and ω_k is accurate enough (see, for example, [6]) to allow a precise estimation of $\dot{p}'_h$. Then, p'_h can be calculated by integrating (5).

III. Voltage Dependent Loads

In practice, the power consumption at a node of the network is the combined effect of several devices, each of them with a potentially different dynamic behavior. In this paper, we focus on a simple but versatile and quite general VDL model which is widely utilized in the literature, as follows:

$$p_{\rm D}(t) = p_{{\rm D},o}\, v_h^{\gamma_p}(t)\,, \qquad q_{\rm D}(t) = q_{{\rm D},o}\, v_h^{\gamma_q}(t)\,, \quad (6)$$

where $p_{{\rm D},o}$ and $q_{{\rm D},o}$ are the load active and reactive powers at the nominal voltage magnitude, i.e., at $v_h = 1$ pu; and γ_p and γ_q are the parameters that define the load behavior.

Three relevant special cases, namely constant power, constant admittance and constant current loads are discussed below. In all

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cases discussed in the remainder of this section, we assume that the VDL is connected to the rest of the grid through a single lossless connection, hence:

$$p_h(t) = B_{hk}\, v_h(t)\, v_k(t)\, \sin\big(\theta_h(t) - \theta_k(t)\big)\,, \tag{7}$$

$$q_h(t) = B_{hk}\, v_h^2(t) - B_{hk}\, v_h(t)\, v_k(t)\, \cos\big(\theta_h(t) - \theta_k(t)\big)\,. \tag{8}$$

Differentiating the expressions above gives:

$$dp_h = c_{hk}(t)\, v_h(t)\, v_k(t)\, d\theta_{hk} + s_{hk}(t)\, dv_{hk}\,, \tag{9}$$

$$dq_h = s_{hk}(t)\, v_h(t)\, v_k(t)\, d\theta_{hk} - c_{hk}(t)\, dv_{hk} + 2B_{hk} v_h(t) dv_h\,, \tag{10}$$

where $dv_{hk} = v_k dv_h + v_h dv_k$, $d\theta_{hk} = d\theta_h - d\theta_k$, $c_{hk} = B_{hk}\cos(\theta_h - \theta_k)$ and $s_{hk} = B_{hk}\sin(\theta_h - \theta_k)$. Note that, according to the the defintion of dp'_h and dp''_h given in (2) and from (9), one has:

$$dp'_h = c_{hk}(t)\, v_h(t)\, v_k(t)\, d\theta_{hk}\,, \tag{11}$$

$$dp''_h = s_{hk}(t)\, dv_{hk}\,. \tag{12}$$

The simple model (7)-(8) is assumed for simplicity but without loss of generality in the remainder of this section. The fully-fledged power system model is then utilized in the case studies.

A. Constant Power

In this case $\gamma_p = \gamma_q = 0$. The active power consumption of the load is thus given by:

$$p_h(t) = -p_{\mathrm{D},o}\,. \tag{13}$$

Differentiating p_h and remembering the defintion of dp'_h and dp''_h given in (2), one obtains:

$$dp_h = 0 \quad \Rightarrow \quad dp'_h = -dp''_h\,, \tag{14}$$

and, dividing by dt:

$$\dot{p}_h(t) = 0\,, \qquad \dot{p}'_h(t) = -\dot{p}''_h(t)\,, \tag{15}$$

which indicates that, even though p_h is constant the two components p'_h and p''_h are not. The ability of a load to maintain a constant active power consumption, in fact, implies that for any variation of the voltage, the current will change to compensate such a variation.

