A PMU-based Method for On-line Thévenin Equivalent Estimation

Pourya Abdi, Mohamad-Esmail Hamedani-Golshan, *Senior Member, IEEE,*
Hassan Haes Alhelou, *Senior Member, IEEE,* and Federico Milano, *Fellow, IEEE*

***Abstract*—The paper proposes a technique for the on-line estimation of Thévenin equivalent (TE) which only needs the present phasor measurements. The technique is based on the wide-area measurements including voltage and current phasors of the system loads and synchronous generators, so that it can simultaneously estimate TE of the system seen from the interested buses. The technique has two main steps. In the first step, the conventional impedance matrix $\bar{Z}_{\rm bus}$ is modified to be suitable for TE estimation. In the second step, a correction coefficient based on the sensitivity theory calculated by using two consecutive phasor measurements is applied to correct the diagonal terms of the $\bar{Z}_{\rm bus}$ matrix. Simulation results carried on the IEEE 39-bus and 118-bus test systems show that the proposed technique can accurately estimate the TE for both generation and consumption buses under any load-increase scenario. The comparison scenarios verify the superiority of the proposed technique over others.**

***Index Terms*—Thévenin equivalent, phasor measurements, bus impedance matrix, sensitivity theory, maximum power transfer.**

I. Introduction

A. Motivation

THE Thévenin equivalent (TE) of the grid as viewed from load and generator buses has an important role in the voltage stability assessment carried out by system operators in control centers [1]. In recent years, the calculation of these equivalents has been facilitated by the ability of phasor measurement units (PMUs) of providing in almost real time local voltage and current phasor measurements. An aspect of the determination of TEs based on PMUs is how to properly cope with the errors and uncertainty that inevitably accompany phasor measurements. Accuracy is key as it can impact the estimation of the conditions of the voltage instability. On the other hand, while the precision of the estimation can be improved using more measurements, another key aspect is to keep the number of measurements as small as possible to reduce computation times and allow for more effective preventive action by the system operators. This paper enhances the current state of art on this topic by proposing a technique to improve the estimation of the equivalent Thévenin impedances while keeping at the minimum the number of required measurements from PMUs.

P. Abdi, and M. E. H. Golshan are with the Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran.

H.H. Alhelou, and F. Milano are with the School of Electrical and Electronic Engineering, University College Dublin, Dublin, Ireland.

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B. Literature Review

The estimation of TE by using a limited PMU measurements is sensitive to transients in the voltage and current measurements and noise and error in the PMU data. To overcome the problem, classical least squares method [2], [3], robust least-squares method [4], recursive least squares-method with variable forgetting factors [5], and constrained least-squares method [6] have been proposed. In [7], an algorithm based on a Kalman filter is utilized to estimate the Thévenin's impedance. Least-squares methods generally need a large data window to minimize the error in TE estimation due to errors and transients in the voltage and current phasors which leads to delay significantly the estimation process.

To reduce the required time in the TE estimation process, a class of the online TE estimation methods that use a limited number of the consecutive phasor measurements has been suggested. In [8], two consecutive phasor measurements are used in equations developed based on Tellegen's theorem and the concept of the adjoined Thévenin network. In [9], [10], the equations of TE estimation based on two consecutive phasor measurements are used considering correction of the phase drift due to the deviation of the power system frequency from its nominal value. The negative effects of the phase drift are corrected by using the rate of change of frequency (RoCoF) values available at the PMU data [9]. For elimination of the phasor drift impact, three consecutive phasor measurements have been used in [10]. An algorithm for online tracking of power system impedance parameters has been proposed in [11] by using three consecutive phasor measurements which does not require synchronized data. A method has been proposed in [12] to avoid the phase drift problem. This method uses the voltage and current magnitude and active and reactive powers derived from five consecutive phasor measurements to find TE. A graphical method for deriving Thévenin equivalent parameters using a large number of online PMU measurements has been developed in [13] based on representing and manipulating the measured voltage phasors in the complex voltage plane.

Using a small number of consecutive phasor measurements has a positive impact on the TE estimation accuracy because it is likely that the more samples do not related to a specific TE of the system. In [14], it is assumed that the equivalent

Thévenin resistance is approximately zero, then equivalent Thévenin reactance is evaluated by taking into account the present voltage and current phasor measurements and the previous estimation values of equivalent Thévenin reactance and voltage. An optimal Thévenin equivalent estimation algorithm has been proposed in [2] in which a range of reasonable values for equivalent reactance is used to calculate equivalent Thévenin voltage. Then, the correct value of equivalent Thévenin reactance is selected. This algorithm uses present voltage and current phasor measurements.

In [15], a voltage stability monitoring scheme has been proposed using a combination of the virtual and the self impedance calculated from the system admittance matrix and synchronous generator voltage and voltage and current measurements at the interested buses. The defined Thévenin impedance remains approximately constant under a proportional increase load scenario. The method proposed in [15] has been modified in [16] for any load variations scenario. To overcome the restrictions of the approach, a new approach based on the main theory in [15] and [16] has been developed to estimate the maximum loading point considering the generator reactive power limits [17]. The node-voltage equation has been used to construct the complex high-dimensional linear system for obtaining equivalent voltage in [18], then, the Gaussian elimination of the complex high-dimensional linear system is converted into the optimized substitution solution using LU factorization so as to realize the rapid and accurate identification of equivalent parameters. In [19], a factor-solve method has been used for the sparse admittance matrix of the power system to compute Thévenin equivalents for power system buses.

