Linear Performance Indices to Predict Oscillatory Stability Problems in Power Systems

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***Abstract*—In this paper, various indices are proposed and studied to detect and predict oscillatory instabilities associated with Hopf bifurcations in power systems. A methodology is also presented to produce a linear profile for these indices. The indices are based on eigenvalue and singular values of the state and extended system matrices. Applications of these indices to several test power systems are presented here, demonstrating the usefulness of some of these indices, particularly for on-line applications.**

***Index Terms*— Hopf bifurcations, on-line performance indices, second order index, eigenvalues, singular values, subsynchronous resonance, power system oscillations.**

I. Introduction

NONLINEAR phenomena, including bifurcations and chaos, occurring in power system models have been the subject of several studies during the last two decades (e.g. [1], [2], [3], [4], [5], [6], [7]). Among the different types of bifurcations, saddle-node, limit-induced, and Hopf bifurcations (HBs) have been identified as pertinent to instability problems in power systems [8], [7].

In the case of saddle-node bifurcations, a singularity of a system Jacobian and/or state matrix results in disappearance of steady state solutions, whereas in the case of certain types of limit-induced bifurcations, the lack of steady state solutions arises from system controls reaching limits (e.g. generator reactive power limits). Both bifurcation modes typically lead to voltage collapse [9]. HBs, on the other hand, produce limit cycles (periodic orbits) that may lead the system to oscillatory instabilities, as it has been shown in a variety of power system models (e.g. [2], [8], [10], [11]), and observed in practice (e.g. [12], [13]).

HBs could arise due to variable net damping, frequency dependence of electrical torque and voltage control issues (e.g. fast acting automatic voltage regulators in generators) [10], [14], and are typically triggered by system contingencies. In most cases, these bifurcations occur on very stressed systems, i.e. heavily loaded systems. This is a concern nowadays, as many current networks operate near their stability limits due to economical and environmental constraints. Incidents of Hopf bifurcation (HB) induced system collapses include the Sri Lankan power system disturbance of May 2, 1995 [12], and the Western System Coordination Council (WSCC) system disturbance of August 10, 1996 [13]. With ways of predicting and controlling HBs, the above incidents conceivably could have been avoided by operator action (e.g. load shedding). Further planning studies can then be pursued to place PSS and/or proper FACTS controllers to take care of the oscillation problems in a more permanent basis [15].

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The attention of researchers reported in the literature has been more in trying to reveal the presence of HBs, rather than on their prediction. An index to determine the proximity of a system to a HB with respect to a given system parameter would be beneficial, especially as a tool in system operation. While there has been some work done in this area using optimization techniques [16], extensive computations and optimizations are needed every time the topology of the system changes. Methods to detect HBs in general dynamical systems based on using the real part of a “critical” eigenvalue as an index are presented in [17]; however, its non-linear behavior in some cases makes it impractical for power system applications, as shown in the present paper. A closed-loop monitoring system for detecting impending instability related to HBs in uncertain nonlinear plants is proposed in [18]; an advantage of this methodology is that it gives a warning at a point close to the instability even when no accurate system model is available. However, earlier prediction of such an event is not possible, as the index is not continuous or smooth. A predictable index with a quasi-linear profile is useful in projecting problematic loading levels, for a given generation and load directions, both in planning and operation stages. By predicting a problem (e.g. HB) well in advance, a measure can be devised (e.g. load shedding) to mitigate the problem. In the current paper, linear profile indices are proposed and their practical benefits are demonstrated.

This paper is organized as follows: Section II explains briefly the basic theory behind HBs from the point of view of power systems. The proposed indices are dicussed in Section III together with a methodology to linearize them. In Section IV, the results obtained for various test systems are presented and discussed, from the point of view of on-line application of the proposed indices. Finally, the main contributions made in this paper are highlighted in Section V.

II. Hopf Bifurcations

Power system oscillations problem are classically associated with a pair of complex eigenvalues of system equilibria (operating points) crossing the imaginary axis of the complex plane, from the left half-plane to the right half-plane, when the system undergoes sudden changes (e.g. line outages) [19]. If this particular oscillatory problem is studied using more gradual changes in the system, such as changes on slow varying parameters like system loading, it can be directly viewed as a HB problem [6], [10]. Thus, by predicting or detecting these types of bifurcations well in advance to the onset of a possible oscillatory instability, this type of problem can be avoided.

HBs are characterized by periodic orbits or limit cycles emerging around an equilibrium point. These types of bifurcation are also known as oscillatory bifurcations. In order to explain the basic theory behind the HB from the point of view of power system, consider the following DAE model of a power system:

$$\begin{aligned} \dot{x} &= f(x, y, \lambda, p) \\ 0 &= g(x, y, \lambda, p) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is a vector of state variables, such as generator internal angles and rotational speed; $y \in \Re^m$ is a vector of algebraic variables, such as load voltage magnitudes; $\lambda \in \Re^\ell$ is a set of uncontrollable parameters, such as active and reactive power load variations; and p is a set of controllable parameters, such as AVR set points. When the parameters λ and/or p vary, the equilibrium points (x_o, y_o) change, and so do the eigenvalues of the corresponding system state matrix. The equilibrium points are asymptotically stable if all the eigenvalues of the system state matrix have negative real parts. The point where a complex conjugate pair of eigenvalues reach the imaginary axis with respect to the changes in (λ, p), say $(x_o, y_o, \lambda_o, p_o)$, is known as a HB point.

At a HB point $(x_o, y_o, \lambda_o, p_o)$, the following conditions are satisfied [17]:

1) $[f(x_o, y_o, \lambda_o, p_o)\ g(x_o, y_o, \lambda_o, p_o)]^T = 0$.
2) The Jacobian matrix evaluated at $(x_o, y_o, \lambda_o, p_o)$ should only have a simple pair of purely imaginary eigenvalues $\mu = \pm j\beta$.
3) The rate of change of the real part of the critical eigenvalues with respect to a varying system parameter, say λ_i, should be nonzero.

If this is the case, a limit cycle appears at $(x_o, y_o, \lambda_o, p_o)$ with an initial period of $T_o = 2\pi/\beta$.

These conditions imply that a HB corresponds to a system equilibrium state with a pair of purely imaginary eigenvalues with all other eigenvalues having non-zero real parts, and that the pair of bifurcating or critical eigenvalues cross the imaginary axis as the parameters (λ, p) change, yielding oscillations in the system.

III. Hopf Bifurcation Indices

A. Eigenvalue Index (EVI)

In [17], the real part of the complex pair of eigenvalues crossing the imaginary axis (critical eigenvalues) is used as an index to predict the HB point. Hence, in this paper the index

$$EVI = |\alpha| \tag{2}$$

where α is the real part of the critical eigenvalue μ, is also used to study its behavior in power system applications.