B. Constant Admittance

In this case $\gamma_p = \gamma_q = 2$. To obtain a condition on $\dot{p}'_h$ and $\dot{p}''_h$, it suffices to observe that, from the frequency divider formula presented in [7] and assuming that the load is connected to the grid as indicated in (7) and (8), the frequencies at the sending and receiving ends of the line satisfy the condition:

$$\omega_h(t) \approx \omega_k(t)\,, \tag{16}$$

or, equivalently:

$$\dot{\theta}_h(t) \approx \dot{\theta}_k(t) \quad \Rightarrow \quad d\theta_h \approx d\theta_k\,. \tag{17}$$

From (17) it descends that the expression of dp_h in (9) does not depend on $d\theta_{hk}$ and that the terms c_{hk} and s_{hk} are constant. Thus, by the definition given in (2), one has:

$$dp'_h \approx 0 \quad \Rightarrow \quad dp_h \approx dp''_h\,, \tag{18}$$

and, dividing by dt:

$$\dot{p}'_h(t) \approx 0\,, \qquad \dot{p}_h(t) \approx \dot{p}''_h(t)\,. \tag{19}$$

C. Constant Current

In this case $\gamma_p = \gamma_q = 1$. The load active and reactive powers are given by:

$$\begin{aligned} p_h(t) &= -i_{\mathrm{D},o}\, \cos\phi_{\mathrm{D},o}\, v_h(t)\,, \\ q_h(t) &= -i_{\mathrm{D},o}\, \sin\phi_{\mathrm{D},o}\, v_h(t)\,, \end{aligned} \tag{20}$$

where $i_{\mathrm{D},o}$ and $\cos\phi_{\mathrm{D},o}$ are the constant current magnitude and constant power factor, respectively, of the load. Equalling the expression of the active power in (20) into (7) and dividing by v_h leads to:

$$-i_{\mathrm{D},o}\, \cos\phi_{\mathrm{D},o} = s_{hk}(t)\, v_k(t)\,, \tag{21}$$

whose differentiation gives:

$$0 = c_{hk}(t)\, v_k(t)\, d\theta_{hk} + s_{hk}(t)\, dv_k\,. \tag{22}$$

Multiplying (22) by v_h and imposing the resulting equality into (9), one obtains:

$$dp_h = s_{hk}(t)\, v_k(t)\, dv_h\,. \tag{23}$$

Similarly, equalling the expression of the reactive power in (20) into (8) and dividing by v_h leads to:

$$-i_{\mathrm{D},o}\, \sin\phi_{\mathrm{D},o} = B_{hk} v_h(t) - c_{hk}(t)\, v_k(t)\,, \tag{24}$$

whose differentiation leads to:

$$0 = B_{hk}\, dv_h - c_{hk}(t)\, dv_k + s_{hk}(t)\, v_k(t)\, d\theta_{hk}\,. \tag{25}$$

If v_k is the voltage of Thevenin equivalent of the rest of the grid, one can assume that v_k is almost constant and, without lack of generality, $v_k \approx 1$ pu. Then, (25) leads to:

$$dv_h \approx -\frac{s_{hk}(t)}{B_{hk}}\, d\theta_{hk}\,, \tag{26}$$

and, finally, substituting the obtained expression of dv_h in (26) into (23), one obtains:

$$dp_h \approx -\frac{s_{hk}^2(t)}{B_{hk}}\, d\theta_{hk}\,, \tag{27}$$

which is an expression that does not depend on the bus voltage magnitudes. Thus, if the load is connected to a “strong” network, and by the definition given in (2), one has:

$$dp''_h \approx 0 \quad \Rightarrow \quad dp_h \approx dp'_h\,, \tag{28}$$

and, dividing by dt:

$$\dot{p}''_h(t) \approx 0\,, \qquad \dot{p}_h(t) \approx \dot{p}'_h(t)\,. \tag{29}$$

D. Estimation of the Voltage Exponents of VDLs

Table I summarizes the discussion above and shows the equivalence of the active power component of the VDL depending on the exponent γ_p. Assuming the approximations introduced above, the following relationship can be deduced:

$$\frac{\dot{p}'_h(t)}{\dot{p}_h(t)} \approx \frac{2-\gamma_p}{\gamma_p}\,, \tag{30}$$

which is valid for any value of the voltage exponent of the VDLs.