TE estimation has gained a considerable attention in recent years due to its applications in modern power systems, including stability and security assessment studies [20]–[23]. However, there are some practical and technical issues related to estimating TE, such as low accuracy of existing methods in estimating the TE resistance, reactance, and voltage source. Depending on the method, other technical challenges include the need of a considerable variation in the load to perform the estimation and/or a wide data-window and, consequently, a large computational burden. Moreover, existing methods generally do not consider the dynamic model of system. This work tackles these research gaps and proposes a novel method that can be effectively implemented in practice.

C. Contribution

This paper proposes a novel TE parameter estimation method that utilizes wide-area measurements including voltage and current phasors of loads and synchronous generators to modify the bus impedance matrix of the system for estimation of the TE seen from the buses of interest. The modified bus impedance matrix includes impedance of all system loads except the interested bus for TE estimation and transient reactance of the synchronous generators. To overcome inaccuracy of the estimated reactance of the generators model, a correction coefficient is calculated based on the sensitivity of the bus voltage to the reactive power. The determination of the correction coefficient only needs present phasor measurements for each TE calculation process. Therefore, this paper suggests estimating TE parameters in the first stance based on the model of the system, then in the second stance, the effect of uncertainties and inaccuracies are minimized by introducing a novel correction coefficient in order to modifying the proposed method. The method can calculate the TE at load and generator buses and is independent from the hard limits and nonlinearities of generators and loads.

The main contributions of this paper are as follows.

- To estimate the TE of the system using the basic theory of relation between $\bar{Z}_{\text{bus}}$ elements and Thévenin equivalent impedances at the system buses.
- Using the present set of wide-area measurements, to construct an appropriate $\bar{Z}_{\text{bus}}$ and determine TE of system at different buses simultaneously.

The most relevant features of the proposed method are as follows.

- It can be applied to all load and generator buses under any load variation scenario.
- It includes the calculation of a correction coefficient that increases the accuracy and minimizes the impact of nonlinearitie (e.g., reactive power limits of PV buses) and uncertainties.
- It takes into account the impact of uncertainties as well as system and load variations on the estimation of TE.

D. Organization

The remainder of this paper is organized as follows. Section II introduces the main steps used for estimating Thévenin-equivalent parameters in the proposed procedure and suggests a new technique for improving the accuracy based on the sensitivity method. Section III presents the simulation results and discussions, while Section IV concludes.

II. Proposed Method for TE Estimation

This section recalls the definition of Thévenin equivalent of a grid (Section II-A); defines a modified bus impedance matrix (Section II-B); describes the proposed procedure to correct the diagonal elements of the bus impedance matrix based on PMU measurements (Section II-C); and finally describes the proposed method to calculate TEs based on solely present PMU measurements (Section II-D).

A. Thévenin Equivalent

In view of each load bus i, the power system can be modeled as a two-bus system including TE voltage and impedance as shown in Fig 1. Based on the model, voltage $\bar{V}_{Li}$ can be written in terms of the Thévenin voltage, $\bar{E}_{\text{Th}}$, and Thévenin impedance, $\bar{Z}_{\text{Th}}$, as

$$\bar{V}_{Li} = \bar{E}_{\text{Th}} - \bar{Z}_{\text{Th}}\bar{I}_{Li} \,. \tag{1}$$

According to the developed theory in this section, TE parameters, $\bar{E}_{\text{Th}}$ and $\bar{Z}_{\text{Th}}$, can be determined using the network bus impedance matrix and phasor samples of the current and voltage measured at the system buses.

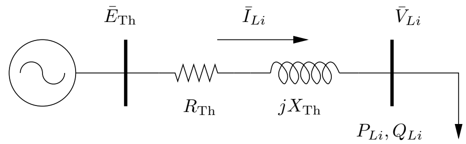


Fig. 1. Thévenin equivalent circuit of a system seen from ith bus load

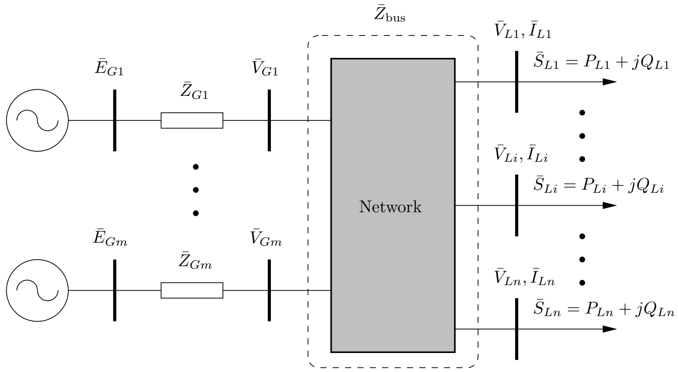


Fig. 2. Power system model as a network and current injections.

B. Modified $\bar{Z}_{\text{bus}}$ for TE Estimation

A power system including m generators and n loads can be represented as a network and a set of the injection currents as shown in Fig. 2. The system buses voltage are related to the injection currents by the bus impedance matrix $\bar{Z}_{\text{bus}}$ as

$$\bar{V} = \bar{Z}_{\text{bus}}\,\bar{I}\ , \tag{2}$$

or, equivalently,

$$\begin{bmatrix}\bar{V}_L\\ \bar{V}_T\\ \bar{V}_G\end{bmatrix} = \bar{Z}_{\text{bus}}\begin{bmatrix}-\bar{I}_L\\ 0\\ \bar{I}_G\end{bmatrix}, \tag{3}$$

where $\bar{V}_L, \bar{I}_L \in \mathbb{C}^n$ are the vectors of load voltages and currents; $\bar{V}_G, \bar{I}_G \in \mathbb{C}^m$ are the vectors of generator voltages and currents; and $\bar{V}_T \in \mathbb{C}^k$ is the vector of voltages at zero-injection buses. The vectors $\bar{V}$ and $\bar{I}$ and the bus impedance matrix $\bar{Z}_{\text{bus}}$ in (2) have thus dimensions $(n+m+k)\times(1)$, $(n+m+k)\times(1)$, and $(n+m+k)\times(n+m+k)$, respectively.

