On the Initialization of Transient Stability Models of Power Systems with the Inclusion of Stochastic Processes

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***Abstract*—This letter addresses the initialization problem of power systems modeled as Stochastic Differential Algebraic Equations (SDAEs) and proposes a novel efficient initialization method. The method involves finding the co-variance matrix for the system state variables and initializing the stochastic state variables of the system randomly based on their probability distribution. Deterministic states and algebraic variables are then initialized based on the initial stochastic processes and the system co-variance matrix. The proposed method is illustrated through Monte Carlo simulations of a one-machine infinite-bus system and its numerical performance discussed through a dynamic 1479-bus model of the all-island Irish transmission system. It is shown that by utilizing the proposed method the computation time needed for the Monte Carlo simulations of the system can be reduced significantly compared to other initialization methods used in the literature.**

***Index Terms*—Stochastic differential algebraic equations (SDAEs), transient stability analysis, Fokker-Planck equation, Lyapunov equation, co-variance matrix, initialization.**

I. Introduction

Traditionally, the power system model for angle and voltage transient stability analysis is formulated as a set of deterministic Differential Algebraic Equations (DAEs). However, such a model does not capture the volatility and uncertainty due, for example, to loads and converter-interfaced renewable generation. The source of uncertainty can be conveniently modeled as Stochastic Differential Equations (SDEs). Thus, the resulting power system model becomes a set of SDAEs.

The use of SDEs for power system studies has been a trending topic over the last few years with the increased integration of variable renewable sources, such as wind and solar generation [1]–[4]. The inclusion of stochastic processes in the dynamic model of power systems requires solving multiple simulations, using the Monte Carlo method, to determine the statistical properties of the system parameters. These simulations are computationally expensive.

An issue that has not been adequately addressed in the literature is the initialization of power systems modeled as SDAEs. Generally, they have either been initialized as deterministic or the SDEs and DAEs are initialized separately. In this letter, this issue is discussed and a method is proposed that allows for the efficient initialization of SDAE systems.

II. Mathematical Formulation

A. Power System Model

The power system model considered in this work is the following set of SDAEs:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{1}$$

$$\mathbf{0}_m = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{2}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{a}(\boldsymbol{\eta}) + \mathbf{B}(\boldsymbol{\eta})\, \boldsymbol{\xi} \,. \tag{3}$$

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The transient behavior of electric power systems is traditionally described through the set of DAEs in (1)-(2). $\boldsymbol{f}$ ($\boldsymbol{f} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^n$) are deterministic differential equations; $\boldsymbol{g}$ ($\boldsymbol{g} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^m$) are the algebraic equations; $\boldsymbol{x}$ ($\boldsymbol{x} \in \mathbb{R}^n$) are the deterministic state variables and $\boldsymbol{y}$ ($\boldsymbol{y} \in \mathbb{R}^m$) are the algebraic variables.

The stochastic processes $\boldsymbol{\eta}$ ($\boldsymbol{\eta} \in \mathbb{R}^p$) are introduced through (3), where $\boldsymbol{\xi}$ ($\boldsymbol{\xi} \in \mathbb{R}^q$) is the vector of white noise processes that represent the time derivatives of the Wiener processes. Each stochastic process can be defined through their drift $\boldsymbol{a}$ ($\boldsymbol{a} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^p$) and diffusion term $\mathbf{B}$. $\mathbf{B}$ is a $p \times q$ matrix. Its elements are, in general, non-linear functions of $\boldsymbol{\eta}$ [1]. The stochastic processes $\boldsymbol{\eta}$ are characterized by their probability distribution and autocorrelation. The probability distribution describes all the possible values and likelihoods that a process can take and allows determining the statistical properties of the process, such as mean and variance. The autocorrelation describes how a process changes over time. Its mathematical definition for a stationary stochastic process η is:

$$R(\tau) = \frac{\mathrm{E}[(\eta_t - \mu)(\eta_{t+\tau} - \mu)]}{\sigma^2}, \tag{4}$$

where E is the expected value operator or *expectation*, η_t is the stochastic process at time t and τ stands for the time lag. μ and σ^2 are the mean of and variance of the stochastic process η.