TABLE I
SYNOPTIC SCHEME OF THE ACTIVE POWER COMPONENTS FOR VDLS

γ_p	$\dot{p}'_h$	$\dot{p}''_h$	$\dot{p}_h$
0	$-\dot{p}''_h$	$-\dot{p}'_h$	0
1	$\dot{p}_h$	0	$\dot{p}'_h$
2	0	$\dot{p}_h$	$\dot{p}''_h$

Assuming that the measurements of the power p_h and of the frequencies at bus h and its neighboring buses are available, (30) allows estimating the exponent γ_p as follows:

$$\breve{\gamma}_p(t) \approx 2\,\frac{\dot{p}_h(t)}{\dot{p}_h(t)+\dot{p}'_h(t)} \approx 2\,\frac{\Delta p_h(t)}{\Delta p_h(t)+\Delta p'_h(t)}\,, \qquad (31)$$

where $\breve{}$ indicates estimated quantities and the time derivatives can, in practice, be approximated with finite differences. The expression (31) holds for ideal lossless networks and for VDLs with $\gamma_p=\gamma_q$. In the case study below, we show that, on one hand, the impact of $\gamma_p \neq \gamma_q$ is negligible, and that lossy branches, on the other hand, can be taken into account by adjusting (31) as follows:

$$\breve{\gamma}_p(t) \approx (2-\rho)\,\frac{\Delta p_h(t)}{\Delta p_h(t)+\Delta p'_h(t)}\,, \qquad (32)$$

where the factor ρ is a function of the ratio of the branches that connect the VDL to the rest of the grid, i.e. $\rho=\rho(R_{hk}/X_{hk})$.

Finally, the coefficient γ_q of the reactive power of the VDL can be deduced from $\breve{\gamma}_p$ and active and reactive power measurements by merging the two equations in (6) as follows:

$$v_h(t) = \left(\frac{p_{\rm D}(t)}{p_{{\rm D},o}}\right)^{1/\gamma_p} = \left(\frac{q_{\rm D}(t)}{q_{{\rm D},o}}\right)^{1/\gamma_q}\,. \qquad (33)$$

By taking logarithms at the latter two equalities, and assuming that $p_h=-p_{\rm D}$ according to (1), we obtain:

$$\ln\left[\left(\frac{-p_h(t)}{p_{{\rm D},o}}\right)^{1/\gamma_p}\right] = \ln\left[\left(\frac{-q_h(t)}{q_{{\rm D},o}}\right)^{1/\gamma_q}\right]\,. \qquad (34)$$

Finally, grouping together both terms at one side and multiplying by $\gamma_p\,\gamma_q$, the following expression to estimate $\breve{\gamma}_q$ based on the previous estimation of $\breve{\gamma}_p$ from (32) is obtained:

$$\breve{\gamma}_q(t)\ln\left(\epsilon-\frac{p_h(t)}{p_{{\rm D},o}}\right)-(\breve{\gamma}_p(t)-\eta)\ln\left(\epsilon-\frac{q_h(t)}{q_{{\rm D},o}}\right)=0\,, \qquad (35)$$

where $\epsilon \ll 1$ and $\eta \ll 1$ are positive coefficients to prevent initialization issues when $p_h/p_{{\rm D},o} \approx 1$ and/or $q_h/q_{{\rm D},o} \approx 1$, and the trivial solution $\breve{\gamma}_p=\breve{\gamma}_q=0$, respectively.

IV. Case Study

This section presents two examples based on the WSCC 9-bus, 3-machine system, which represents a transmission system with $R_{hk}/X_{hk} \ll 1$ ratios, and the IEEE 14-bus system, a region of which represents a distribution grid with lines characterized by $R_{hk}/X_{hk} \approx 1$. All simulations were carried out using Dome [8] and were executed on a 64-bit Linux Ubuntu 18.04 operating system running on an 8 core 3.40 GHz Intel© Core i7™ with 16 GB of RAM. The simulation time is 30 seconds in all scenarios considered, with a time step of 10 ms. The times required to complete the time domain simulations for the WSCC and IEEE 14-bus test systems are, approximately, 5.1 s and 4.7 s, respectively.