To be suitable for transient studies similar to models used for the stability studies, the $\bar{Z}_{\text{bus}}$ in (3) is modified as follows.

- The $\bar{Z}_{\text{bus}}$ in (3) is modified by adding the load equivalent impedance at consumption buses except bus i calculated by the measured voltage and current phasors $\bar{V}_{Lj}$ and $\bar{I}_{Lj}$, $j = 1, .., n; j \neq i$, and setting the elements of vector $\bar{I}_L$ equal to zero except $\bar{I}_{Li}$; and
- The transient reactance of the synchronous machines is used to construct the $\bar{Z}_{\text{bus}}$.

These assumptions lead to rewrite (3) as (see also Fig. 3):

$$\begin{bmatrix}\bar{V}_L\\ \bar{V}_T\\ \bar{E}_G\end{bmatrix} = \bar{Z}'_{\text{bus}}\begin{bmatrix}-\bar{I}'_L\\ 0\\ \bar{I}_G\end{bmatrix}, \tag{4}$$

where

$$\bar{I}'_L = \begin{bmatrix}0\ldots0\ \bar{I}_{Li}\ 0\ldots0\end{bmatrix}\ . \tag{5}$$

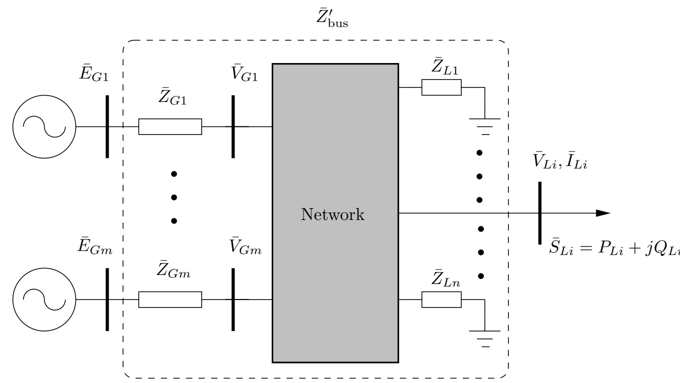


Fig. 3. Modified power system using the proposed corrected $\bar{Z}'_{\text{bus}}$.

In (4), all load buses have been modeled by the load equivalent impedance except load bus i which is modeled by the injection current. Based on (4) and (5), the voltage equation corresponding to each load bus can be written as follows:

$$\bar{V}_{Li} = -\bar{Z}_{ii}\bar{I}_{Li} + \bar{Z}_{i(n+k+1)}\bar{I}_{G1} + \cdots + \bar{Z}_{i(n+k+m)}\bar{I}_{Gm}\ , \tag{6}$$

where $\bar{Z}_{ij}$ is the (i, j)-th element of $\bar{Z}'_{\text{bus}}$.

Comparing (6) with (1), and considering the property of the bus impedance matrix in presentation of Thévenin equivalent impedances, one has

$$\bar{Z}_{\text{Th}} = \bar{Z}_{ii}\ , \tag{7}$$

and

$$\bar{E}_{\text{Th}} = \bar{Z}_{i(n+k+1)}\bar{I}_{G1} + \cdots + \bar{Z}_{i(n+m+k)}\bar{I}_{Gm}\ . \tag{8}$$

Note that the estimated value of $\bar{Z}_{\text{Th}}$ from the modified $\bar{Z}_{\text{bus}}$ obtained in the first step of the algorithm is based only on the present phasor measurements (one sample). For this reason, the phasor drift is not assumed to be an error factor in the estimated $\bar{Z}_{\text{Th}}$.

C. Correction of TE

There are always some errors in parameters of the network components and inaccuracy in the transient reactances of the machines, which change during a transient. To compensate these errors, the Thévenin impedance determined by (7) is corrected by a scalar coefficient α, so that

$$\bar{Z}_{\text{Th,corr}} = \alpha\,\bar{Z}_{\text{Th}}\ . \tag{9}$$

Then, $\bar{E}_{\text{Th,corr}}$ is calculated as

$$\bar{E}_{\text{Th,corr}} = \bar{V}_{Li} + \bar{Z}_{\text{Th,corr}}\bar{I}_{Li}\ . \tag{10}$$

Although phasor measurements provide the possibility of defining a complex-valued correction factor, this can lead to unnecessary complexities. Moreover, since $\bar{Z}_{\text{Th}}$ of the transmission system is dominantly ractive (note that, in the literature, only X_{Th} is generally estimated), we utilise a real-valued correction to estimate R_{Th}.

The sensitivity analysis, specifically the sensitivity of voltage $\bar{V}_{Li}$ at bus i to reactive power of load at bus i is employed to determine coefficient α.

Based on the two-bus equivalent system shown in Fig. 1, the voltage phasor V_{Li} can be related to the load active and reactive powers by TE parameters as follows:

Considering $\bar{V}_{Li}$ as the reference phasor or $\bar{V}_{Li} = V_{Li}\angle 0$, the current phasor $\bar{I}_i$ can be written as

$$\bar{I}_{Li} = \frac{P_{Li} - jQ_{Li}}{\bar{V}_{Li}} . \tag{11}$$

Then, the voltage drop across TE impedance is obtained as

$$\begin{aligned}\Delta\bar{V}_i &= (R_{\mathrm{Th},i} + jX_{\mathrm{Th},i})(\frac{P_{Li} - jQ_{Li}}{\bar{V}_{Li}}) \\ &= \frac{R_{\mathrm{Th},i}P_{Li} + X_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} + j\frac{X_{\mathrm{Th},i}P_{Li} - R_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} .\end{aligned} \tag{12}$$