In lay language, the autocorrelation gives information on how a stochastic process evolves over time, i.e., how the current value of a process is related to its past and future values. At zero lag, a process is fully correlated with itself and therefore $R(0) = 1$ always holds. Then, as the lag grows, the autocorrelation can take values in the range $[1, -1]$. Most physical processes have an autocorrelation that decays exponentially similar to the time response of a first-order differential equation but other behaviors are also possible, e.g., periodic oscillations or superposition of several exponentials [5].

In the remainder of this letter, we assume that the probability distributions and autocorrelations of all $\boldsymbol{\eta}$ are known, and they are used to set up the SDEs that defines the dynamic behavior of $\boldsymbol{\eta}$. Without loss of generality, we also assume that the set of deterministic DAEs can be obtained from (1)-(2), by imposing $\boldsymbol{\eta} = \boldsymbol{\eta}_o$, i.e.:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}_o) \,, \tag{5}$$

$$\mathbf{0}_m = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}_o) \,, \tag{6}$$

where $\boldsymbol{\eta}_o$ is a vector whose elements are the expectations or, equivalently, the mean values of the stochastic process $\boldsymbol{\eta}$.

B. Initialization

The initialization of deterministic power system models, described through the DAEs in (5)-(6), consists in finding a (stable) equilibrium point $(\boldsymbol{x}_o, \boldsymbol{y}_o)$ that satisfies the condition:

$$\mathbf{0}_n = \boldsymbol{f}(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{\eta}_o) \,, \tag{7}$$

$$\mathbf{0}_m = \boldsymbol{g}(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{\eta}_o) \,. \tag{8}$$

The initial values of the deterministic state and algebraic variables are then set as $\boldsymbol{x}(0) = \boldsymbol{x}_o$ and $\boldsymbol{y}(0) = \boldsymbol{y}_o$, respectively.

Initializing the stochastic power system model (1)-(3), on the other hand, is not as straightforward. In the literature, two methods are commonly used, as follows.

- **Method I:** The SDAEs are initialized in a deterministic way. That is the deterministic state and the algebraic variables are initialized through (7)-(8) and the initial values of the stochastic processes are set equal to their expectation, i.e. $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_o$.
- **Method II:** The initialization of the DAEs in (1)-(2) is the same as in Method I, thus leading to $\boldsymbol{x}(0) = \boldsymbol{x}_o$ and $\boldsymbol{y}(0) = \boldsymbol{y}_o$, which are obtained using $\boldsymbol{\eta} = \boldsymbol{\eta}_o$. Once this step is completed, the stochastic processes $\boldsymbol{\eta}$ are initialized in a probabilistic way. That is, the initial value of the stochastic processes are selected at random based on their probability distributions, say $\boldsymbol{\eta}_s$. The complete initial point is thus represented by $\boldsymbol{x}(0) = \boldsymbol{x}_o$, $\boldsymbol{y}(0) = \boldsymbol{y}_o$ and $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_s$.

Neither of these two methods provide the correct initialization of the whole SDAE system. The method, presented below, proposes an alternative and efficient way to initialize the SDAE system as a whole. Note also that, in general, the initial point obtained with Method II yields $\boldsymbol{g}(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{\eta}_s) \neq \mathbf{0}_m$. This, however, is not an issue as, at the first step of the time domain simulation, the condition $\boldsymbol{g}(\boldsymbol{x}_o, \boldsymbol{y}(0), \boldsymbol{\eta}_s) = \mathbf{0}_m$ is recovered by means of the internal loop of the numerical integration scheme.[1]

C. Proposed Initialization Method

The starting point of the proposed initialization method is the set of SDAEs linearized at the equilibrium point $(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{\eta}_o)$ as per Method I above. The linearization of (1)-(3) gives:

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}} \\ \mathbf{0}_m \\ \dot{\tilde{\boldsymbol{\eta}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{\boldsymbol{x}} & \boldsymbol{f}_{\boldsymbol{y}} & \boldsymbol{f}_{\boldsymbol{\eta}} \\ \boldsymbol{g}_{\boldsymbol{x}} & \boldsymbol{g}_{\boldsymbol{y}} & \boldsymbol{g}_{\boldsymbol{\eta}} \\ \mathbf{0}_{p,n} & \mathbf{0}_{p,m} & \boldsymbol{a}_{\boldsymbol{\eta}} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}} \\ \tilde{\boldsymbol{y}} \\ \tilde{\boldsymbol{\eta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n,q} \\ \mathbf{0}_{m,q} \\ \mathbf{B}(\boldsymbol{\eta}_o) \end{bmatrix} \boldsymbol{\xi}\,, \tag{10}$$

where $\boldsymbol{f}_{\boldsymbol{x}}$, $\boldsymbol{f}_{\boldsymbol{y}}$, $\boldsymbol{f}_{\boldsymbol{\eta}}$, $\boldsymbol{g}_{\boldsymbol{x}}$, $\boldsymbol{g}_{\boldsymbol{y}}$, $\boldsymbol{g}_{\boldsymbol{\eta}}$, $\boldsymbol{a}_{\boldsymbol{\eta}}$ are the Jacobian matrices of the system calculated at $(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{\eta}_o)$. $\tilde{\boldsymbol{x}}$ and $\tilde{\boldsymbol{\eta}}$ represent the deterministic and the stochastic states of the linearized system. Eliminating the algebraic variables from (10) and defining $\boldsymbol{z} = (\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{\eta}})$ lead to a set of linear SDEs, as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{\boldsymbol{x}}} \\ \dot{\tilde{\boldsymbol{\eta}}} \end{bmatrix} &= \begin{bmatrix} \boldsymbol{f}_{\boldsymbol{x}} - \boldsymbol{f}_{\boldsymbol{y}} \boldsymbol{g}_{\boldsymbol{y}}^{-1} \boldsymbol{g}_{\boldsymbol{x}} & \boldsymbol{f}_{\boldsymbol{\eta}} - \boldsymbol{f}_{\boldsymbol{y}} \boldsymbol{g}_{\boldsymbol{y}}^{-1} \boldsymbol{g}_{\boldsymbol{\eta}} \\ \mathbf{0}_{p,n} & \boldsymbol{a}_{\boldsymbol{\eta}} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}} \\ \tilde{\boldsymbol{\eta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n,q} \\ \mathbf{B}(\boldsymbol{\eta}_o) \end{bmatrix} \boldsymbol{\xi} \\ &= \mathbf{A}_o \boldsymbol{z} + \mathbf{B}_o \boldsymbol{\xi}\,. \end{aligned} \tag{11}$$

Based on the Fokker-Planck equation, as presented in Chapter 5 of [6], the probability distribution of all state variables in stationary condition satisfy:

$$\mathbf{P}(\boldsymbol{z}) = (\det \mid 2\pi \mathbf{C} \mid)^{-1/2} \cdot \exp\left(-\frac{1}{2} \boldsymbol{z}^T \mathbf{C}^{-1} \boldsymbol{z} \right), \tag{12}$$

where $\mathbf{C}$ is the co-variance matrix of the state variables in (11). This matrix is symmetric and satisfies the Lyapunov equation:

$$\mathbf{A}_o \mathbf{C} + \mathbf{C} \mathbf{A}_o^T = -\mathbf{B}_o \mathbf{B}_o^T\,, \tag{13}$$

[1]This point can be better understood considering an example. For simplicity but without lack of generality, let us assume a set of deterministic DAEs to be integrated through the implicit trapezoidal method. Then, one has, for the ith point of the simulation and using same notation as in the letter:

$$\begin{aligned} \mathbf{0}_n &= \boldsymbol{x}^i - \boldsymbol{x}^{i-1} - 0.5h[\boldsymbol{f}(\boldsymbol{x}^i, \boldsymbol{y}^i) - \boldsymbol{f}(\boldsymbol{x}^{i-1}, \boldsymbol{y}^{i-1})] \\ \mathbf{0}_m &= \boldsymbol{g}(\boldsymbol{x}^i, \boldsymbol{y}^i)\,, \end{aligned} \tag{9}$$

where $\boldsymbol{x}^i$ and $\boldsymbol{y}^i$ are the unknowns, h is the time step, and $\boldsymbol{x}^{i-1}$ and $\boldsymbol{f}^{i-1}$ are the values of the state variables and of the differential equations, respectively, calculated at the previous step. The solution of (9) is achieved only if the condition $\boldsymbol{g} = \mathbf{0}_m$ is satisfied (with a given tolerance). Further details on the integration of SDAEs are given in [1].

which is a special case of the Riccati equation. This equation can be solved numerically for a system of arbitrary size.