B. First Index (HBI_1)

Since at a HB point the system Jacobian has a simple pair of purely imaginary eigenvalues, the problem can be restated as follows: For the system state matrix A, a complex pair of eigenvalues can be rewritten as

$$A\ [v_R \pm jv_I] = [\alpha \pm j\beta]\ [v_R \pm jv_I] \tag{3}$$

where $A = D_x f|_0 - D_y f|_0 D_y g|_0{}^{-1} D_x g|_0$; α and β are the real and imaginary parts of the critical eigenvalue μ, respectively; and $v_R \pm jv_I$ is the associated eigenvector. If real and imaginary parts are separated from equation (3), it follows that

$$\begin{aligned} (A - \alpha I_n)v_R + \beta v_I &= 0 \\ (A - \alpha I_n)v_I - \beta v_R &= 0 \end{aligned}$$

$$\Rightarrow \left(\underbrace{\begin{bmatrix} A & +\beta I_n \\ -\beta I_n & A \end{bmatrix}}_{A_m} - \alpha I_{2n} \right) \begin{bmatrix} v_R \\ v_I \end{bmatrix} = 0 \tag{4}$$

Since $[v_R\ v_I]^T \neq 0$, at a HB where $\alpha = 0$, the modified matrix A_m becomes singular. Observe that this matrix is also singular at a saddle-node bifurcation, as $\alpha = \beta = 0$ in this case.

Following the same criteria previously proposed to define indices for saddle-node bifurcations [9], the singular value of the modified state matrix is used here as an index for detecting HBs. Thus, the first Hopf bifurcation index (HBI) is defined as follows:

$$HBI_1(A, \beta) = \sigma_{min}(A_m) \tag{5}$$

where σ_{min} is the minimum singular value of the modified state matrix A_m, which becomes zero at a Hopf or saddle-node bifurcation point.

C. Second Index (HBI_2)

The HBI_1 index has the problem that it requires the computation of the state matrix A, which is computationally expensive, as it requires the inverse of the jacobian $D_y g|_0$. This problem can be avoided if the full system matrix is used. Thus, using a generalized eigenvalue formulation, for a complex pair of eigenvalues, it follows that

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} v_{1_R} \pm jv_{1_I} \\ v_{2_R} \pm jv_{2_I} \end{bmatrix} = [\alpha \pm j\beta] \begin{bmatrix} v_{1_R} \pm jv_{1_I} \\ 0 \end{bmatrix} \tag{6}$$

where $J_1 = D_x f|_0$, $J_2 = D_y f|_0$, $J_3 = D_x g|_0$, and $J_4 = D_y g|_0$. By separating the real and imaginary parts and rearranging these equations:

$$\left(\underbrace{\begin{bmatrix} J_1 & J_2 & \beta I_n & 0 \\ J_3 & J_4 & 0 & 0 \\ -\beta I_n & 0 & J_1 & J_2 \\ 0 & 0 & J_3 & J_4 \end{bmatrix}}_{J_m} - \alpha \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} v_{1_R} \\ v_{2_R} \\ v_{1_I} \\ v_{2_I} \end{bmatrix} = 0 \tag{7}$$

Since $\alpha = 0$ at a HB, the matrix J_m becomes singular; notice that this also holds at a saddle-node bifurcation point. Therefore, the minimum singular value of the modified full Jacobian matrix J_m can be used as another index to indicate proximity to a Hopf or a saddle-node bifurcation. Thus,

$$HBI_2(A, \beta) = \sigma_{min}(J_m) \tag{8}$$

This index is computationally more efficient than HBI_1, as full advantage can be taken of the sparsity of J_m.

Observe that both indices HBI_1 and HBI_2 behave similarly to the singular value index of A and J, respectively, when the critical eigenvalue is real (the critical eigenvalue is defined here as the eigenvalue closest to the imaginary axis as the system parameter changes). Thus, for β=0:

$$A_m = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$$

$$\Rightarrow HBI_1 = \sigma_{min}(A_m) = \sigma_{min}(A) \tag{9}$$

and

$$J_m = \begin{bmatrix} J_1 & J_2 & 0 & 0 \\ J_3 & J_4 & 0 & 0 \\ 0 & 0 & J_1 & J_2 \\ 0 & 0 & J_3 & J_4 \end{bmatrix} = \begin{bmatrix} J & 0 \\ 0 & J \end{bmatrix}$$

$$\Rightarrow HBI_2 = \sigma_{min}(J_m) = \sigma_{min}(J) \tag{10}$$

D. Linearization

Indices based on first order information such as critical eigenvalues and minimum singular values may be inadequate to predict possible instability problems in practical power systems, due to large discontinuities in the presence of system control limits (e.g. generator limits). However, this problem can be reduced by considering a "second order" index, i.e. an index defined by the index divided by its gradient with respect to the parameter under study, which exploits additional information embedded in these indices, as suggested in [20]. Thus, it has been observed that the critical eigenvalue and minimum singular value of a power system Jacobian can be approximated using the following equation [9]:

$$\gamma = (a - b\lambda)^{1/c} \tag{11}$$

where γ stands for the critical eigenvalue μ or minimum singular value σ_{min}, with suitable values of the scalar positive constants a, b and c, and λ being a given varying parameter. These type of functions can be made linear by dividing the function by its gradient at each point [9], since

$$\frac{\gamma}{d\gamma/d\lambda} = c\lambda - \frac{ac}{b} \tag{12}$$

Hence, by following the same idea, a linearized version for each of the EVI, HBI_1 and HBI_2 indices is proposed here:

$$LEVI = \frac{EVI}{|dEVI/d\lambda|} \tag{13}$$

$$LHBI_1 = \frac{HBI_1}{|dHBI_1/d\lambda|} \tag{14}$$

$$LHBI_2 = \frac{HBI_2}{|dHBI_2/d\lambda|} \tag{15}$$

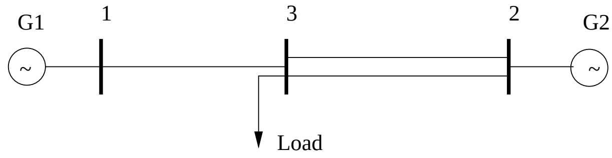

Fig. 1. Three-bus test system.

It is worth mentioning that these indices can be applied to any nonlinear dynamical system to predict and detect HBs. However, the application to power systems is the primary concern of this paper.

IV. Simulation Results

The Hopf bifurcation indices EVI, HBI_1 and HBI_2, as well as $LEVI$, $LHBI_1$ and $LHBI_2$, were applied to several power system examples. Although most of the examples presented in this section are related to low frequency oscillations, an example of oscillatory problem associated with subsynchronous resonance phenomena is also presented and discussed.

A. Low Frequency Oscillations

In all the examples discussed in this section, loads were modeled as constant impedance models for stability studies, since these are the less onerous loads for the system (if HB problems are observed with these types of load models, similar problems could be reasonably expected with more stressful loads such as constant power load models). Furthermore, most of the commercial power system analytical tools use this as a default load model for dynamic analyses. For the associated power flow studies, all the loads were represented as typical constant real and reactive power models, and these are assumed to change according to

$$\begin{aligned} P_L &= P_{Lo}(1+\lambda) \\ Q_L &= Q_{Lo}(1+\lambda) \end{aligned} \tag{16}$$

where P_{Lo} and Q_{Lo} are the initial or base real and reactive power levels, respectively, and λ is the varying parameter representing the loading factor.

1) Three-bus System: A one line diagram of the simplest test system used in this paper is shown in Fig. 1 [21]. Both generators were modeled in detail with standard exciters type AC4a [21], and hydraulic governors. The nominal load is 900 MW, and 300 Mvar.