Substituting (12) into (1), one has:

$$\bar{E}_{\mathrm{Th},i} = \frac{R_{\mathrm{Th},i}P_{Li} + X_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} + j\frac{X_{\mathrm{Th},i}P_{Li} - R_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} + \bar{V}_{Li} , \tag{13}$$

or

$$\bar{E}_{\mathrm{Th},i} = \frac{V_{Li}^2 + R_{\mathrm{Th},i}P_{Li} + X_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} + j\frac{X_{\mathrm{Th},i}P_{Li} - R_{\mathrm{Th},i}Q_{Li}}{\bar{V}_{Li}} . \tag{14}$$

After some algebraic manipulations, the nonlinear relationship of the voltage magnitude $|V_{Li}|$ in terms of load active and reactive powers and TE parameters is derived as

$$\begin{aligned}V_{Li}^4 + (2R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}Q_{Li} - \left|\bar{E}_{\mathrm{Th},i}\right|^2)V_{Li}^2 + \\ \left|\bar{Z}_{\mathrm{Th},i}\right|^2(P_{Li}^2 + Q_{Li}^2) = 0 .\end{aligned} \tag{15}$$

Differentiating (15), the change in voltage magnitude (ΔV_{Li}) is derived in terms of the change in active and reactive powers of load, (ΔP_{Li}) and (ΔQ_{Li}), for a given Thévenin equivalent circuit, as

$$\begin{aligned}4V_{Li}^3\Delta V_{Li} + 2(2R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}Q_{Li} - \left|\bar{E}_{\mathrm{Th},i}\right|^2) \\ V_{Li}\Delta V_{Li} + 2R_{\mathrm{Th},i}V_{Li}^2\Delta P_{Li} + 2X_{\mathrm{Th},i}V_{Li}^2\Delta Q_{Li} + \\ 2\left|\bar{Z}_{\mathrm{Th},i}\right|^2 P_{Li}\Delta P_{Li} + 2\left|\bar{Z}_{\mathrm{Th},i}\right|^2 Q_{Li}\Delta Q_{Li} = 0 .\end{aligned} \tag{16}$$

The variation of load active power, (ΔP_{Li}), can be related to the variation of load reactive power, (ΔQ_{Li}), by coefficient γ_i.

$$\Delta P_{Li} = \gamma_i \Delta Q_{Li} . \tag{17}$$

The coefficientd γ_i can be calculated with two consecutive PMU measurements (the present and previous phasor) at bus i as

$$\gamma_i(t) = \frac{\Delta P_{Li}(t)}{\Delta Q_{Li}(t)} = \frac{P_{Li}(t) - P_{Li}(t-1)}{Q_{Li}(t) - Q_{Li}(t-1)} . \tag{18}$$

From (17) in (16), the sensitivity of voltage magnitude at bus i to load variation in this bus, $\frac{\Delta V_{Li}}{\Delta Q_{Li}}$, is obtained as

$$(\frac{\Delta V_{Li}}{\Delta Q_{Li}})_{\mathrm{calc}} = \frac{(2V_{Li}^2(R_{\mathrm{Th},i}\gamma_i + X_{\mathrm{Th},i}) + 2\left|\bar{Z}_{\mathrm{Th},i}\right|^2(P_{Li}\gamma_i + Q_{Li}))}{(-4V_{Li}^3 - 2(2R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}Q_{Li} - \left|\bar{E}_{\mathrm{Th},i}\right|^2)V_{Li})} . \tag{19}$$

The index $\frac{\Delta V_{Li}}{\Delta Q_{Li}}$ is a very appropriate index to evaluate accuracy of the two-bus equivalent model at bus i. The $\frac{\Delta V_{Li}}{\Delta Q_{Li}}$ calculated from the system model by (19) can be compared with the $\frac{\Delta V_i}{\Delta Q_{Li}}$ obtained from the measurements at bus i.

$$(\frac{\Delta V_i}{\Delta Q_i})_{\mathrm{meas}} = \frac{V_{Li}(t) - V_{Li}(t-1)}{Q_{Li}(t) - Q_{Li}(t-1)} . \tag{20}$$

The algorithm uses the data of the present time to estimate TE parameters in the first step. Then, the two-bus model is derived from these parameters and the sensitivities of voltage magnitude at bus i to load variation in this bus is computed. In addition, the $(\frac{\Delta V_i}{\Delta Q_{Li}})_{\mathrm{meas}}$ is calculated from the two consecutive samples related to the present time and the just previous time, so two sensitivities computed from the two-bus model and two consecutive samples are concerned to the present time.

The inaccuracy in TE parameters determined using the bus impedance matrix leads to the fact that $\frac{\Delta V_i}{\Delta Q_i}$ obtained from the model (19) is different from $\frac{\Delta V_i}{\Delta Q_i}$ obtained from (20) using PMU measurements. This difference can be used for improving the accuracy of TE parameters estimation method based on the introduced impedance matrix-based. To bring these two values closer to each other and reducing the mismatch between the sensitivity calculated from the equivalent model of the power system and real sensitivity, $\bar{Z}_{\mathrm{Th}}$ in (16) can be replaced by $\alpha\bar{Z}_{\mathrm{Th}}$. Then, the modifying factor α can be calculated by setting

$$(\frac{\Delta V_i}{\Delta Q_i})_{\mathrm{meas}} = (\frac{\Delta V_i}{\Delta Q_i})_{\mathrm{calc}} . \tag{21}$$

Setting $(\frac{\Delta V_i}{\Delta Q_i})_{\mathrm{meas}} = \beta$, a second order equation is obtained for α from (21).