A. VDLs in Transmission Systems

The WSCC system is considered in this section. Figure 1 shows the ratio $\Delta p'_8/\Delta p_8$ for a wide range of values of γ_p and γ_q of a VDL at bus 8. The curves indicate that the dependence of $\breve{\gamma}_{p,8}$ on $\gamma_{q,8}$ is negligible. This result is general and we have confirmed it through several tests on different systems and loading conditions. From the results in Fig. 1, one can also deduce that the corrective factor in (32) is $\rho \approx 0.1$ which, in this case, is about the ratio R_{hk}/X_{hk} of the lines that connect bus 8 to the rest of the system. Figures 2 and 3 shows the performance of the expressions (32) and (35) for several values of $\gamma_{p,8}$ and $\gamma_{q,8}$.

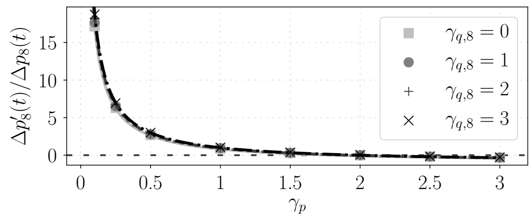

Fig. 1. WSCC system – Ratios of active power variations as functions of the voltage exponents.

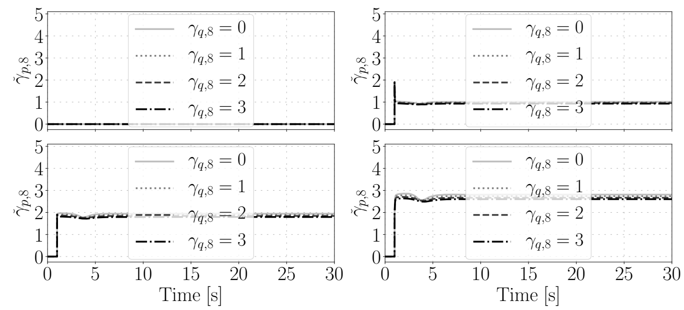

Fig. 2. WSCC system – From left to right and from top to bottom, estimated voltage exponent $\breve{\gamma}_{p,8}$ ($\gamma_{p,8} \in \{0,\,1,\,2,\,3\}$) for different values of $\gamma_{q,8}$.

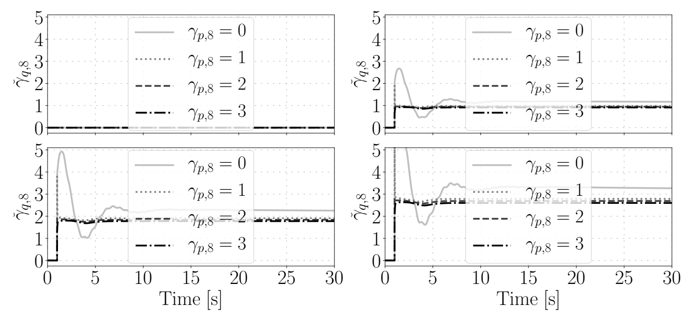

Fig. 3. WSCC system – From left to right and from top to bottom, estimated voltage exponent $\breve{\gamma}_{q,8}$ ($\gamma_{q,8} \in \{0,\,1,\,2,\,3\}$) for different values of $\gamma_{p,8}$.

B. VDLs in Distribution Systems

This scenario considers the case of VDLs connected at the distribution system level, for which the condition $B_{hk} \approx G_{hk}$ holds. With this aim, the IEEE 14-bus is considered for simulations. This system includes a region at 13.8 kV (buses 7 and 9-14). The lines of this region are characterized by a ratio $R_{hk}/X_{hk} \in [0.47, 0.63]$, with some lines with a ratio greater than 1 (line 12–13).