Figure 2 shows the corresponding P-V curves at bus 3, for the base case and also for a line 2-3 outage. The real and reactive power required by the system are shared equally by both generators, as the system load increases up to the nose point. The HB points are also depicted in this figure, as well as the load line associated with an impedance load model; the crossing points of this line with the corresponding P-V curves define an equilibrium point of the associated dynamic system model.

Figures 3 shows the locus of the critical eigenvalue for the base case. As it can be seen in this figure, only the critical eigenvalue moves to the right as the loading parameter λ changes,

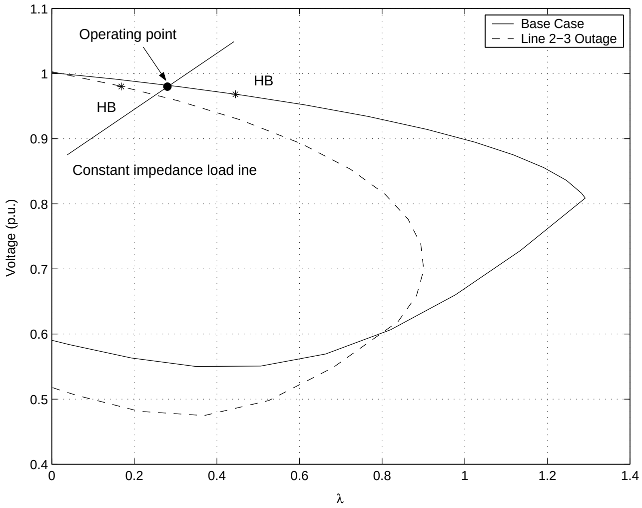

Fig. 2. P-V curves at bus 3 for the three-bus system.

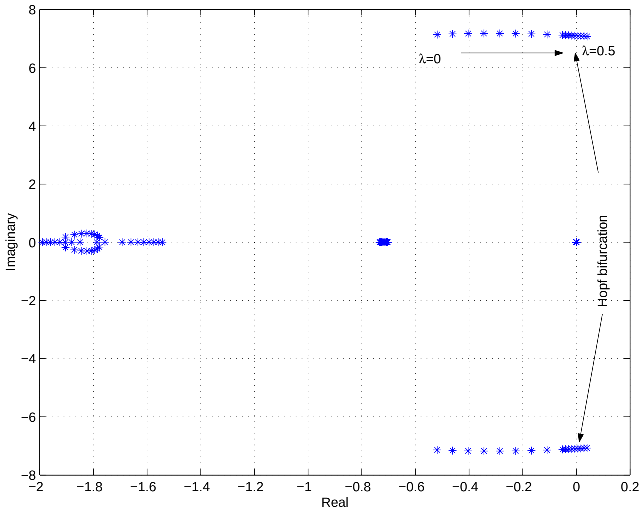

Fig. 3. Locus of the critical eigenvalue for the three-bus system base case.

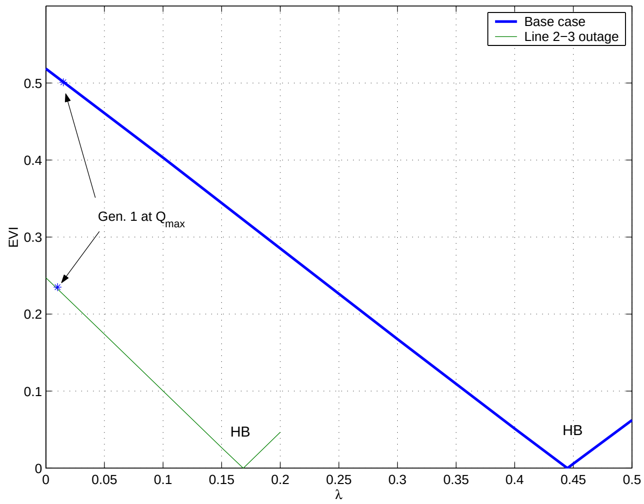

Fig. 4. Eigenvalue index for the three-bus system.

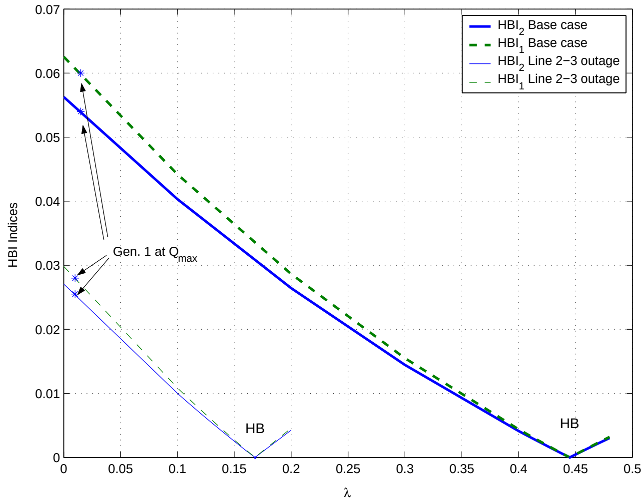

Fig. 5. Hopf bifurcation indices for the three-bus system.

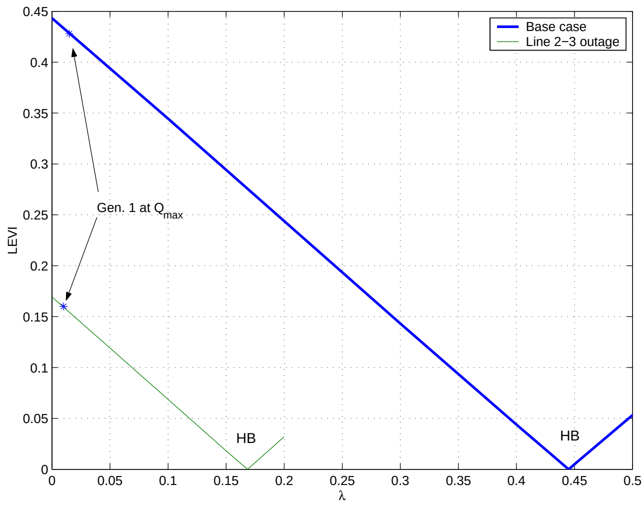

Fig. 6. Linearized eigenvalue index for the three-bus system.

and crosses the imaginary axis creating a HB condition; the rest of the eigenvalues do not change significantly with changes in λ (the zero eigenvalue is due to the absence of an infinite bus in the eigenvalue computations). A participation factor analysis indicates that the dominant state variables associated with the critical mode (HB modes) are the δ and ω of G2.

Figures 4, 5, 6 and 7 depict the Hopf bifurcation indices EVI, HBI_1 and HBI_2, and their linearized versions $LEVI$, $LHBI_1$ and $LHBI_2$, respectively. In all these figures, the points at which generators reach reactive power limits are clearly highlighted. The second order or gradient information of the Hopf bifurcation indices required for the computation of $LEVI$, $LHBI_1$ and $LHBI_2$ are calculated numerically. All these indices, especially the linearized ones, show linear profiles with respect to the loading factor λ. According to the indices, loading the base system beyond 0.45 p.u. is problematic, showing also that a line 2-3 outage could lead the system to an oscillatory unstable condition when the system operates beyond a 0.163 p.u. loading factor level. All these results were confirmed through time domain simulations.