The co-variance matrix can be clustered as:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{\boldsymbol{xx}} & \mathbf{C}_{\boldsymbol{x\eta}} \\ \mathbf{C}_{\boldsymbol{\eta x}} & \mathbf{C}_{\boldsymbol{\eta\eta}} \end{bmatrix}, \tag{14}$$

where $\mathbf{C}_{\boldsymbol{xx}}$ and $\mathbf{C}_{\boldsymbol{\eta\eta}}$ are the co-variance matrices of $\tilde{\boldsymbol{x}}$ and $\tilde{\boldsymbol{\eta}}$, respectively. The two remaining sub-matrices $\mathbf{C}_{\boldsymbol{x\eta}}$ and $\mathbf{C}_{\boldsymbol{\eta x}} = \mathbf{C}_{\boldsymbol{x\eta}}^T$ represent the co-variance between the deterministic $\tilde{\boldsymbol{x}}$ and stochastic $\tilde{\boldsymbol{\eta}}$ state variables.

The proposed method for initializing the SDAE power system model in (1)-(3) begins with the initialization of the stochastic state variables, $\boldsymbol{\eta}$. The initial stochastic state variables are set independently at random, based on their probability distribution, i.e. $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_s$. That is, in the same way as in Method II. Then, the state variables $\boldsymbol{x}$ are initialized using $\boldsymbol{\eta}_s$ and $\mathbf{C}$ [7]:

$$\boldsymbol{x}_s = \mathbf{C}_{\boldsymbol{x\eta}} \mathbf{C}_{\boldsymbol{\eta\eta}}^{-1} \boldsymbol{\eta}_s\,. \tag{15}$$

As discussed for Method II, there is no need to update the algebraic variables as they will be determined by the time integration routine. Hence the resulting initial point is $\boldsymbol{x}(0) = \boldsymbol{x}_s$, $\boldsymbol{y}(0) = \boldsymbol{y}_o$ and $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_s$.

All the initialization methods discussed in this letter only impact on the initial values for the system SDAE equations. The model utilized to solve the time domain integration of the system is exactly the same for all three methods. That is, the full non-linear system modeled as presented in (1)-(3).

III. Case Study

This case study is divided into two parts. In the first part, the proposed method is compared to Methods I and II for a One-Machine Infinite-Bus (OMIB) system. In the second part, the all-island Irish transmission system is utilized to demonstrate that the proposed method scales well and can be applied to large systems.

All simulations were obtained using Dome, a Python-based power system software tool and solved exploiting parallelism on a 8 core 3.60 GHz Intel Xeon with 12 GB of RAM. Equation (13) is solved using the open-source library SLICOT.

A. OMIB System

The test system used in this part is the simple three bus OMIB system where the machine is a wind power plant. The OMIB test system utilized in this section is shown in Fig. 1. Note that, in this case study, the individual wind turbines are not modeled. Thus, an averaged model is used to model the whole wind farm [8].

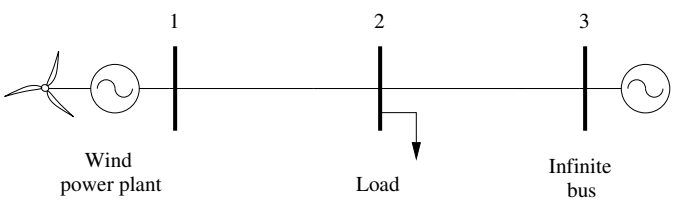

Fig. 1: The OMIB test system.

The wind speed input for the wind power plant is modeled as the following Ornstein-Uhlenbeck stochastic process.

$$\dot{\eta}(t) = \alpha(\mu - \eta(t))dt + \beta\xi, \tag{16}$$

where $\mu = 9.67$ m/s is the mean wind speed, $\alpha = 0.01$ is the mean reversion speed and $\beta = 0.05$ is the diffusion constant of the process.