$$\begin{aligned}&[4V_{Li}^3\Delta V_{Li} - 2((2R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}Q_{Li})\alpha - \left|\bar{E}_{\mathrm{Th},i}\right|)^2 V_{Li}] \\ &= \left[2V_{Li}^2(R_{\mathrm{Th},i}\gamma_i + X_{\mathrm{Th},i})\alpha + 2\left|\bar{Z}_{\mathrm{Th},i}\right|^2\alpha^2(P_{Li}\gamma + Q_{Li})\right]\beta ,\end{aligned} \tag{22}$$

or

$$\begin{aligned}&2\left|\bar{Z}_{\mathrm{Th},i}\right|^2(P_{Li}\gamma + Q_{Li})\beta\alpha^2 + 2V_{Li}^2(R_{\mathrm{Th},i}\gamma_k + X_{\mathrm{Th},i})\beta \\ &+ 2(R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}V_{Li})\alpha + 4V_{Li}^4 + \left|\bar{E}_{\mathrm{Th},i}\right|^2 V_{Li} = 0 ,\end{aligned} \tag{23}$$

where

$$\begin{aligned}A &= 2\left|\bar{Z}_{\mathrm{Th},i}\right|^2(P_{Li}\gamma + Q_{Li})\beta , \\ B &= [2V_{Li}^2(R_{\mathrm{Th},i}\gamma_i + X_{\mathrm{Th},i})\beta \\ &\quad + 2(R_{\mathrm{Th},i}P_{Li} + 2X_{\mathrm{Th},i}Q_{Li})]V_{Li} , \\ C &= 4V_{LK}^4 + \left|\bar{E}_{\mathrm{Th},i}\right|^2 V_{Li} .\end{aligned} \tag{24}$$

Equation (23) con be rewritten in compact form as:

$$A\alpha^2 + B\alpha + C = 0 , \tag{25}$$

and its solution yields the correction coefficient α.

On one hand, the required sensitivities in the algorithm should be computed by the derivative formulas (18) and (20) based on two consecutive PMU measurements. On the other hand, PMU measurements can be polluted by noise and outlier, so relying on two PMU measurements can lead to inaccurate estimations. To solve this problem, the values of correction factor calculated in the previous and present times are passed from a moving average filter to smooth the fluctuations and eliminate the outlier values. Thus, the filtered value of α at the present time is determined by averaging the previous filtered α's and current α.

D. Steps of the Proposed Technique

Table I shows the steps of the proposed technique.

1) The system real-time operation status is gathered based on the system $\bar{Z}_{\mathrm{bus}}$ and voltage and current measurements at system buses from PMUs.
2) The state estimation program as a common tool in all control center is executed. Therefore, the voltage and current phasors related to all system buses become available in the control center due to measurements or estimation. Note that, in general, control centers have acces to a limited number of phasor measurements obtained with PMUs. On the other hand, the proposed algorithm needs the voltage and current phasors at many system buses, which can be obtained, e.g., with a state estimation algorithm. Thus, the role of the state estimation is only the estimation of the required phasors, when these phasors are not directly provided by PMU.
3) The voltage and current phasors are used to required calculations in the proposed algorithm. The first, $\bar{Z}_{\mathrm{load}}$ at all buses $j = 1, 2, ..., n, j \neq i$ are calculated by phasors $\bar{V}_{Lj}$ and $\bar{I}_{Lj}$ as

$$\bar{Z}_{\mathrm{load},j} = \frac{\bar{V}_{Lj}}{\bar{I}_{Lj}}\,. \tag{26}$$

4) The defined $\bar{Z}'_{\mathrm{bus}}$ is obtained by modifying the $\bar{Z}_{\mathrm{bus}}$ existing in the control center by adding $\bar{Z}_{\mathrm{load},j}$ and the impedance of synchronous generators including transient reactance to original $\bar{Z}_{\mathrm{bus}}$.
5) The system equivalent in view of load bus i is determined by using (7) and (8). In this step, it is possible to calculate the Thévenin impedance and voltage by using (7) and (8) for each generation bus i by setting $\bar{I}_L = 0$ and constructing $\bar{Z}'_{\mathrm{bus}}$ by equivalent impedance calculated for all load buses.
6) Based on the nonlinear model $V_{Li}(P_{Li}, Q_{Li}, \bar{E}_{\mathrm{Th},i}, \bar{Z}_{\mathrm{Th},i})$ of bus i, the sensitivity of voltage V_{Li} respect to ΔQ_{Li} is determined according to (19). Based on the sensitivity calculated from the model and the sensitivity computed from the measurements, correction coefficient α_i is determined for present time.
7) Using the proposed correction coefficient α_i, the $\bar{Z}_{\mathrm{Th}}$ and $\bar{E}_{\mathrm{Th}}$ are corrected by (9) and (10) for TE estimation at interested buses.

TABLE I
Proposed algorithm to estimate TEs

Step No. #	Algorithm Process
Step I	**Receive** PMU data at WAMS center and Z_{bus} from control center
Step II	**Execute state estimation**
Step III	**Calculate on-line** of $\bar{Z}_{\mathrm{load},j}$ using (26) for $j = 1, 2, ..., n, j \neq i$
Step IV	**Modify** $\bar{Z}_{\mathrm{bus}}$ to obtain $\bar{Z}'_{\mathrm{bus}}$ using (4)
Step V	**Calculate** the system equivalent using (7) & (8)
Step VI	**Obtain** $V_{Li}(P_{Li}, Q_{Li}, \bar{E}_{\mathrm{Th},i}, \bar{Z}_{\mathrm{Th},i})$ for buses of interest; **Determine** the sensitivity coefficient, β_i, using (20); and **Calculate** correction factor α_i.
Step VII	**Find** the corrected $\bar{Z}_{\mathrm{Th}}$ and $\bar{E}_{\mathrm{Th}}$ using (9) and (10).
Step VIII	**End**

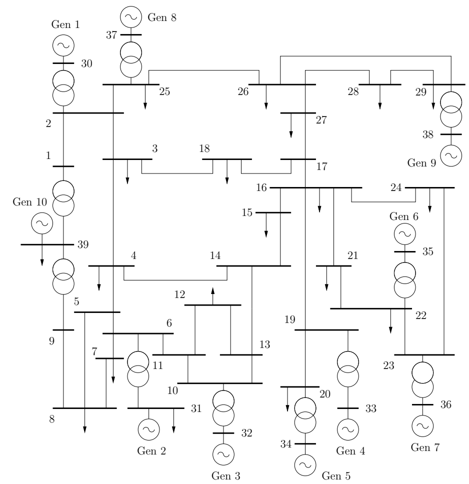


Fig. 4. IEEE 39-bus network.