Figure 4 shows the ratio $\Delta p'_{14}/\Delta p_{14}$ calculated at the end of the time domain simulation for a range of values of γ_p. The contingency is the disconnection of the load at bus 5. Based on several tests, we have determined that $\rho \approx 0.35$ for systems where $R_{hk}/X_{hk} \approx 1$.

Figures 5 and 6 depict the estimated voltage exponent of the active power component of the VDLs at buses 14 and 12, respectively. The accuracy of the estimation slightly decreases for higher values of γ_p but, overall, provides a very good estimation of $\breve{\gamma}_p$. Note also that typical values of γ_p are usually in the range $[0, 2]$ (see e.g., [9], and the references therein).

V. Concluding Remarks

The letter proposes a novel technique to estimate the parameters of voltage dependent loads using non-confidential measurements. The theory developed in the letter indicates that such estimation can be obtained through power and frequency measurements, exclusively. Simulation results confirm the accuracy of this approach.

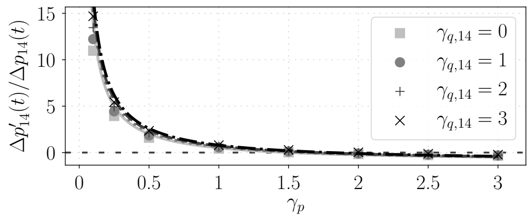

Fig. 4. IEEE 14-bus system – Ratio of active power variations as functions of the voltage exponents.

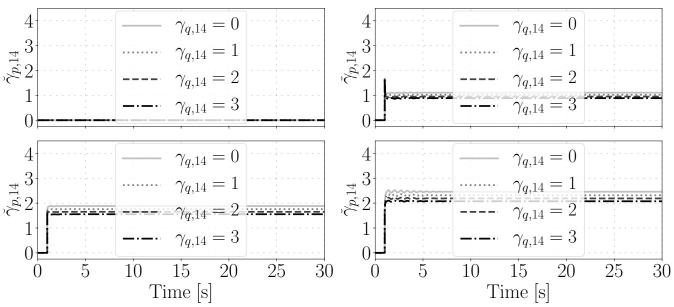

Fig. 5. IEEE system – From left to right and from top to bottom, estimated voltage exponent $\check{\gamma}_{p,14}$ $(\gamma_{p,1} \in \{0,\ 1,\ 2,\ 3\})$ for different values of $\gamma_{q,14}$.

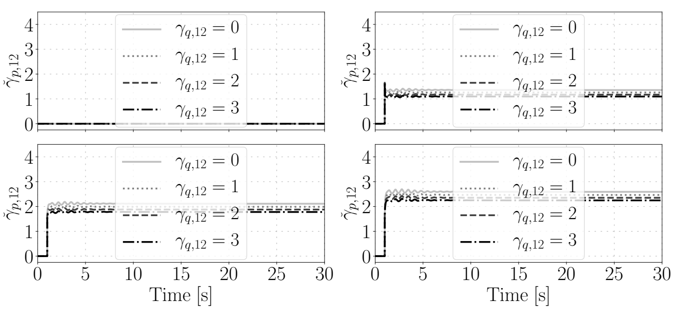

Fig. 6. IEEE system – From left to right and from top to bottom, estimated voltage exponent $\check{\gamma}_{p12}$ $(\gamma_{p,12} \in \{0,\ 1,\ 2,\ 3\})$ for different values of $\gamma_{q,12}$.

Note that the definition of ρ in the two systems presented in the case study has been based on the observation of numerous tests that have been carried out using several transmission and distribution networks. Based on these tests, we have concluded that the value of ρ appears to be always between 0.1 and 0.35. Thus, we trust that the expressions (32) and (35) are reliable tools to estimate VDL coefficients.

Note also that, since expressions (32) and (35) are a function of time, we expect that they can also provide an estimation of the transient active and reactive voltage exponents of VDLs with exponential recovery. The estimation of the parameters of such dynamic VDLs as well as of frequency dependent and ZIP loads will be the focus of future work.

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