The OMIB system is utilized to test the three initialization methods discussed above, that is Method I, II and the proposed method, presented in Section II. For each method, 1000 Monte Carlo simulations are solved for 60 s with a time step of 0.1 s. As an example, the active power flow from bus 2 to 3 is shown in Figs. 2-4. The system base is 100 MW. In these figures, μ and σ represent the mean and standard deviation of the 1000 simulated time series. The process is said to have reached stationarity when the standard deviation becomes constant. At this point, the simulations represent the full probability distribution of the variable.

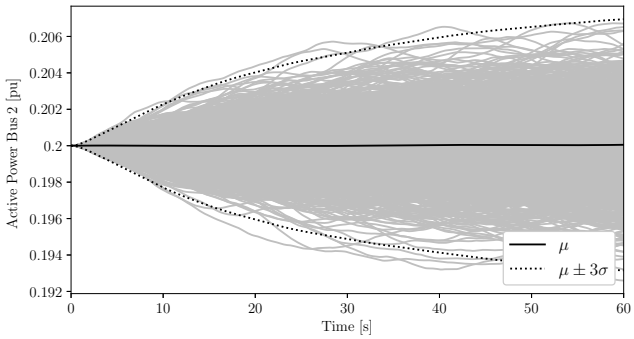

Fig. 2: Illustration of Method I. In this case the stochastic processes have not reached stationarity as the standard deviation is still continuously growing at time 60 s.

Figure 2 shows that the standard deviation of the bus voltage is still growing at the end simulation time. Thus, it has not reached stationarity. The time it takes the system variables to reach stationarity for Method I is dictated by the autocorrelation of the stochastic processes in the system. In this case the autocorrelation of the single stochastic process in (16) is $\exp(-\alpha t)$. Therefore, this particular system needs to be simulated for at least $2/\alpha = 200$ s to reach stationarity, which requires a relatively large computing time.

To reduce the computing time required to reach stationarity, an option is to increase the time step. However, in general, this is not a feasible solution for power systems. The transient stability model, in fact, is stiff, i.e., it combines fast and slow dynamics. Fast dynamics quickly reach stationarity but need a small time step. On the other hand, the slow dynamics dictate when the system trajectories reach stationarity.

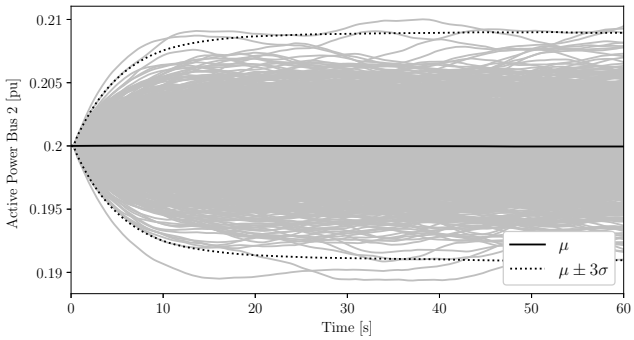

Fig. 3: Illustration of Method II. In this case the processes reach stationarity at time 20 s. From there on, the standard deviation is not continuously growing.

Figure 3 shows the behavior of Method II. In this method, the slowest dynamics of the differential equation $\boldsymbol{f}$ (in this case, wind power plant dynamics) determine the time that the system takes to reach stationarity. This occurs, in this case, in approximately 20 s of simulated time. Thus, if the stochastic processes have a slow autocorrelation, Method II requires less computing time than Method I to reach stationarity. Note that, for systems with slow dynamics, Methods I and II have an equivalent computational burden.

The results of the Monte Carlo simulations obtained with the proposed method are shown in Fig. 4. In this case, the initial conditions are stationary as confirmed by the fact that the standard deviation is constant along the whole simulation. This method significantly reduces the required computation time as no part of the simulated processes need to be discarded.

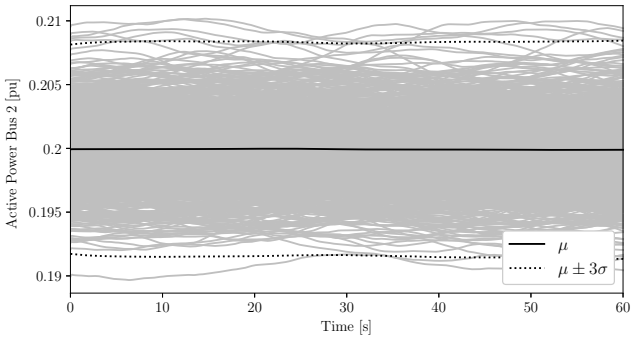

Fig. 4: Illustration of the proposed initialization method.

B. All-Island Irish Transmission System

A case study of the all-island Irish transmission system is utilized to demonstrate the application of the proposed method to a bigger system. The model includes 1479 buses, 1851 transmission lines and transformers, 245 loads, 22 conventional synchronous power plants with AVRs and turbine governors, 6 PSSs and 169 wind power plants. Further details on the system and the stochastic models utilized in this case study are provided in [9].

The Irish test system is simulated using a Monte Carlo method, i.e. 1000 simulations with a simulated time of 60 s with a time step of 0.1 s using the three different initialization methods discussed above. Figure 5 shows the standard deviation of the voltage at a centrally located bus (Athlone in county Westmeath) in the system. Method I does not reach stationarity within the 60 s time frame. Method II reaches stationarity in approximately 10 s.

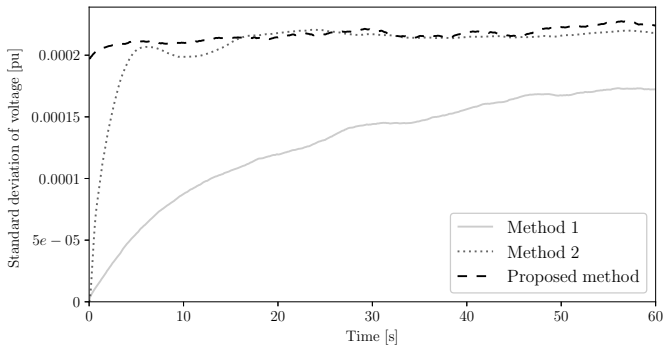

Fig. 5: Standard deviation of a voltage at a centrally located bus of the all-island Irish transmission system.

As expected the proposed initialization method allows starting the time domain simulations with points that are in stationary conditions. Only a very short and, effectively, negligible transient of the standard deviation can be observed due to the approximations introduced by the linearization and the solution of (13) obtained with the SLICOT library.

C. Discussion

Table I shows the computing time for the three initialization methods, defined as the time that each method takes to reach a stationary condition. For example, for the OMIB system, the computing time for Method I is the time required to solve 1000 simulation for 200 s. Similarly, the computing time of Method II for the OMIB system is the time required to simulate 1000 trajectories for 20 s. On the other hand, the computational burden of the proposed initialization method is driven, in large part, by the time required to solve (13). However, the Lyapunov equation needs to be solved only once for all Monte Carlo simulations of a given scenario.

TABLE I: Computing time for each initialization method

	Computation time [s]	
	OMIB	**Irish system**
Method I	28.38	1317.77
Method II	7.00	76.50
Proposed method	0.08	16.05

The speedup of using the proposed method as apposed to Method I and 2 depends on a trade-off between the number of trajectories simulated with the Monte Carlo approach and the size of the system. If the number of trajectories is sufficiently high the proposed method will always safe time compared to the conventional methods.

Another benefit of the proposed method, besides saving time is that less disc space is required. This is because the initial non-stationary part of the Monte-Carlo simulations for Method I and II are typically discarded as the full probability distribution of the variables is not simulated. With the proposed approach the whole trajectories are meaningful.

Finally, when using Method I and II, the slowest processes in the system need to be identified to be able to know before-hand how long a simulation time is needed to reach stationarity. This can be involved and time consuming for large systems. With the proposed method, on the other hand, this analysis is not required.

IV. Conclusions

The proposed method enables the efficient initialization of power systems modeled as SDAEs. The method scales well and can be applied to large systems. It reduces the computational burden of Monte Carlo simulations of SDAE power systems compared to the methods utilized so far in the literature.

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