III. Case Study

This section discusses a case study that is based on the IEEE 39-bus network (see Fig. 4) and aims at verifying the performance of the proposed method using Digsilent Power Factory and Matlab software tools. The data of the system under investigation is available in [24]–[26]. The purpose of this study is to estimate TE parameters of the network as seen from different buses. The TE parameters are updated every 20 ms according to the rate of PMU reporting equal to one power system cycle. It is important to note, however, that the proposed procedure can run continuously in longer intervals, say, once for every 5 s because the procedure is used to evaluate long-term voltage stability.

The results of the proposed method are compared with those of two-sample method. The basic two-sample method usually leads to relatively large errors due to its simple estimation approach (see the Appendix). For fair comparison, the two-sample method used in this case study includes a multi-stage pre-processing of the PMU data, and a multi-stage post-processing of the computed parameters by two-sample equation. This algorithm overcomes high sensitivity of the basic two-sample method to the noise of PMU measurements. The presentation of this comprehensive algorithm is out of scope of the paper, however, it should be noted that the estimated values by this improved two-sample method have been thoroughly verified with other methods, e.g., those proposed in [9], [10] and [11], and with both simlated and real-world data.

A. Scenario 1: Load Ramp leading to Voltage Collapse

The active and reactive powers of the load connected to bus 28 are gradually and monotonically increased. A random variation is also superimposed on the increasing load as shown in Fig. 5. The voltage collapse occurs at the 915th phasor sample when the load at bus 28 consumes 1353 MW and 181.3 MVAr. According to the theory of impedance-match [1], the magnitude of the TE impedance of the system at the bus 28 is equal to the magnitude of the load impedance at the maximum loading condition point.

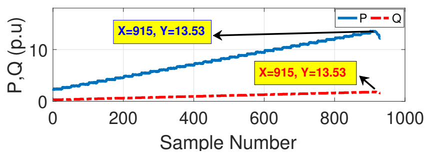


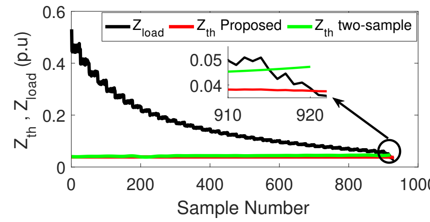
Fig. 5. Scenario 1: Active and reactive power of the load at bus 28.

The magnitude of the Thévenin impedances during the load increase obtained from the modified impedance matrix generated according to the proposed method in section II and the values obtained using the well-known two-sample method are depicted in Fig. 6(a). The amount of Thévenin impedance magnitudes obtained using the proposed method is almost identical to the results obtained using the two-sample method. The plot of Thévenin impedances magnitude obtained from two-sample method intersects the plot of the load impedances magnitude at 916th sample while the intersection of the plots of Thévenin impedances magnitude obtained from the proposed method and load impedances magnitude occurs at 920th sample that presents a relatively good accuracy of the impedance matrix based results. Fig. 6(b) also shows the values of Thévenin impedances parameters obtained by the proposed method are approximately equal to the results of the two-sample method. The values of the Thévenin voltage magnitude obtained using the proposed method and the two-sample method are also plotted in Fig 6(c).

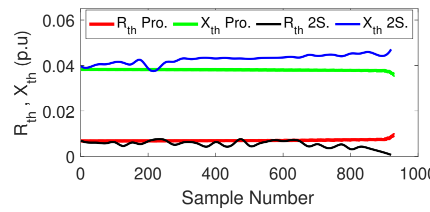
We now apply the proposed correction factor α to correct the results of the Z_{bus}-based method. Figure 7(a) shows that the calculated α values have fluctuations so that there are some outliers data. Therefore, the measured values of α are filtered. In this scenario, filtered α values are slightly bigger than 1. The values of the corrected Thévenin impedance are plotted in Fig. 7(b). Since α is close to unity in this case, the corrected Thévenin impedances are similar to those obtained using the two-sample method. Applying the correction coefficient α causes the Thévenin impedances curve intersects the load impedances curve in the 916th sample which is the accurate point of load maximum. Fig. 7(c) shows the corrected Thévenin impedances parameters by applying correction coefficient, which is almost identical to the results of the two-sample method. The values of the modified Thévenin voltage magnitude as well as the values of the Thévenin voltage magnitude obtained using the two-sample method are shown in Fig. 7(d). It is worth to mention that E_{Th} in Fig. 7(d) has been computed with two different methods, i.e., the modified two-sample method and the proposed algorithm. As observed in Fig. 7(c), the parameters (R_{Th} and X_{Th}) have small fluctuations as the load bus 28 increases. These small fluctuations are weighted by load current I_L to compute E_{Th}. With the increase of load power of bus 28, load current increases and causes the effect of small fluctuations of R_{Th} and X_{Th} is magnified in the computation of E_{Th}.

Figure 8 shows the Thévenin impedance magnitudes of the grid as seen from bus 7 when the load of bus 28 changes according to the Scenario 1. In this case, the difference between the proposed method and the two-sample method mistakenly are apparent. This is because the consecutive samples measured at bus 7 are similar and the two-sample method fails to estimate Thévenin impedance under this condition (see Fig. 8(a)). Methods such as the two-sample method, in fact, cannot calculate the Thévenin impedance at buses with constant load. On the other hand, using the proposed bus impedance matrix-based method, the values of Thévenin equivalent can be obtained for loads that are unchanged during system variation (see Fig. 8(b)). The values presented in Fig. 8(a) and 8(b) are related to the first step of the algorithm (without applying the correction factor α). The calculation of α requires γ, which, in turn, needs two PMU measurements as mentioned above (see discussion right before (18)). Thus, when the load is constant, α cannot be computed. In this case, differently from the two-sample method that computes the load impedance instead of $\bar{Z}_{\text{Th}}$, the proposed algorithm yields an approximated (yet quite precise) value of $\bar{Z}_{\text{Th}}$. To increase accuracy, one can use the last value of α. For example, considering the variation of the load at bus 7, one obtains the value of α shown in Fig. 8(c). Therefore, α of bus 7 is set to 1.22 for the case in which load of bus 7 is constant. Using this value for α leads to TE parameters shown in Figs. 8(d) and 8(e).

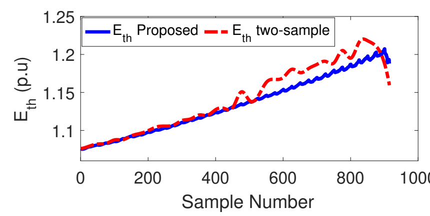
The proposed method also has the ability to determine Thévenin equivalent seen from generation buses. The follow-



(a)



(b)



(c)

Fig. 6. Scenario 1: (a) Thévenin and load impedance magnitudes; (b) Thévenin resistance and reactance; (c) Thévenin voltage magnitude values.

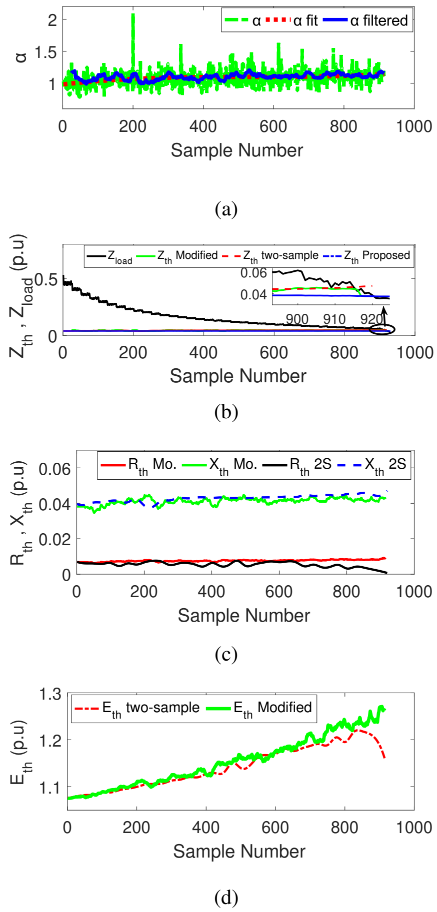


Fig. 7. Scenario 1: (a) α values, (b) Modified Thévenin impedance magnitude and load impedance magnitude, (c) Modified Thévenin resistance and reactance values, (d) Modified Thévenin voltage magnitude values.

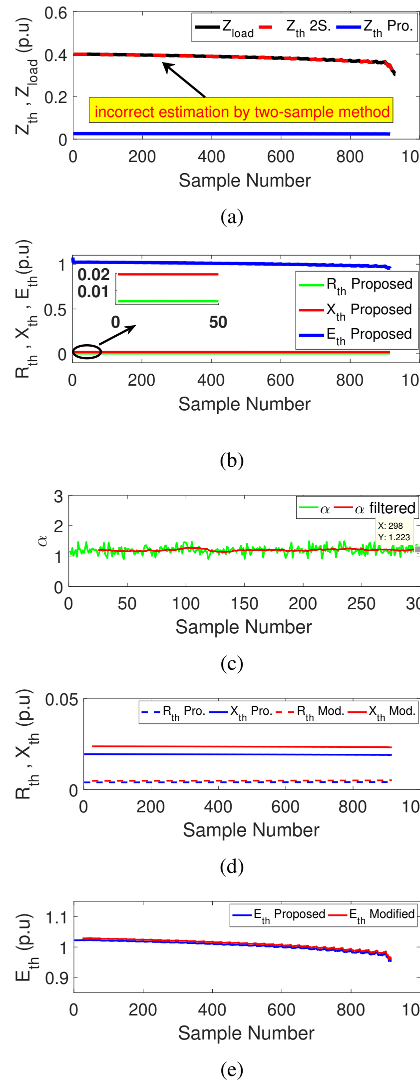


Fig. 8. Scenario 1: TE seen from bus 7. (a) Thévenin and load impedance magnitudes; (b) Thévenin parameters; (c) α values; (d) TE parameters using modified method; and (e) TE voltage source.

ing results are related to busbar 31 connected to generator G_2 in Fig. 4. The values of Thévenin impedance obtained using the proposed method are depicted in Fig. 9.

B. Scenario 2: Load Ramp and Line Outage

In this second scenario, the active and reactive load powers connected to bus 28 increase monotonically as in Scenario 1. Moreover, at $t = 10$ s, the line 16-17 is disconnected. The voltage collapse point, in this case, is reached at the 652nd sample after the occurrence of the line outage.

The magnitudes of the Thévenin impedances obtained with the proposed method and the two-sample method are shown in Fig. 10(a). Without applying the correction coefficient α, the value of the Thévenin impedance magnitude obtained by the proposed method intersects the load impedance magnitude values at the 677th sample. This does not match the maximum power point. Fig. 10(b) shows the values of the correction coefficient α. The values of the corrected Thévenin impedance are shown in Fig. 10(c). When applying the correction, the results of proposed method show a better accuracy than the two-sample method. In fact, the curve of the magnitude of the corrected $\bar{Z}_{\mathrm{Th}}$ intersects the curve of the magnitude of the load impedance at 652 sample. Fig. 10(d) shows the modified Thévenin impedance parameters. The values of the Thévenin voltage are shown in Fig. 10(e).

C. Scenario 3: Ramps and Random Variations of All Loads

In this third and last scenario, the active and reactive powers of all system loads are gradually increased. A random noise is also applied to all loads. For illustration, the trajectory of the active power of the load at bus 28 are shown in Fig. 11. The voltage collapse occurs in the 192nd sample.

Figure 12 shows the values of the Thévenin impedance magnitude obtained using the impedance matrix without corrections in the proposed method as well as the those obtained with the two-sample method. In this scenario, the accuracy of the conventional $\bar{Z}_{\mathrm{bus}}$-based results is not acceptable, however, when the correction coefficient is applied the results of the proposed method is significantly improved (see Fig. 13(a)). The values of α obtained using the proposed correction coefficient method are shown in Fig. 13(b). The corrected Thévenin impedance curve intersects the load impedance curve

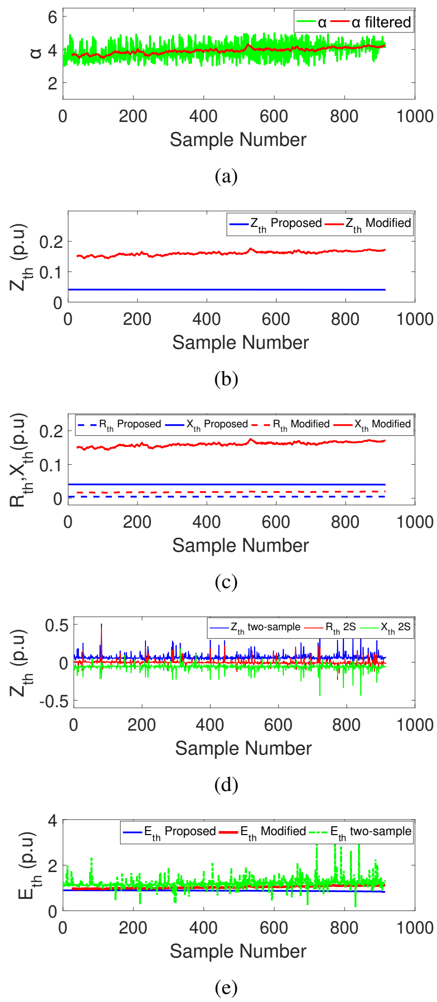


Fig. 9. Scenario 1: TE seen from bus of generator 2 during scenario 1, (a) α values; (b) Thévenin impedance magnitudes; (c) Thévenin impedance parameters; (d) Thévenin impedance magnitudes and parameters by 2S method; (e) Thévenin voltage value using the proposed, modified and two-sample methods.

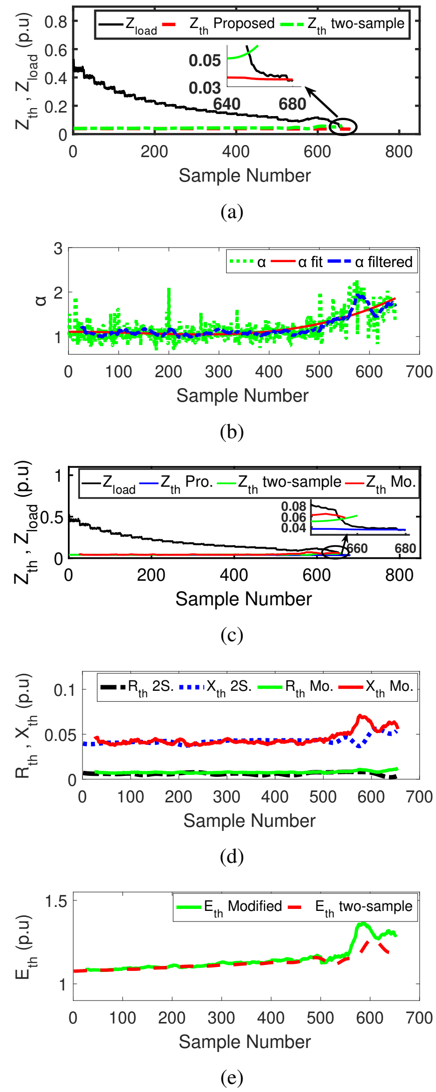


Fig. 10. Scenario 2: (a) Thévenin and load impedance magnitudes; (b) α values; (c) magnitude of the Thévenin impedances and the load impedances; (d) Thévenin resistances and reactances; (e) Thévenin voltage magnitudes.

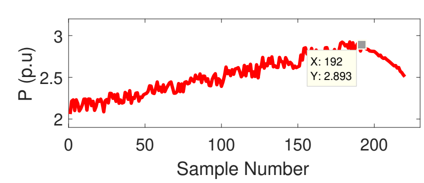


Fig. 11. Scenario 3: Active power of the load at bus 28.

in 192nd sample which is an evidence of improving the accuracy of the proposed method. The corrected Thévenin impedance parameters as well as the parameters obtained with the two-sample method are shown in Fig. 13(c). Figure 13(d) shows the modified Thévenin voltage magnitude and the Thévenin voltage magnitude determined by the two-sample method.

D. