Transmission Congestion Management and Pricing in Simple Auction Electricity Markets

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Abstract— **This paper presents a novel technique to analyze, manage and price transmission congestion in electricity markets based on a simple auction mechanism. The proposed technique is basically an iterative rescheduling algorithm, relying on an "on-line" evaluation of the transmission system congestion, as defined by a "System-wide" Available Transfer Capability (SATC), and associated sensitivities, which are all computed based on formulas that account for voltage stability constraints as well as thermal and bus voltage limits. The methodology is tested using a 3-area test system, a 6-bus test system with both demand-side bidding and inelastic demand, as well as a 129-bus model of the Italian High Voltage transmission system with demand-side bidding. The results obtained for some of these test systems with the proposed technique are compared with similar results obtained from an optimization-based method.**

Keywords— **Electricity markets, electricity pricing, simple auction, transmission system congestion, Available Transfer Capability (ATC), voltage stability.**

I. Introduction

THE WIDESPREAD and rapid deregulation and/or privatization of electricity markets throughout the world has led to the implementation of a variety of competitive market structures that could be categorized into three main groups [1], [2], [3], namely, pool or centralized markets (e.g. the "old" U.K. market, Chile, and PJM), simple auction or decentralized markets (e.g. Spain and the former California markets), and spot pricing or hybrid markets (e.g. New Zealand, Ontario, and the current U.K. and New England markets).

Centralized markets can be basically viewed as unit commitment problems where a "central" broker/operator takes care of "dispatching" the market participants based on their bids, while accounting somewhat for the transmission system and network security [4]. Decentralized markets, on the other hand, are considered "transparent" markets that are run by a central broker or market operator, and where only the participants' bids are used to determine a market clearing price using a simple auction mechanism, without considering system constraints; the results of this auction are passed on to a system operator who may approve, modify and/or reject the transactions, depending on the market rules and system constraints [3]. Finally, hybrid markets are based on spot pricing techniques and associated Optimal Power Flow (OPF) methods [5], so that price signals can be given to all market participants from an optimal system operation perspective through the use of "nodal" or "locational" marginal prices (LMP) [6].

The costs associated with ensuring transaction feasibility in the various types of markets is typically referred to as Transaction Security Cost (TSC), and is an important component of unbundled transaction costs [7]. From an economic point of view, accurate costs are needed to provide the correct price signals to foster adequate services and fair competition, as well as create a stable market and potential profits. Thus, correct TSC information can help market operators determine fair locational prices, and coordinate and manage transactions by generating proper feedback signals to encourage market participants to make competitive and profitable market decisions that at the same time allow to maintain system security [8], [9], [10].

Determining the costs associated with system security has been of great interest in power systems, especially under the framework of competitive electricity markets [11], [12], [13], [14]. In most of these references, Lagrange multipliers generated during optimization procedures are used to analyze the different cost components of power system operation and determine the LMPs, based on spot pricing techniques. In centralized and hybrid markets, these methodologies can be readily applied as these markets are based on optimization procedures. Thus, by introducing inequalities constraints in the optimization processes to represent system security, the Lagrange multipliers are used to determine the TSC and associated prices [6], [15]. However, in decentralized markets, locational prices that account for system security costs cannot be obtained as a byproduct of the auction process on which these markets are based, given the simple mechanisms used to obtain a clearing price; in this case, system security can only be introduced in the market solution process using iterative techniques [3]. Hence, the present paper concentrates on developing techniques to introduce security costs in decentralized market structures, so that locational prices that account for the TSC can be generated based on a rescheduling technique somewhat similar to the one described in [16].

In all market structures, rather conservative limits on the transmission system power flows and bus voltage magnitudes are typically used to somewhat represent system security; these limits are typically determined through off-

Original manuscript submitted to *IEEE Trans. Power Systems*, October 2001. Modified and resubmitted on November 2002.

This research work was carried out under the financial support of NSERC, Canada, and the E&CE Department at the University of Waterloo.

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line studies by making fairly conservative assumptions to account for operating uncertainties. When some of these limits are reached, the system is considered to have a transmission congestion problem, and hence operating and pricing signals are generated to ensure the operational feasibility of the proposed transactions. However, representing system security through the use of these limits may lead to incorrect price signals and may even compromise system security, as these limits most likely do not reflect the security margins for the actual operating conditions. Hence, in the current paper, an on-line technique that accounts for voltage stability constraints as well as thermal and bus voltage limits is used to approximately compute a "System-wide" Available Transfer Capability (SATC), and thus properly evaluate the transmission system congestion. The proposed SATC computation methodology, which is similar to the ones proposed in [17], [18], also generates a series of sensitivity factors that are used here to define Nodal Congestion Prices (NCPs) that account for the TSC.

The paper is structured as follows: In Section II, the basic concepts behind the computation of the proposed SATC and associated sensitivity factors are presented and discussed. A rescheduling technique is presented in Section III for the determination of the proposed NCPs. In Section IV, the results of applying the proposed methodologies to a 3-area test system, a 6-bus test system with inelastic demand and demand-side bidding, and a 129-bus model of the Italian system with supply and demand bids are used to illustrate the proposed NCP calculation. The advantages and disadvantages of this technique are highlighted in this section in comparison to an standard OPF-based analysis, since the proposed transmission congestion management and pricing technique is an iterative procedure that could be basically considered a sub-optimal solution to a security constrained market auction. Finally, Section V discusses the main contributions of this paper and possible future research directions.

II. SATC and Sensitivity Computations

The pricing methodology presented in this paper is basically an implementation of a simple auction market where SATC computations and sensitivity analyses, which are based on voltage stability constraints as well as thermal and bus voltage limits, and carried out for the given bidding conditions, are proposed to determine nodal prices that reflect TSCs, assuming equal firmness of all potential transactions. The methodology presented here is based on sensitivity formulas developed for the stability limit conditions defined by a SATC. Hence, this section concentrates on describing the techniques used for the proposed SATC and sensitivity calculations.

A. SATC Calculations

The ATC, as defined by NERC, is a "measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses" [19]. Thus, mathematically it is defined as

$$\text{ATC} = \text{TTC} - \text{ETC} - \text{TRM} \tag{1}$$

where

$$\text{TTC} = \min(P_{\max_{I_{lim}}}, P_{\max_{V_{lim}}}, P_{\max_{S_{lim}}})$$

represents the Total Transfer Capability, i.e. the maximum power that the system can deliver given the security constraints defined by thermal limits (I_{lim}), voltage limits (V_{lim}) and stability limits (S_{lim}), typically based on an N-1 contingency criterion (worst single, "believable" contingency of the transmission system). ETC stands for the Existing Transmission Commitments, and TRM corresponds to the Transmission Reliability Margin, which includes a Capacity Benefit Margin (CBM) and is meant to account for uncertainties in system operation.

From the ATC definition (1), it is clear that its value changes with the system operating conditions, and hence has to be computed for every bidding condition. However, in most of the current implementations of electricity markets, the ATC is determined in off-line studies and represented in the bidding process as rather conservative limits on the power flowing through the main transmission "corridors" or interchange paths. These power limits are unrealistic and could easily lead to either fictitious congestion problems, with the corresponding pricing implications, or unsecured operating conditions, which may lead to stability problems.

In this paper, the basic ATC concept is extended to the system domain [20], and is referred here as "System-wide" ATC or SATC, which is basically a transfer capability limit computed for the whole system using the same concept defined in (1), as opposed to a given interchange path. It is important to highlight the fact that the SATC concept is *only* used to properly represent system security in the congestion management and pricing technique proposed here, whereas "standard" ATC values are typically used to define area interchange limits for a wide variety of market applications.

The problem in the computation of the SATC is the actual determination of the stability limits, which require costly time domain simulations. Hence, in the present paper, the stability limits are approximately represented using voltage stability margins, which can be readily and quickly computed, giving a good idea of the "relative" stability of the network [21]. In this case, a continuation power flow approach is used to determine the maximum system loading, which represents a "System-wide" TTC or STTC, used to compute the SATC value [17], [20].

Thus, consider that the system can be represented in steady state with the following set of nonlinear equations:

$$f(x, \lambda, p) = 0 \tag{2}$$

where $x \in \Re^n$ stands for the state and algebraic system variables, such as bus voltage magnitudes and angles; $\lambda \in \Re$ is a loading parameter used to represent the system loading margin, as the load powers are modeled as

$$P_L = P_{L_o} + \lambda P_D \tag{3}$$

with P_{L_o} representing the power levels of loads that *do not* directly bid in the market (e.g. "non-dispatchable" loads in the Ontario market [22]), and hence define the initial loading conditions, and P_D corresponding to the demand power bids; all loads are assumed to have constant power factors. In this analysis, generator powers are modeled as

$$P_G = P_{G_o} + (\lambda + k_G)P_S \tag{4}$$

where P_{G_o} stands for the generator power levels that *do not* directly bid in the market (e.g. "must-run" generators in the Ontario market [22]), and k_G is a variable used to represent a distributed slack bus. The parameters $p \in \Re^m$ correspond to "controllable" market or system parameters, such as the supply and demand power bids P_S and P_D, respectively. Equations (2) typically correspond to a set of "modified" power flow equations, which basically result from modeling system controls and limits in greater detail than in the typical power flow equations [21].

The voltage stability limits for the system represented by equations (2) are basically associated with saddle-node and limit-induced bifurcations of the corresponding set of nonlinear equations [21]; at these bifurcation points, the system collapses. Thermal and voltage limits, on the other hand, can be treated mathematically, for the purpose of sensitivity analyses, in a similar way as limit-induced bifurcations, although the system does not collapse when these limits are reached. Hence, by considering that a "System-wide" ETC (SETC) is represented through the base system conditions, i.e. $\text{SETC} = \sum P_{L_o}$, and assuming that a "System-wide" TRM (STRM) is basically a fixed value, i.e. $\text{STRM} = K$, where K is a given MW value used to represent contingencies that are not being considered during the SATC computations (e.g. N-2 contingencies), the SATC for (2) can then be defined as

$$\text{SATC} = \lambda_c \cdot T - K \tag{5}$$

where T represents the total transaction level, and λ_c represents the "critical" (maximum) loading parameter at which the system is at a limit condition due to voltage stability, thermal or bus voltage constraints for a worst contingency scenario.

B. Sensitivity Formulas

Since voltage stability constraints as well as thermal and voltage limits are used to determine the SATC value, one can also readily determine the sensitivities of the SATC with respect to various system parameters, especially with respect to the participants' bids, i.e.

$$\frac{\text{dSATC}}{\text{d}p} = \frac{\text{d}\lambda_c}{\text{d}p} \cdot T \tag{6}$$

The required sensitivity formulas can be obtained from the definition of the STTC and the use of basic voltage stability concepts [21].

B.1 Saddle-Node Bifurcations

Saddle-node bifurcations (SNB) are characterized by a pair of equilibrium points coalescing and disappearing as the parameter λ "slowly" changes. Mathematically, the SNB point is an equilibrium point (x_c, λ_c, p_c) with a singular Jacobian $D_x f|_c$ and associated unique right and left "singular" eigenvectors v and w, respectively, i.e. $D_x f|_c v = D_x^T f|_c w = 0$.

By taking the derivatives of (2), one has at the SNB point that

$$\begin{aligned} D_x f|_c \,\text{d}x + D_\lambda f|_c \,\text{d}\lambda + D_p f|_c \,\text{d}p &= 0 \\ \Rightarrow \quad w^T D_x f|_c \,\text{d}x + w^T D_\lambda f|_c \,\text{d}\lambda + w^T D_p f|_c \,\text{d}p &= 0 \end{aligned}$$

Hence, from these equations and as proposed in [23], one has that the sensitivities of the system loading λ with respect to changes in the parameters p at the SNB point can be determined by using

$$\frac{\text{dSATC}}{\text{d}p} = -\frac{1}{\omega^T D_\lambda f|_c} \, \omega^T D_p f|_c \cdot T \tag{7}$$

B.2 Limits

Limit-induced bifurcations (LIB) are equilibrium points where a system control limit is reached, which in *some* cases may lead to a system collapse characterized by a pair equilibrium points coalescing and disappearing for slow changes of the parameter λ. At a LIB, as opposed to a SNB, the system Jacobian is not singular at the bifurcation point (x_c, λ_c, p_c); hence, equation (7) does not directly apply at this point. Furthermore, sensitivities of system limits that are not necessarily associated with stability problems but are rather the result of equipment limitations, such as thermal limits on transmission lines, cannot be studied using (7) either.

In general, a system reaching any particular limit at an equilibrium point (x_c, λ_c, p_c), such as a bus voltage, thermal or reactive power limit, can be characterized by two different sets of equations, i.e.

$$\begin{aligned} f_1(x_c, \lambda_c, p_c) &= 0 \\ f_2(x_c, \lambda_c, p_c) &= 0 \end{aligned} \tag{8}$$

where the first set $f_1(\cdot)$ corresponds to the "original" system equations, whereas the second set $f_2(\cdot)$ corresponds to a modified set of equations where the limit is active. For example, when a reactive power generator limit is reached at a bus i, a generator voltage control equation, say $V_i - V_{i_c} = 0$, may be replaced by $Q_{G_i} - Q_{G_{lim}} = 0$ at the limit condition. Hence, taking the derivatives of (8) at the equilibrium point where the limit becomes active,

$$\begin{aligned} D_x f_1|_c \,\text{d}x + D_\lambda f_1|_c \,\text{d}\lambda + D_p f_1|_c \,\text{d}p &= 0 \\ D_x f_2|_c \,\text{d}x + D_\lambda f_2|_c \,\text{d}\lambda + D_p f_2|_c \,\text{d}p &= 0 \end{aligned}$$

Eliminating $\text{d}x$ from these equations, leads to

$$\frac{\text{dSATC}}{\text{d}p} = \frac{1}{\mu^T \mu} \, \mu^T \left(D_x f_2|_c \, D_x f_1|_c^{-1} \, D_p f_1|_c - D_p f_2|_c \right) \cdot T \tag{9}$$

where

$$\mu = D_\lambda f_2|_c - D_x f_2|_c \, D_x f_1|_c^{-1} \, D_\lambda f_1|_c$$

Observe that the sensitivity formula (9), which can be shown to be equivalent to a formula proposed in [24], applies to any limit condition, independent of whether it corresponds to a LIB or a thermal or voltage limit. Hence, equation (9) together with (7) are used to determine the sensitivity of the SATC with respect to the system parameters p, which for the purpose of this paper correspond to the supply and demand bids P_S and P_D, respectively.

In this paper, all SATC and required sensitivity values are computed based on the results generated by UWPFLOW [25], which is a continuation power flow program capable of representing various power system elements using "detailed" steady state models.

III. Rescheduling Technique

The rescheduling technique proposed in this paper, which considers both generation redispatch and load curtailment, follows similar general criteria as the one discussed in [16], where rescheduling is used to address the issue of a simple auction mechanism that yields a market clearing condition that violates certain congestion criteria, as defined by power flow limits on the transmission system. The costs resulting from dispatching a participating unit (non-bidding units are not considered), which might be more expensive than the market clearing price (MCP), or curtailing loads to solve the congestion problem are then "distributed" among the different participants. The idea here is to redispatch units or curtail load based only on the effect that these have on transmission system congestion, without too much regard for costs, as at this point system security takes precedence over economic considerations, especially in view that, in most markets, bidding results can be rejected based on security criteria. Observe that by choosing the units or loads based on this criterion, there will be minimum impact on the desired transactions, thus probably reducing the costs associated with achieving the required security criteria, as demonstrated in one of the examples in Section IV.

The main improvements of the technique proposed here with respect to known methodologies are:

- Thermal, bus voltage and voltage stability limits are all accurately considered to compute an SATC value "on-line", as opposed to just using much simpler approximate MW flow constraints, which are typically determined through off-line studies for particular system conditions that might not correspond to the actual transactions under consideration.
- The congestion costs are distributed among the market participants based on the actual impact that each one of them has on the SATC value.
- Nodal Congestion Prices (NCP) are calculated using the sensitivities of SATC with respect to the generator and load bids. Thus, the method presented here could be considered as a sub-optimal way of determining congestion prices, with the advantage that contingencies can be directly considered in the SATC computations, which so far is not feasible with OPF-based techniques.

The suggested rescheduling technique to address the problem of market clearing conditions that do not meet the actual SATC requirements, as defined in the previous section, is summarized in the flow-diagram shown in Fig. 1. The depicted methodology is based on a series of linearizations; however, the SATC does change nonlinearly as the system parameters change due to the highly nonlinear behavior of the system. The latter is the main reason for using an iterative process. The proposed technique determines the transaction costs for the different market participants as follows:

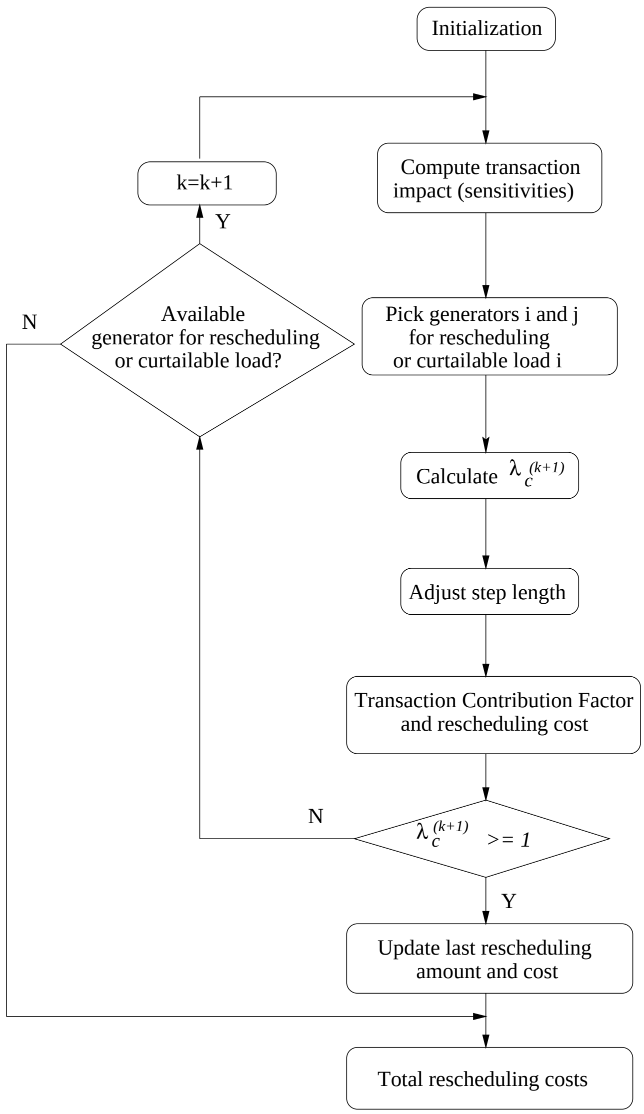

Fig. 1. Rescheduling technique for a simple auction system.

A. MCP Computation

Using a simple auction mechanism, the MCP and associated total transaction power level T are determined, together with the load and generator power levels that clear the market, namely, P_D and P_S for all loads and generators, respectively. The values of P_D and P_S are used in the determination of the SATC, as these define the load and generator direction used for the computation of λ_c in UWPFLOW [25].

B. Transaction Impact and Rescheduling

If the SATC is violated, i.e. if $\lambda_c < 1$, the impact of each possible system transaction is determined using (7) or (9), depending on the limiting factor that defines the SATC. Thus, the generator i with the most positive impact on the SATC that has not been fully dispatched in the bidding process, and the generator j dispatched in the market clearing process with the most negative or least positive impact on the SATC are chosen for rescheduling. Thus, the corresponding increase and decrease in generation is defined as

$$\Delta P_{S_i}^{(k)} = -\Delta P_{S_j}^{(k)} = \Delta P_S^{(k)}$$

where k is the number of iteration in the redispatch process, and $\Delta P_S^{(k)}$ is chosen depending on the value that one wants for the SATC, since the new value of the SATC may be approximated using

$$\lambda_c^{(k+1)} \approx \lambda_c^{(k)} + \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{S_i}}\right|_c^{(k)} \Delta P_{S_i}^{(k)} + \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{S_j}}\right|_c^{(k)} \Delta P_{S_j}^{(k)} \tag{10}$$

It is assumed here that $\mathrm{d}\lambda/\mathrm{d}P_{S_i}|_c^{(k)} > \mathrm{d}\lambda/\mathrm{d}P_{S_j}|_c^{(k)}$, otherwise no SATC improvements can be attained by redispatching generation.

Since the whole process is based on a linearization, one cannot make large changes in generated power, otherwise this might have a large effect on the actual SATC value, which changes nonlinearly as the parameters change; hence the need for an iterative process. The amount of generation chosen for redispatch $\Delta P_S^{(k)}$ may be readjusted when determining the actual value of $\lambda_c^{(k+1)}$ using the full nonlinear system.

Observe that system losses are not directly considered in the proposed rescheduling models. As described in [26], the losses can be assigned to a "slack bus"; shared proportionally among the suppliers according to their power bids, as assumed in this paper by means of a "distributed slack bus"; or by any other methodologies without significantly affecting the proposed rescheduling procedure.

Only if there are no adequate generators available for redispatch, is the load considered for curtailment. This approach is to be expected when the load is inelastic, as these types of loads require that the forecasted load be dispatched, given the high "costs" of load curtailment. In the case of elastic demand associated with demand-side bidding or loads with curtailment bids, however, the load could be considered for rescheduling in the same way as the generators, i.e. the load with the most negative impact on the SATC, say i, may be reduced by an amount that has a "significant" impact on the SATC value, as per approximation

$$\lambda_c^{(k+1)} \approx \lambda_c^{(k)} - \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{D_i}}\right|_c^{(k)} \Delta P_{D_i}^{(k)} \tag{11}$$

Observe that in the case of demand-side bidding, one may assume that there is no security cost for load curtailment, since the loads are intrinsically running a risk of not being dispatched by participating in the market. However, this means that overall system revenues are lost, as loads that could be served by redispatching available generation would not be dispatched, even if these loads were willing to pay the higher costs of rescheduling generated by the proposed market process. This can be also observed in OPF-based market models.

When loads are rescheduled, the transaction level T is affected by the load reduction; thus,

$$T^{(k+1)} = T^{(k)} - \Delta P_{D_i}^{(k)}$$

Furthermore, generator power bids must be reduced in this case to compensate for the reduction in demand. This reduction of excess power generation will depend on the particular market rules. Here, we assume a reduction that considers the original generator's power bid, the amount to be rescheduled from previous iterations, as well a the transaction level; thus,

$$\Delta P_{S_i}^{(k)} = \Delta P_{D_i}^{(k)} \frac{P_{S_i}^{(0)} - \sum_{j=1}^{k-1} \Delta P_{S_i}^{(j)}}{T^{(k)}}$$

This could be considered a reasonable and fair mechanism to share the load curtailment; however, other mechanisms could be readily implemented, such as rescheduling the load reduction among the participating generators based on their SATC sensitivities.

C. Rescheduling Adjustment

The step changes in generation or load are readjusted by computing the actual value of $\lambda_c^{(k+1)}$ and comparing it to the approximated value computed using (10) or (11). If the difference is greater than a chosen tolerance, the previous step and this one are repeated with smaller changes in the supply and demand until a desired tolerance is met. This step is required to account for the system nonlinearities.

D. Rescheduling Pricing

Based on the definitions of P_{G_o} and P_{L_o}, which are the power levels that define the base system conditions (SETC), we will assume that the security cost incurred by potential transactions will not be distributed among these "must-run" generators and "must-serve" loads, as their prices are determined by different market mechanisms (e.g. Must-run Contracts and averaging market prices in the Ontario market [22]). Under this assumption, the rescheduling costs for the given iteration k are determined based on a Transaction Contribution Factor (TCF) as defined by

$$\mathrm{TCF}_i^{(k)} = \frac{\mathrm{d}\lambda/\mathrm{d}p_i|_c^{(k)}\ p_i^{(k)}}{\sum_j \mathrm{d}\lambda/\mathrm{d}p_j|_c^{(k)}\ p_j^{(k)}} \tag{12}$$

where i stands for the bus number, and $p_i^{(k)}$ corresponds to the value of the corresponding parameter, i.e. the value of $P_{S_i}^{(k)}$ or $P_{D_i}^{(k)}$. Only buses with negative impact on the SATC, i.e. buses with $\mathrm{d}\lambda/\mathrm{d}p_i|_c^{(k)} < 0$, are considered in this computation; buses with positive impact are given a zero TCF value, so that market participants that do not create the security problem are not charged for the cost of keeping the system secure (OPF-based techniques, as shown in the examples discussed in Section IV, may yield "negative" congestion prices). The parameter values $p_i^{(k)}$ are included in this "normalization" process to account for the "size" of the corresponding transactions in the security cost.

The total generator redispath security cost of the k^{th} iteration may be defined as

$$\mathrm{SC}_k = (C_{S_i} - \mathrm{MCP})\ \Delta P_{S_i}^{(k)} \tag{13}$$

where i is the generator chosen in step B, with bid C_{S_i}, and MCP is the market clearing price obtained from the simple bidding process. In the case of non-curtailable loads or inelastic loads, one can assume that there is a bid or a high cost associated with curtailing the load that should be considered as part of the cost of keeping the system secure; thus, one would have in this case that

$$\mathrm{SC}_k = A_{D_i}\ \Delta P_{D_i}^{(k)} \tag{14}$$

where A_{D_i} is the "cost" of curtailing the load at the chosen bus i, which could be negotiated or "imposed", making the load elastic. Observe that the full SC_k amount in this case should be given back to the curtailed load to compensate it for the "forced" reduction in demand.

E. Convergence Check and Final Rescheduling Adjustment

If the SATC requirements are met, i.e. if $\lambda_c^{(k+1)} > 1$, then the iterative process stops, say at $k = m$. At this point, the final generator or load reschedules are adjusted based on (10) or (11), respectively, so that the SATC and final transaction level are the same, i.e. $\lambda_c^{(m+1)} \approx 1$. Thus,

$$\Delta P_{S_i}^{(m)} = -\Delta P_{S_j}^{(m)} = \frac{1-\lambda_c^{(m)}}{\mathrm{d}\lambda/\mathrm{d}P_{S_i}|_c^{(m)} - \mathrm{d}\lambda/\mathrm{d}P_{S_j}|_c^{(m)}}$$

or

$$\Delta P_{D_i}^{(m)} = \frac{1-\lambda_c^{(m)}}{\mathrm{d}\lambda/\mathrm{d}P_{D_i}|_c^{(m)}}$$

F. Computation of Nodal Congestion Prices (NCP)

The final transaction levels and NCP for each i node are readily determined as follows:

- Generators:

$$\begin{aligned} P_{S_i} &= P_{S_i}^{(0)} + \sum_{k=1}^{m} \Delta P_{S_i}^{(k)} \\ \mathrm{NCP}_{S_i} &= \frac{1}{P_{S_i}} \sum_{k=1}^{m} \mathrm{TCF}_i^{(k)}\ \mathrm{SC}_k \end{aligned} \tag{15}$$

- Loads:

$$\begin{aligned} P_{D_i} &= P_{D_i}^{(0)} - \sum_{k=1}^{m} \Delta P_{D_i}^{(k)} \\ \mathrm{NCP}_{D_i} &= \frac{1}{P_{D_i}} \sum_{k=1}^{m} \mathrm{TCF}_i^{(k)}\ \mathrm{SC}_k \end{aligned}$$

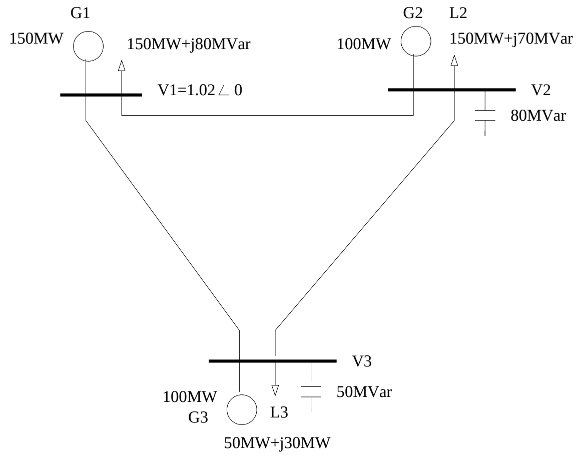

Fig. 2. 3-area test system.

TABLE I
PRICE-QUANTITY BIDS FOR 3-AREA SYSTEM

Bus i	Participant	C_i [\$/MWh]	$P_{\max}$ [MW]
1	G_1	25	150
2	G_2	33	100
3	G_3	32	100
2	L_2	30	100
3	L_3	35	100

It is important to highlight the fact that the proposed technique to compute NCP assumes that all the costs of rescheduling are fully distributed among the market participants, as in [16]. This is not the case in OPF-based methodologies, as shown in the next section, as the total congestion price paid by the loads is greater than what generators are paid, with the difference being kept by the market operator.

IV. EXAMPLES

In this section, the proposed technique is applied to three test systems, namely, a 3-area system representing three different transaction areas, a 6-bus system, and a realistic 129-bus system model of the Italian High Voltage network.

Some of the results obtained with the proposed rescheduling technique are compared to similar results obtained with the standard OPF-based methodology described in the Appendix. Although the results cannot be compared on a bus-by-bus basis, since the two methodologies are essentially different, total transaction levels and security costs obtained from each technique are used here to give the reader a general idea of both benefits and disadvantages of the proposed rescheduling procedure.

A. Three-area Test System

The 3-area test system depicted in Fig. 2 is used here to show some of the advantages of the proposed rescheduing technique. In this case, market participants are represented by suppliers G_1, G_2 and G_3, and buyers L_2 and L_3.

The market participants' price-quantity bids are shown in Table I. High-low bid matching yields an MCP = 30

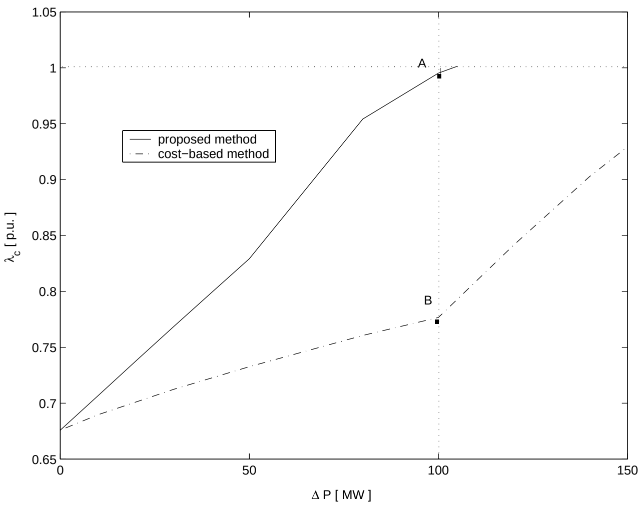

Fig. 3. Generation rescheduling results for 3-area system.

$/MWh. The potential transactions are: G1 sells 150 MW; load L2 buys 50 MW; and load L3 buys 100 MW. Thus, the total level of potential transaction is $T = 150$ MW. The most critical contingency, i.e. Line 1-2 outage, defines the SATC = 101.11 MW, which is less than the total amount of potential transactions T; hence, market rescheduling is needed for secure system operation.

The generation rescheduling results are depicted in Fig. 3. The solid-line curve shows the results obtained from the proposed rescheduling method. The power at G_2 is first chosen for increase, as it has the largest positive impact. When generation on G_2 is increased, the power at G_1, which has the least impact, must be decreased, to keep the total generation unchanged. After using up all the available power at G_2 (point A in Fig. 3), power at G_3 is then increased to facilitate all transactions. The total amount of generation rescheduling is 105 MW, and the total security cost is $SC = 310$ $/h, which can then be distributed among the participants based on the proposed NCP technique, or any other methodology.

If the cheapest available generator G_3 is chosen for rescheduling, as explained in [16], one obtains the dotted-line curve in Fig. 3. In this figure, point B represents the point at which G_3 reaches its maximum power bid, and where G_2 is chosen for rescheduling. The total rescheduling amount is 150 MW at a total rescheduling cost is 350 $/h; however, the system is not yet secure, as $\lambda_c < 1$, and thus load curtailment is needed, resulting in higher security cost.

This particular example clearly shows the advantages of proposed method, not only from the security and rescheduling point of view, but also from the perspective of possible cost reduction.

B. Six-bus Test System

The proposed technique is tested on the 6-bus test system of Fig. 4 [3], with 3 generation companies (GENCOs) that provide supply bids, and three energy supply companies (ESCOs) that provide demand bids, as shown in Table

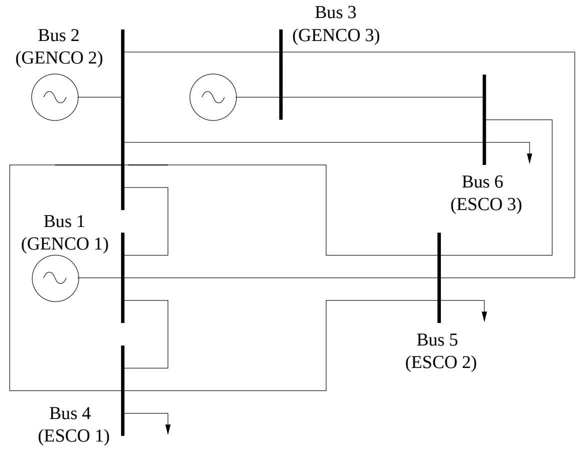

Fig. 4. 6-bus test system.

TABLE II
GENCO and ESCO Bids for 6-bus test system

Bus i	Participant	C_i [$/MWh]	$P_{\max}$ [MW]
1	GENCO 1 (S_1)	9.7	20
2	GENCO 2 (S_2)	8.8	25
3	GENCO 3 (S_3)	7	20
4	ESCO 1 (D_1)	12	25
5	ESCO 2 (D_2)	10.5	10
6	ESCO 3 (D_3)	9.5	20

TABLE III
Bus Data at Base Loading for 6-bus test system

Bus i	V_0 [p.u.]	P_{L_o} [MW]	Q_{L_o} [MVar]	P_{G_o} [MW]	$Q_{G_{\lim}}$ [MVar]	$V_{\max}$ [p.u.]	$V_{\min}$ [p.u.]
1	1.05	0	0	90	±150	1.1	0.9
2	1.05	0	0	140	±150	1.1	0.9
3	1.05	0	0	60	±150	1.1	0.9
4	0.986	90	60	0	0	1.1	0.9
5	0.969	100	70	0	0	1.1	0.9
6	0.991	90	60	0	0	1.1	0.9

TABLE IV
Line Data for 6-bus test system

Line i-j	R_{ij} [p.u.]	X_{ij} [p.u.]	$B_i/2$ [p.u.]	I_{ij}^{max} [A]	P_{ij}^{max} [MW]
1-2	0.1	0.2	0.02	1500	30
1-4	0.05	0.2	0.02	1500	60
1-5	0.08	0.3	0.03	1500	53
2-3	0.05	0.25	0.03	1500	30
2-4	0.05	0.1	0.01	1500	76
2-5	0.1	0.3	0.02	1500	35
2-6	0.07	0.2	0.025	1500	60
3-5	0.12	0.26	0.025	1500	30
3-6	0.02	0.1	0.01	1500	60
4-5	0.2	0.4	0.04	1500	15
5-6	0.1	0.3	0.03	1500	12

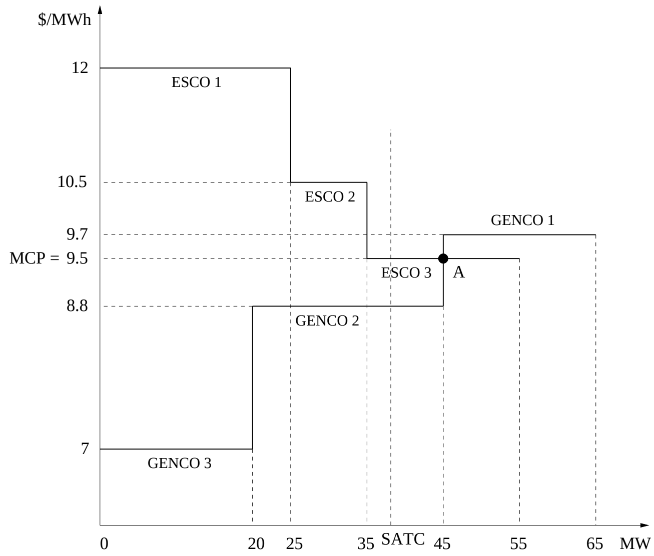

Fig. 5. Simple auction for 6-bus test system with demand-side bidding.

II. All system data, extracted from [3], are shown in Tables III and IV to facilitate the duplication of the results presented here. In Table IV, the maximum power flows in the lines obtained from off-line N-1 contingency studies are also shown; these values are *not* used for the proposed rescheduling technique, only for OPF-based computations used here for comparison purposes. The bus voltage and thermal limits used in this example are also shown in Tables III and IV. The STRM value is assumed to be zero in this case, i.e. $K = 0$ in equation (5), without loss of generality.

B.1 Simple Auction Results

A simple auction mechanism assuming demand-side bidding yields the results depicted in Fig. 5. The maximum loading value of the system in this case is 175.60 MW, which is associated with a minimum bus voltage limit. The SATC, on the other hand, is 38.23 MW, and corresponds to a minimum bus voltage limit for the worst contingency (Line 2-4 out). Hence, since SATC $< T$, where $T = 45$ MW, the rescheduling technique is needed to make the transaction feasible.

For inelastic demand, the total load must be served, i.e. the transaction level is $T = 55$ MW. The simple auction results for this case are depicted in Fig. 6. This load and generation pattern yields a maximum loading condition 219.19 MW, corresponding to a minimum bus voltage limit, and an SATC of 44.74 MW, corresponding to a minimum bus voltage limit for the worst contingency (Line 3-6 out), i.e. SATC $< T$, requiring as well of the rescheduling strategy to make the proposed transaction feasible.

B.2 Rescheduling Results

The rescheduling procedure of Fig. 1 yields the results depicted on Tables V and VI for the case of demand-side bidding. This procedure is based on the sensitivity vector $\mathrm{d}\lambda/\mathrm{d}p|_c^{(k)}$, as previously explained; for example, at the

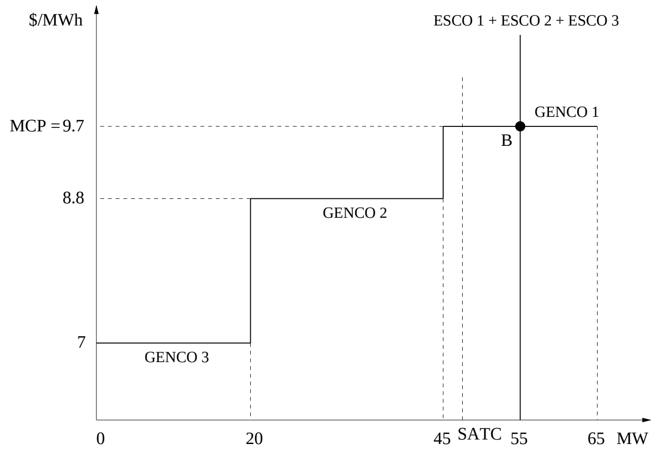

Fig. 6. Simple auction for 6-bus test system with inelastic demand.

TABLE V
RESCHEDULING AND SECURITY COSTS FOR ELASTIC DEMAND, 6-BUS TEST SYSTEM

k	$\Delta P_{G_1}^{(k)}$ [MW]	$\Delta P_{G_2}^{(k)}$ [MW]	$\lambda_c^{(k)}$	SC_k [$/h]
1	5	-5	0.90094	1.00
2	5	-5	0.95647	1.00
3	3	-3	0.992	0.60
4	1	-1	1.0042	0.20

initial iteration $k = 1$, (9) yields

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}p}\right|_c^{(1)} = \begin{bmatrix} \mathrm{d}\lambda/\mathrm{d}P_{S_1}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{S_2}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{S_3}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_1}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_2}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_3}|_c^{(1)} \end{bmatrix} = \begin{bmatrix} 0.95 \\ -0.03 \\ 0.03 \\ -1.55 \\ -0.49 \\ -0.09 \end{bmatrix} \tag{16}$$

which shows that GENCO 1 has the most positive impact on system security. Observe in (16) that curtailing ESCO 1 is more efficient for improving system security than rescheduling generation, which is typically the case.

Table VII shows the final results of the proposed rescheduling procedure in the case of demand-side bidding. Table VIII, on the other hand, depicts the results obtained by means of a standard OPF-based technique [6],

TABLE VI
TCFs FOR ELASTIC DEMAND, 6-BUS TEST SYSTEM

k	1	2	3	4
$\mathrm{TCF}_1^{(k)}$	0	0	0	0
$\mathrm{TCF}_2^{(k)}$	0.0166	0.0607	0.08	0.0813
$\mathrm{TCF}_3^{(k)}$	0	0.0324	0.0772	0.1077
$\mathrm{TCF}_4^{(k)}$	0.8554	0.8198	0.7767	0.7555
$\mathrm{TCF}_5^{(k)}$	0.1082	0.087	0.0662	0.0556
$\mathrm{TCF}_6^{(k)}$	0.0199	0	0	0

TABLE VII
Simple Auction with Rescheduling for Elastic Demand, 6-bus test system

Bus i	Part.	V [p.u.]	NCP [$/MWh]	P_S or P_D [MW]
1	S_1	1.05	0.0	15.14
2	S_2	1.05	0.0129	11.9
3	S_3	1.05	0.005	21.63
4	D_1	0.966	0.0917	25.00
5	D_2	0.956	0.0246	10.00
6	D_3	0.984	0.002	10.00

TABLE VIII
OPF-based Results for Elastic Demand, 6-bus test system

Bus i	Part.	V [p.u.]	NCP [$/MWh]	P_S or P_D [MW]
1	S_1	1.1	-0.8675	9.49
2	S_2	1.1	0.0	3.58
3	S_3	1.1	-0.0874	20.00
4	D_1	1.0224	2.2347	25.00
5	D_2	1.0177	1.0185	4.68
6	D_3	1.0431	0.3450	2.29

[27], which is briefly explained in the Appendix; the negative NCPs in this table indicate that the corresponding market participants are being paid. Thus, using the OPF-based NCPs, the total security costs paid to the generators in this case is 9.98 $/h (the sum of NCP × P_S), as opposed to the total security costs of 2.8 $/h obtained using the proposed rescheduling technique, which is fully distributed among market participants. Loads, on the other hand, pay a total of 61.42 $/h (the sum of NCP × P_D) for congestion in the OPF-based method, with the difference $61.42 - 9.28 = 52.14$$/h going to the market operator (although there is the argument that the difference between what loads paid and generators receive could be used for transmission system upgrades, the actual use of these monies is a point of contention in electricity market design). Furthermore, the total transaction level of the OPF-based solution is $T = 31.97$ MW, which is lower than the $T = 45$ MW value obtained with the proposed rescheduling method. These results show that the rescheduling technique produces, in general, better market conditions than a standard OPF-based methodology, since the overall security costs are lower while the transaction levels are higher, mainly due to the improper representation of system security through the use of line power limits computed off-line in the OPF-based technique, as previously discussed.

A similar procedure for the case of inelastic demand yields the results depicted on Table IX, which shows the NCP resulting from generator redispatching *and* load curtailment, as there are not enough power supply bids to solve the congestion problem. Thus, in this case, demand curtailment was considered assuming the costs or "bids" depicted in Table X. Observe the large NCPs due to the relatively large load curtailment costs. In this case, comparisons are not possible with the standard OPF-based methodology,

TABLE IX
Simple Auction with Rescheduling for Inelastic Demand, 6-bus test system

Bus i	Part.	V [p.u.]	NCP [$/MWh]	P_S or P_D [MW]
1	S_1	1.05	0.6726	18.99
2	S_2	1.05	0.0	23.73
3	S_3	1.05	1.501	9.49
4	D_1	0.9663	1.2597	25.00
5	D_2	0.9555	1.4345	10.00
6	D_3	0.9812	4.6284	13.00

TABLE X
Curtailment Bids for Inelastic Demand, 6-bus test system

Bus i	Participant	A_{D_i} [$/MWh]	$P_{\max}$ [MW]
4	ESCO 1 (D_1)	24	25
5	ESCO 2 (D_2)	21	10
6	ESCO 3 (D_3)	19	20

since this particular technique is not designed to properly handle load curtailment bids/costs.

C. 129-bus Italian HV Transmission System

A 129-bus model of the Italian 400 KV transmission system is depicted in Fig. 7; this system is used to test the proposed technique in a more realistic environment. In this model, 32 generators and 82 consumers are assumed to participate in the market auction. All bids are in the 30 to 40 US$/MWh range, based on the actual operating costs of thermal plants and the average prices over the last few years in other European countries where electricity markets are currently in operation. Fixed generation P_{G_o} and fixed loads P_{L_o} were assumed to be about 80 % of the average power level for a typical working day, based on the fact that most of the generation and load is still directly or indirectly owned by the state-owned utility company ENEL. The power bids are chosen to be about 30 % of the average consumption to force transmission congestion. All system data and most of the security constrains, i.e. voltage limits, generation reactive power limits and transmission line thermal limits, were provided by CESI, the Italian electrical research center. The power flow limits in transmission lines used *only* in a standard OPF-based market computations, which are utilized here for comparison purposes, were computed off-line based on the assumed power bids and through an N-1 contingency analysis.

The bid matching yields the total potential transaction level $T = 3822.49$ MW, while the SATC is determined to be 3440.7 MW; therefore, rescheduling is needed. The results of this process are shown in Table XI; the total rescheduling amount is 60.44 MW, and the total security costs are 127.55 $/h. The NCPs at some relevant nodes are shown in Fig. 8.

Figure 9 depicts, for the same buses shown in Fig. 8, the NCPs obtained by means of a standard OPF-based

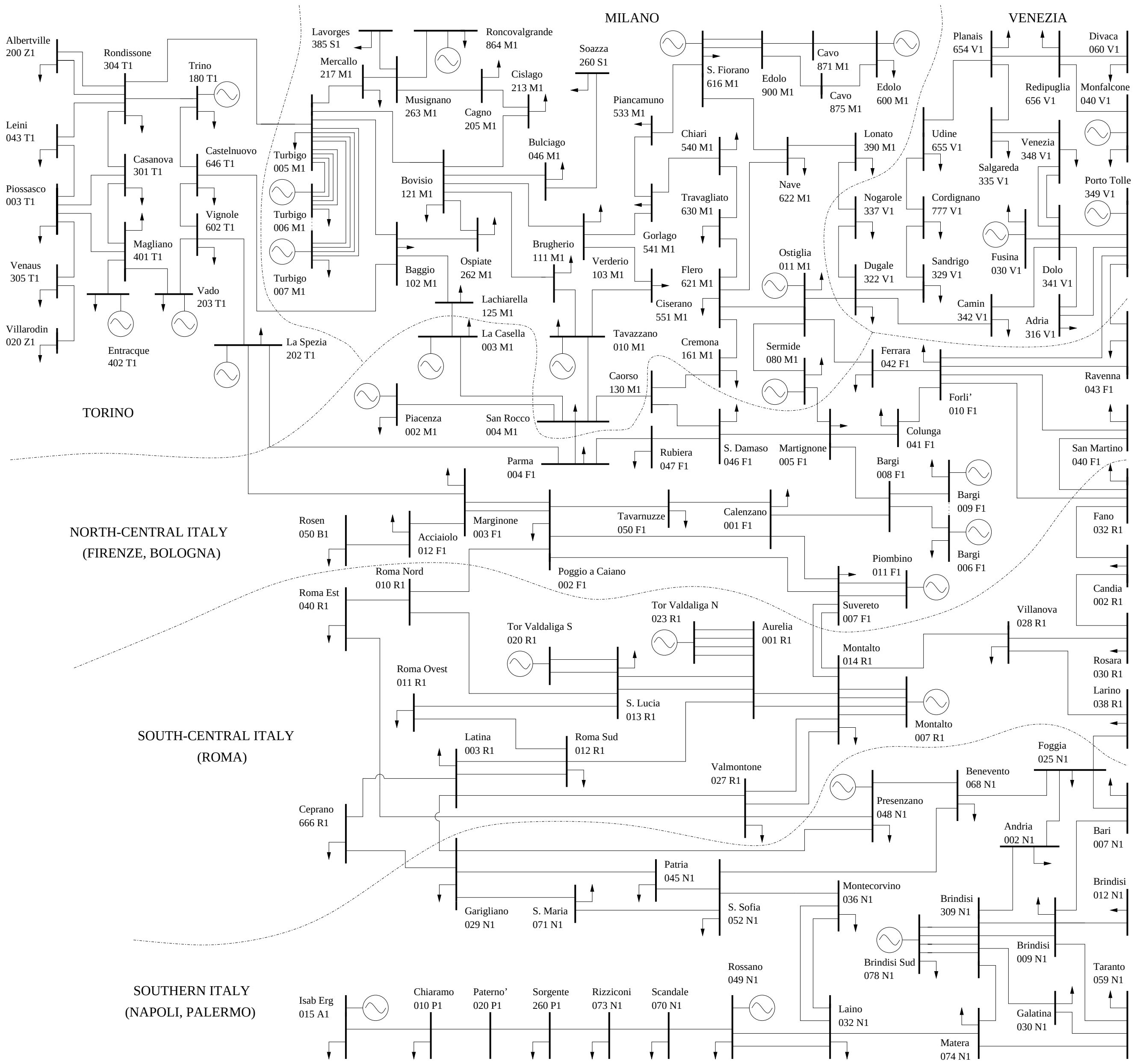

Fig. 7. Italian HV transmission system, 129-bus model.

method. In this case, the total transaction level is $T = 2639$ MW, and the total security costs paid to the generators and paid by the loads are 299.97 \$/h and 4237.10 \$/h, respectively, which in general are worse than those obtained with the proposed rescheduling technique.

V. Conclusions

A rescheduling technique is proposed, described and tested to analyze, manage and price transmission congestion in a simple-auction-based electricity market. The proposed methodology is essentially an iterative generation redispatch and load curtailment technique. Transmission congestion is represented through SATC computations, based on voltage stability constraints as well as thermal and bus voltage limits. This allows for the use of certain sensitivity formulas that form the base for the proposed techniques and pricing methodologies.

The rescheduling technique proposed in this paper could be considered as a feasible and better alternative to other methodologies that have been proposed and are currently being used in simple-auction-based electricity markets to handle transmission congestion, as the full nonlinearities of the system as well as certain system stability issues are considered in the pricing process. The comparison of the results obtained for the various test systems with respect to those obtained with a standard OPF-based technique shows that proper representation of system security limits leads to better overall market conditions, i.e. lower congestion costs and higher transaction levels.

System dynamics are not fully represented in the proposed methodology, and it is an issue that still needs to be addressed. Nevertheless, observe that the proposed itera-

TABLE XI
Simple Auction with Rescheduling for 129-bus Italian system model

k	i	j	$\lambda_c^{(k)}$	$\Delta P_s^{(k)}$ [MW]	SC_k [$/h]
1	12	13	0.92098	10	4.40
2	6	13	0.93026	10	35.60
3	8	13	0.94864	10	8.00
4	6	13	0.96939	10	35.60
5	8	13	0.97846	10	8.00
6	6	13	0.99908	10	35.60
7	8	13	1.0	0.44	0.35

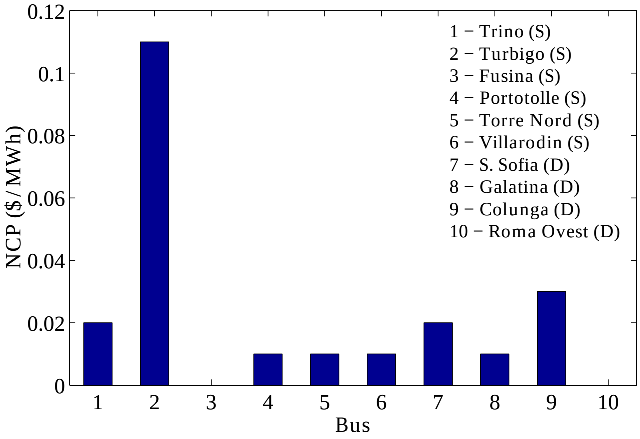

Fig. 8. NCPs of some relevant buses of the Italian system model obtained with the proposed rescheduling technique.

tive technique could be readily adapted to integrate full dynamic SATC computations, as long as the required SATC values and sensitivities can be "quickly" determined.

VI. Acknowledgments

The authors would like to thank Profs. Antonio J. Conejo and Javier Contreras from the University of Castilla-La Mancha for their valuable suggestions and comments throughout the development of this work. Thanks are also due to Dr. M. Pozzi and Mr. A. Danelli from CESI for proving the Italian system data.

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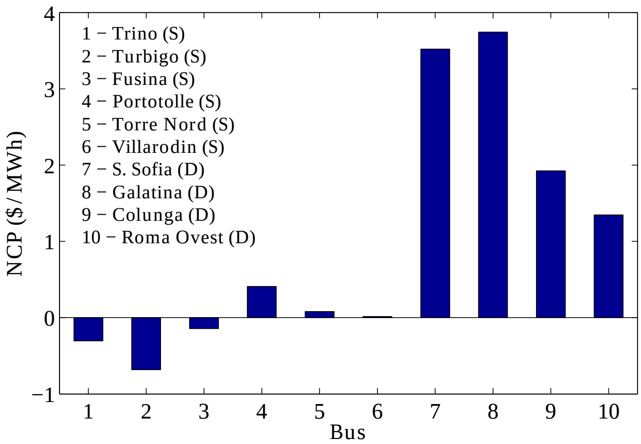

Fig. 9. NCPs of some relevant buses of the Italian system model obtained with a standard OPF-based approach.

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Appendix

A standard OPF-based technique was used to provide a basis for evaluating the performance of the proposed method for pricing transmission congestion. The OPF market problem is typically formulated as follows:

$$\begin{array}{lllll}
\text{Min.} & -\left(C_D^T P_D - C_S^T P_S\right) & \rightarrow & \text{Social benefit} & (17)\\
\text{s.t.} & f(\delta, V, Q_G, P_S, P_D) = 0 & \rightarrow & \text{PF equations} & \\
 & 0 \le P_S \le P_{S_{\max}} & \rightarrow & \text{Sup. bid blocks} & \\
 & 0 \le P_D \le P_{D_{\max}} & \rightarrow & \text{Dem. bid blocks} & \\
 & \mid P_{ij}(\delta, V) \mid \le P_{ij_{\max}} & \rightarrow & \text{Power transfer lim.} & \\
 & \mid P_{ji}(\delta, V) \mid \le P_{ji_{\max}} & & & \\
 & I_{ij}(\delta, V) \le I_{ij_{\max}} & \rightarrow & \text{Thermal limits} & \\
 & I_{ji}(\delta, V) \le I_{ji_{\max}} & & & \\
 & Q_{G_{\min}} \le Q_G \le Q_{G_{\max}} & \rightarrow & \text{Gen. } Q \text{ lim.} & \\
 & V_{\min} \le V \le V_{\max} & \rightarrow & V \text{ ``security'' lim.} &
\end{array}$$

where C_S and C_D are vectors of supply and demand bids in \$/MWh, respectively; Q_G stand for the generator reactive powers; V and δ represent the bus phasor voltages; P_{ij} and P_{ji} represent the power flowing through the lines in both directions, and are used to model system security by limiting the transmission line power flows, togheter with line current I_{ij} and I_{ji} thermal limits and bus voltage limits V_{min} and V_{max}; and P_S and P_D represent bounded supply and demand power bids in MW. In this model, which is typically referred to as a security constrained OPF, P_{ij} and P_{ji} limits are obtained by means of off-line stability studies, considering an N-1 contingency criterion.

Using the decomposition formula for LMPs proposed in [6], [27], one can define the NCPs as follows:

$$\text{NCP} = \left(\frac{\partial f^T}{\partial y}\right)^{-1} \frac{\partial h^T}{\partial y}(\mu_{\max} - \mu_{\min}) \qquad (18)$$

where $y = [\delta \ V]^T$, h represents the inequality constraint functions (e.g. transmission line powers), and $\mu_{\max}$ and $\mu_{\min}$ are the dual variables or shadow prices associated with the inequality constraints. Equation (18) is a vector of active and reactive nodal congestion prices.

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