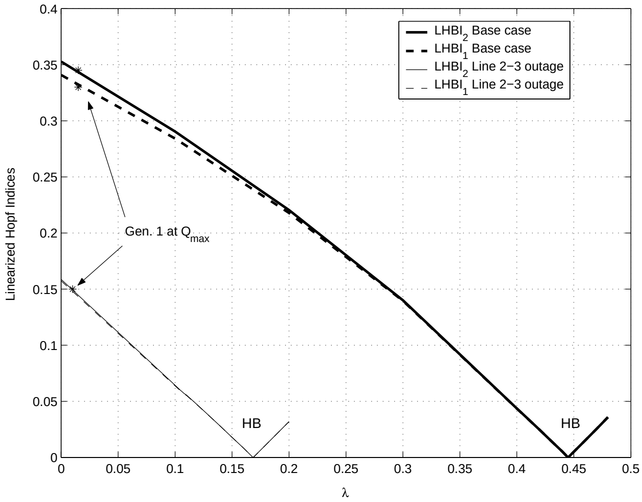

Fig. 7. Linearized Hopf bifurcation indices for the three-bus system.

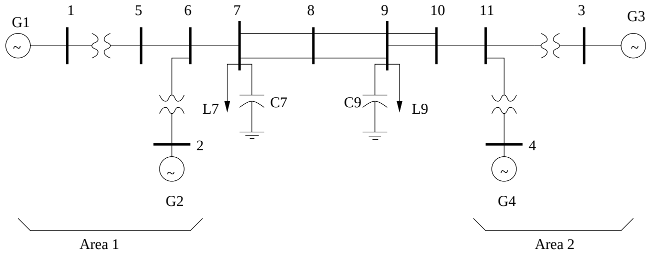

Fig. 8. Two-area test system.

2) Two-area System: Figure 8 is the one-line diagram of the "classical" two-area system proposed in [19] for oscillation studies. The topology of the system is symmetrical with respect to bus 8; however, the loads and the limits of individual generator are not equal in both areas. All of the generators were modeled in detail with a simple exciter, except G2. A simple turbine governor is also considered in each of the generators, except G3. The total system load is 2734 MW and 200 Mvar.

Figures 9, 10 and 11 show the corresponding P-V curves, and eigenvalue and HB indices, for the base case and for a line 9-10 outage, and Figs. 12 and 13 shows the linearized version of the eigenvalue and HB indices, respectively; no generator reactive power limits are reached in this case. Once again, the gradient information, which is necessary for obtaining of the linearized Hopf bifurcation indices, was calculated numerically. Observe the linear behavior of the indices. According to the results, loading the system beyond 0.081 p.u. is not feasible. A participation factor analysis indicates that the dominant state variables associated with the critical mode (HB mode) are δ and ω for G3. These results were verified using the time domain analyses.

3) IEEE 50-machine System: The IEEE 50-machine system shown in Fig. 14 is an approximated model of an actual power system, and was originally developed for stability studies [22]. It consists of 145 buses and 453 lines, including 52 fixed tap transformers. There are 60 loads for a total of 2.83 GW and 0.80 Gvar.

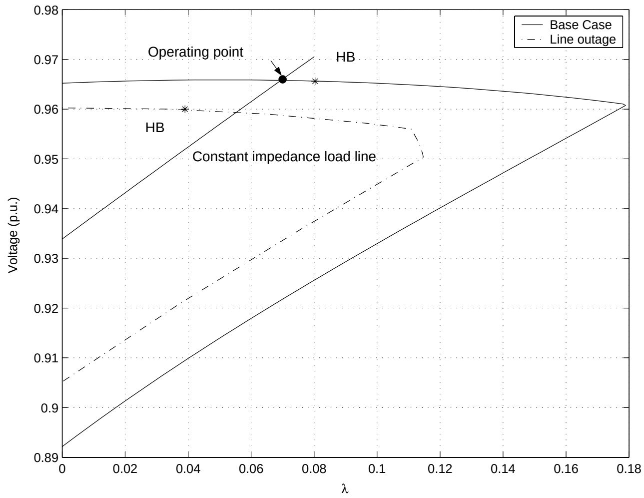

Fig. 9. P-V curves at bus 11 for the two-area system.

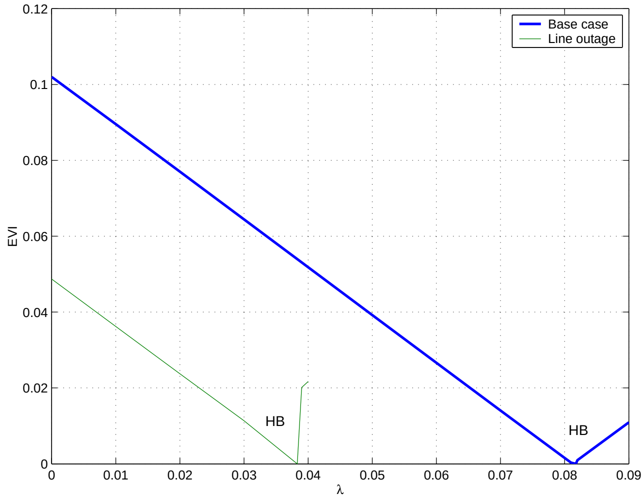

Fig. 10. Eigenvalue index for the two-area system.

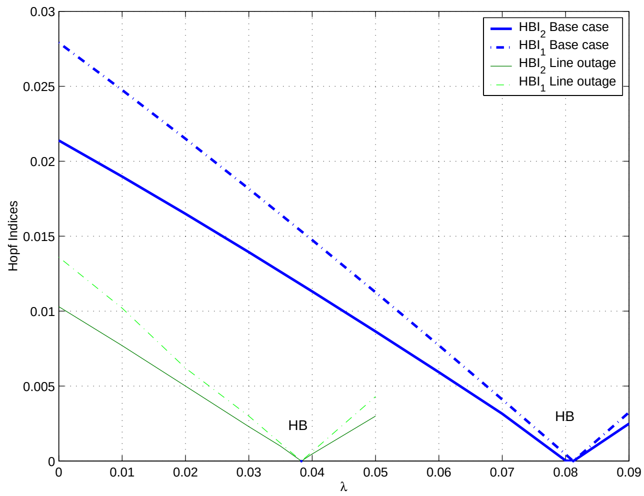

Fig. 11. Hopf bifurcation indices for the two-area system.

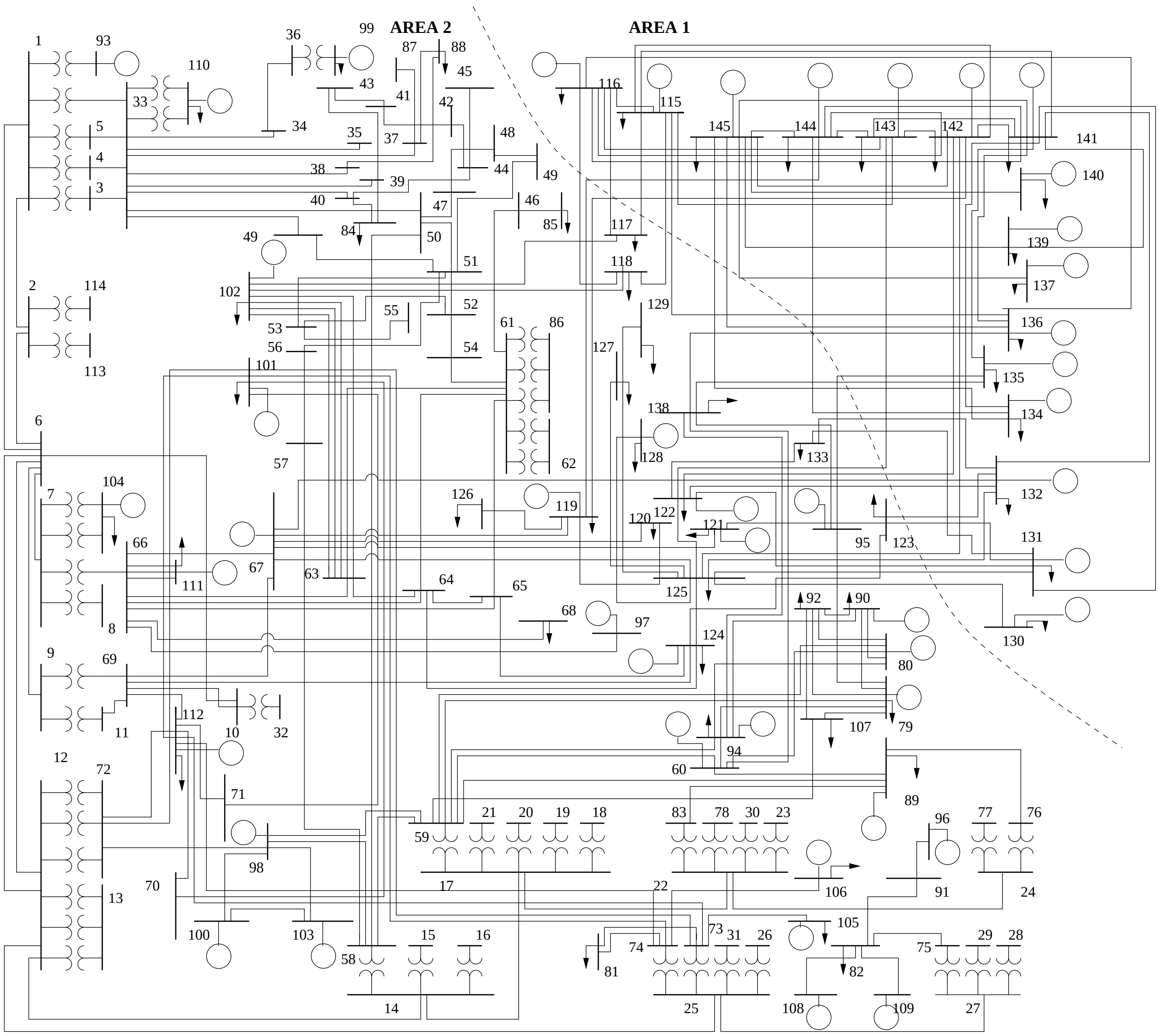

Fig. 14. IEEE 50-machine test system.

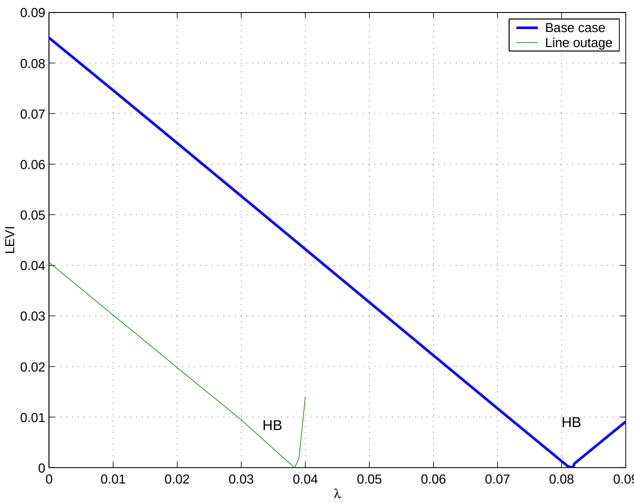

Fig. 12. Linearized eigenvalue index for the two-area system.

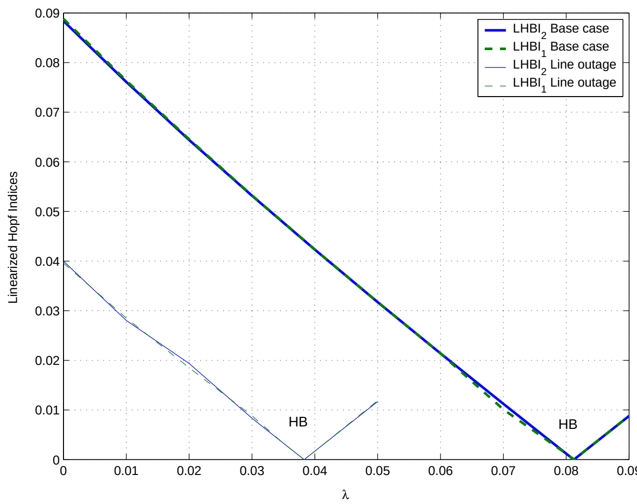

Fig. 13. Linearized Hopf bifurcation indices for the two-area system.

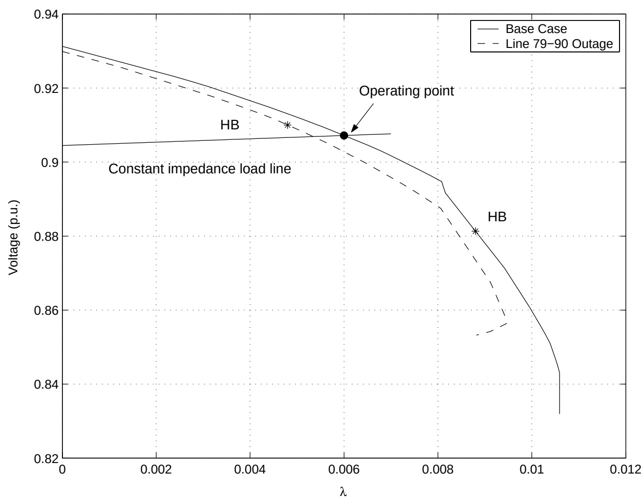

Fig. 15. P-V curve at bus 92 for the IEEE 50-machine system, Case I.

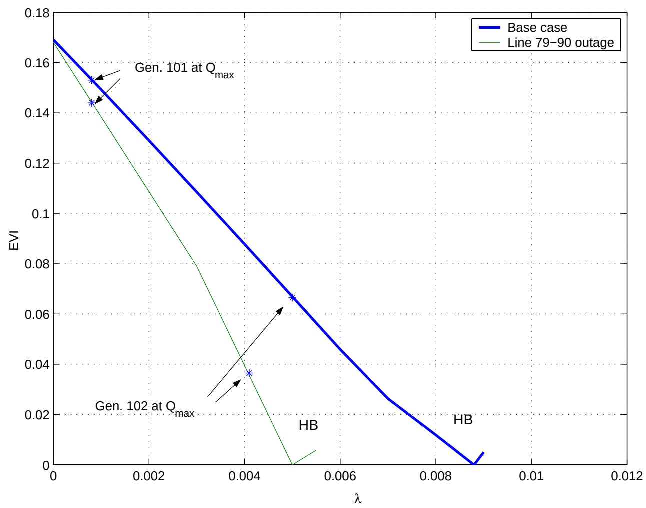

Fig. 16. Eigenvalue index for the IEEE 50-machine system, Case I.

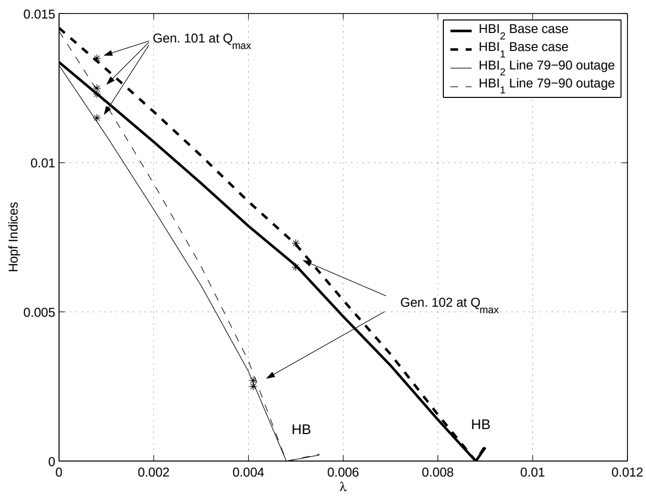

Fig. 17. Hopf bifurcation indices for the IEEE 50-machine system, Case I.

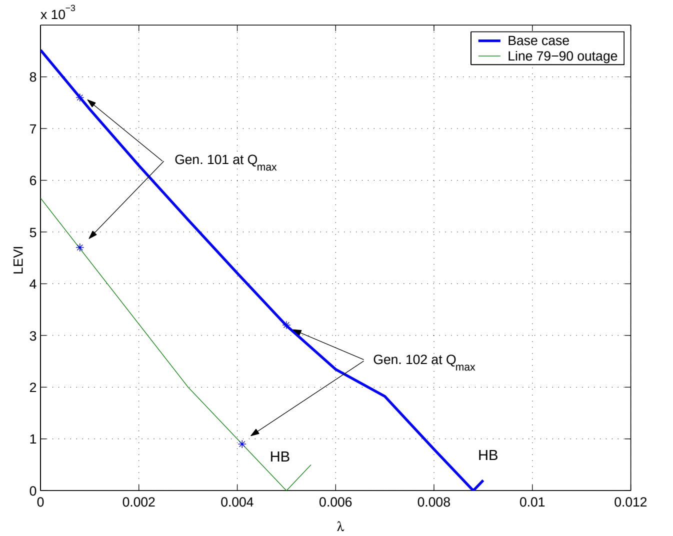

Fig. 18. Linearized eigenvalue index for the IEEE 50-machine system, Case I.

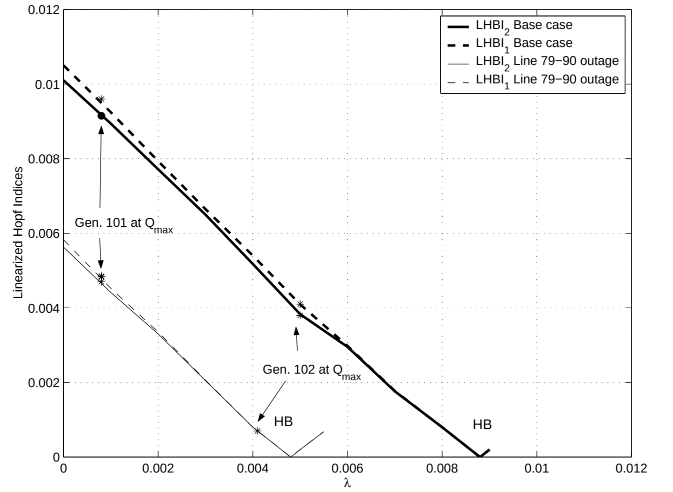

Fig. 19. Linearized Hopf bifurcation indices for the IEEE 50-machine system, Case I.

1) *Case I*: Seven of the generators are modeled in detail in this case, including complete models of their exciters. P-V curves including HB points for the base case and for a line 79-90 outage are shown in Fig. 15. The HB indices and the linearized version of the indices are given in Figs. 16, 17, 19 and 19, respectively; some of the points at which generator reach reactive limits are also highlighted in all these graphs. The line 79-90 outage was chosen since this line is one of the most heavily loaded lines in the weakest area of the system. Once again, observe the linear or quasi-linear profile of these indices with respect to the loading factor λ. In this case too, the gradient information of the HB indices with respect to the parameter was calculated numerically. All of the HB points were confirmed by time domain simulations.
2) *Case II*: In this case, six generators are modeled in detail with simple exciter models. Figures 20, 21, 23 and 22 depict the EVI, HBI_1 and corresponding $LEVI$ and $LHBI_1$ results. Observe the general nonlinear profile of

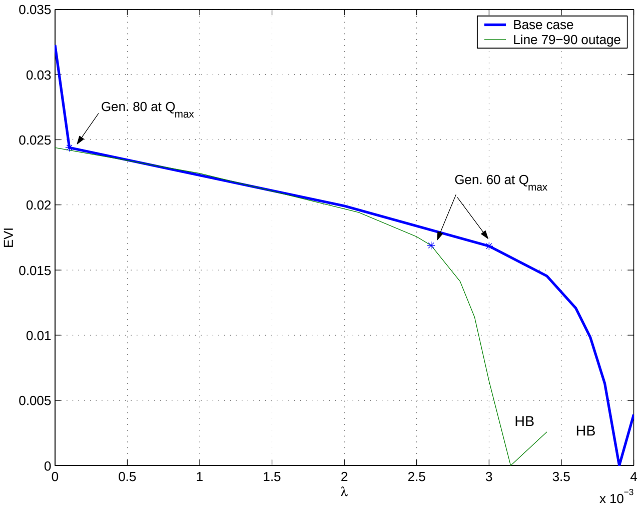

Fig. 20. Eigenvalue index for the IEEE 50-machine system, Case II.

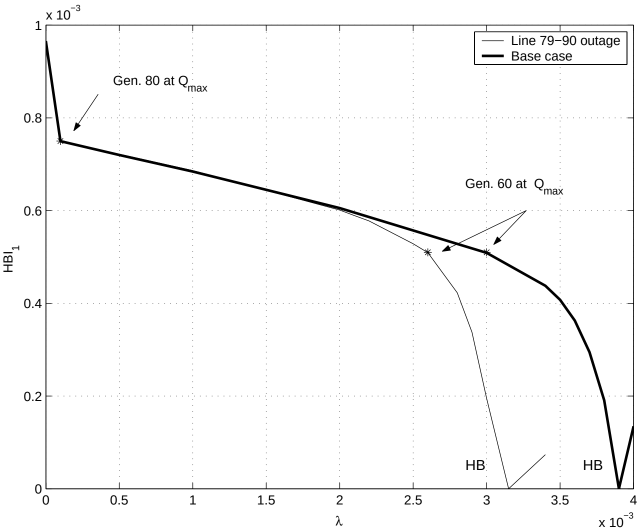

Fig. 21. Hopf bifurcation index for the IEEE 50-machine system, Case II.

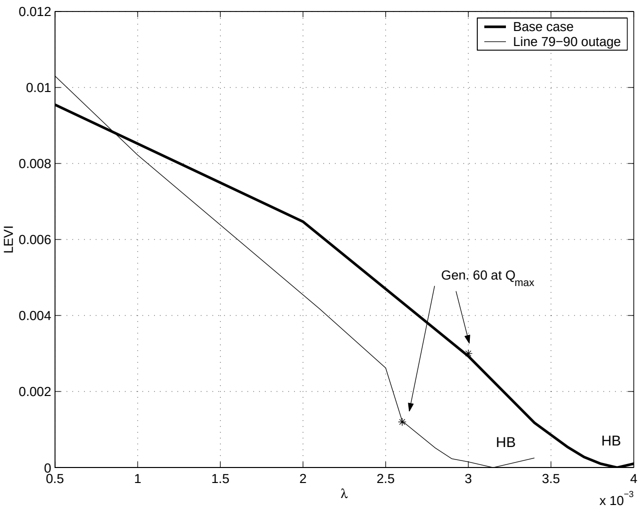

Fig. 22. Linearized eigenvalue index for the IEEE 50-machine system, Case II.

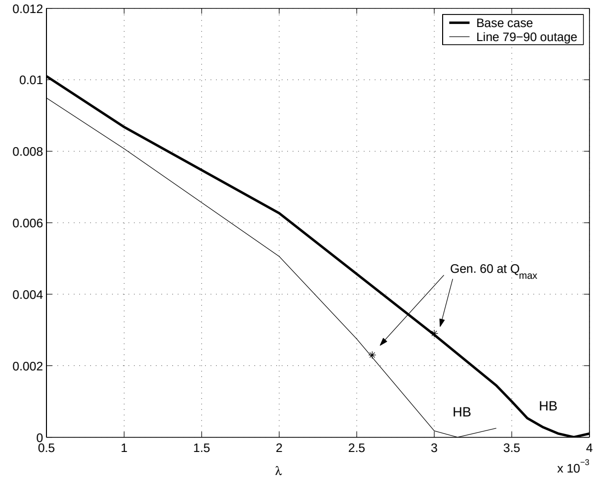

Fig. 23. Linearized Hopf bifurcation index for the IEEE 50-machine system, Case II.

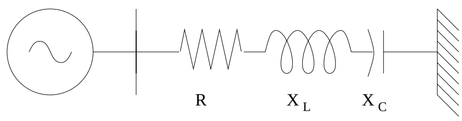

Fig. 24. Generator-∞ bus system with compensated line.

the EVI, HBI_1 and $LVEI$ indices, whereas $LHBI_1$ presents a much better profile. (The HBI_2 and corresponding $LHBI_2$ indices are not presented in this particular case due to the limitations of the software package used to obtain the results discussed in this particular section, as the program did not generate the extended matrix information required to compute these indices.)

B. Subsynchronous Resonance Example

Subsynchronous resonance (SSR) is a well-known problem of undamped oscillations that may occur when the transmission line to which the machine is connected is compensated by a series capacitor, as depicted in Fig. 24. In this case, the dynamics of the RLC circuit cannot be neglected, since the line presents two modes whose frequency can be roughly estimated as $\omega_b(1 \pm \sqrt{X_C/X_L})$, where ω_b is the base frequency in rad/sec. For typical values of the inductive and capacitive reactances, the lower of these two frequencies can be close to one of the mechanical oscillations of the generator shaft. Thus, beyond a certain value of the compensation level, the machine may experience a negative damping of one of the mechanical modes that results in dangerous stress to the shaft.

The Hopf bifurcation indices EVI and HBI_1 are used here to predict the SSR point, with respect to the capacitor compensation X_c, which in this case is the parameter λ. The system model used in this paper is identical to the one presented in [3]. Thus, a complex mode crosses the imaginary axis creating a Hopf bifurcation as λ increases, as shown in Fig. 25; observe that a pair of complex eigenvalues with a frequency of about 26 Hz cross the imaginary axis at $\lambda = X_c = 0.158$p.u..

The EVI and $LEVI$, and HBI_1 and $LHBI_1$ are illustrated in Figs. 26 and 27, respectively. Notice the quasi-linear and smooth behavior of the $LHBI_1$, which can be used to predict

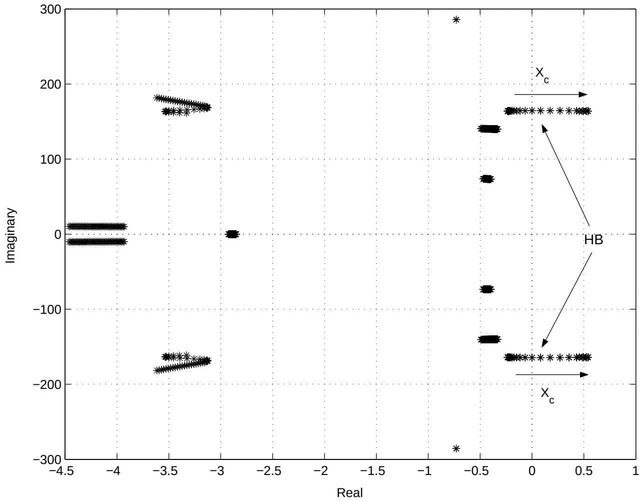

Fig. 25. Locus of the critical eigenvalue for the SSR example.

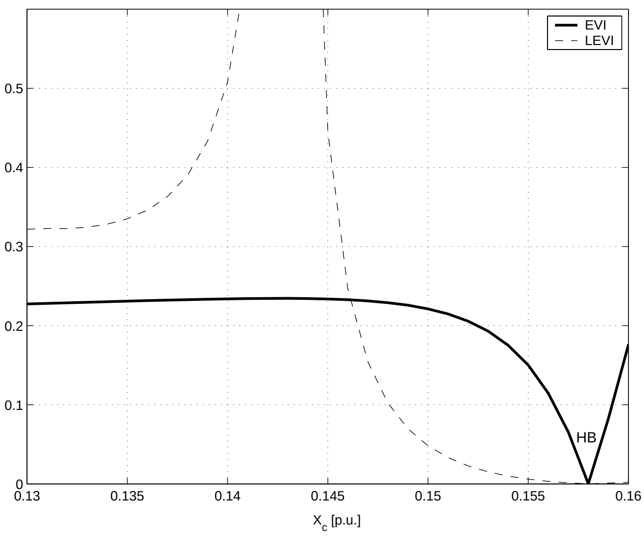

Fig. 26. Eigenvalue index and its linearization for the SSR example.

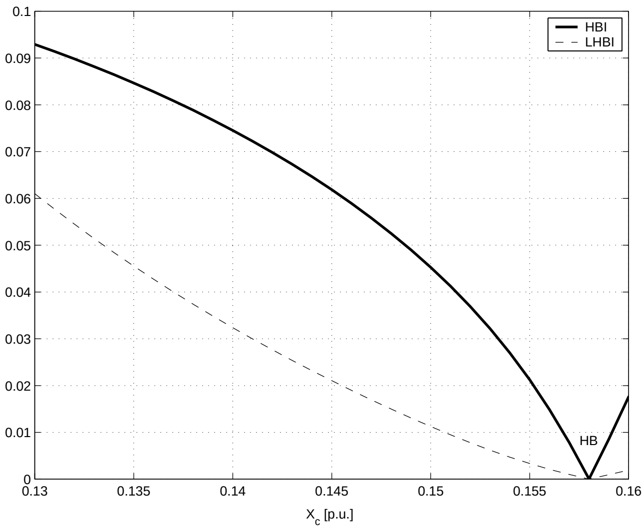

Fig. 27. Hopf bifurcation index and its linearization for the SSR example.

TABLE I
TIME SAVINGS OF COMPUTING HBI_2 WITH RESPECT TO HBI_1

System	Buses	Machines	% Time Savings
Three-bus	3	2	none
Two-area	11	4	4
IEEE 50-machine	145	50	20

the problematic compensation level and thus avoid the onset of oscillations. In this case, since the model used does not include algebraic equations, there is no difference between HBI_1 and HBI_2, and $LHBI_1$ and $LHBI_2$.

C. Discussion

Among all indices, the less expensive computationally is clearly EVI; on the other hand, HBI_2 is computationally less demanding than HBI_1, as shown in Table I, where the time savings of computing HBI_2 with respect to the computation of HBI_1 are presented. Observe that the savings increase with the size of the power system, as expected, since the calculation of the inverse of the matrix J_4 is not needed, exploiting the sparsity of the jacobian in the HBI_2 calculations. It is also important to highlight the fact that the computational cost of obtaining the minimum singular values needed for computing HBI_2 is not very significant, once the critical eigenvalue is determined, since a simple inverse iteration technique can be readily applied to the extended matrix J_m to determine this value in a few iterations. Furthermore, the numerical linearization of all these indices is not significant from the computational point of view, as it only requires simple operations with adjacent index values.

The calculation of the proposed indices on-line requires the state matrix A or the extended system matrix J, as well as the value of the critical eigenvalue μ. The information needed for the construction of these matrices are a fast power flow solution, along with the system topology update, models and data of machines and other controllers of interest. Typically, these system data, both static and dynamic, are available in modern control centers, and getting fast and multiple power flow solutions is not an issue nowadays. Once the matrices are setup, any efficient eigenvalue algorithm can be used to quickly compute μ. Thus, a reliable detection of HBs using these indices is certainly feasible.

As shown in the several examples discussed in this paper, EVI, HBI_1 or HBI_2 are adequate for detection of HBs, as they show a smooth profile with respect to the varying parameter in all cases. However, these indices do not present a linear profile in all cases (e.g. 50-machine Case II, and SSR example), and hence are not appropriate for making reliable predictions of the onset of HBs. Hence, based on the quasi-linear profile of the proposed linearized indices, particularly the $LHBI$ indices, in most cases, and given the fact that the additional computational costs are not excessive, especially for $LHBI_2$, one can reasonably argue the $LHBI_2$ index appears to be the best option for a variety of practical power system applications.

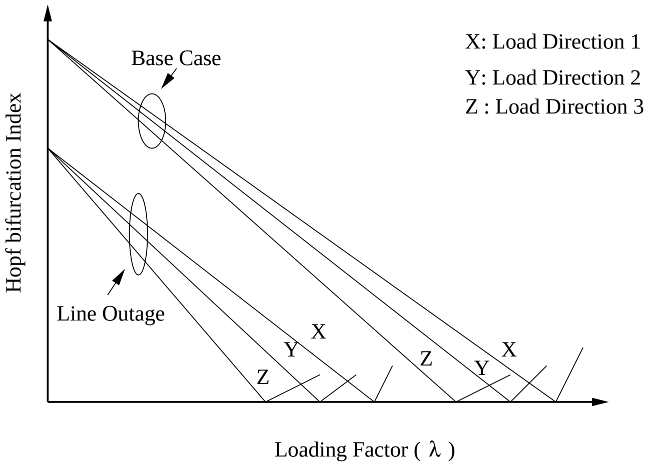

Fig. 28. Hopf bifurcation index for various operating conditions.

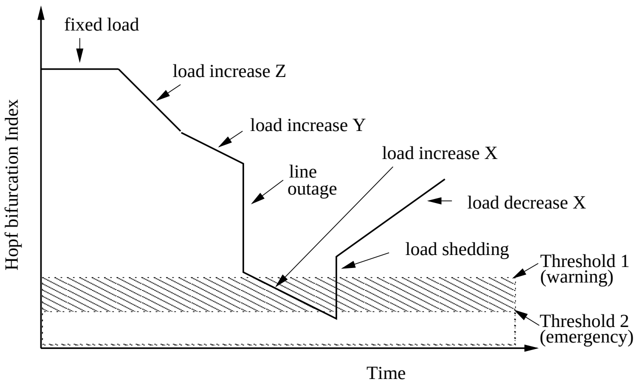

Fig. 29. Hopf bifurcation index as an on-line operating tool.

The power system examples presented here, with different dynamic characteristics as well as varying size and complexity, resemble practical power systems. Hence, the application of the proposed indices to the on-line monitoring of a practical power system is shown here to be a feasible proposition. Furthermore, in practical systems, information of the value of β or the frequency of the problematic modes are typically known (e.g. WSCC inter-area modes; Ontario Hydro local modes); thus, by inserting the expected value of β in the state matrices, the HBI and $LHBI$ indices can be calculated without actually having to compute the critical eigenvalue, thus reducing the computational costs associated with determining the value of these indices .

In a real scenario, a power system faces varying operation conditions (e.g. load increase with different directions; line outages). Figure 28 illustrates possible profiles of Hopf bifurcation indices for various system conditions, i.e. base case and a line outage case with different load directions. In the case of a change in operating conditions, the proposed Hopf bifurcation indices follow a different profile. A possible profile of the Hopf bifurcation indices as an operator would see it in real-time is depicted in Fig. 29. By monitoring the index on-line and making predictions based on linear extrapolations of the index, timely remedial measures such as load shedding can be implemented when the indices hit a given threshold value. The determination of adequate threshold values, however, is certainly not a simple task, as it requires multiple simulations off-line for a variety of system operating conditions, as well as a good operational knowledge of the system. Furthermore, observe that the linear extrapolations that would allow operators to somewhat adequately predict possible stability problems at a given operating condition, have to be continuously reassessed and weighed against the operators' knowledge of the system before taking any remedial emergency actions, given the ever changing network conditions. Thus, all these indices must be considered as additional, albeit important, tools on the operator's toolbox to help him/or her make adequate operating decisions on-line.

V. Conclusions

This paper studies and proposes various indices to predict and detect proximity to the HB points in nonlinear systems, and hence the onset of oscillatory stability problems such as inter-area, local mode and subsynchronous resonance oscillations in power systems. It is shown that one of the proposed indices ($LHBI_2$) presents significant advantages that make it suitable for on-line applications, as the smooth and quasi linear behavior of this index allows to reliably predict the onset of oscillatory problems in a variety of practical power system applications.

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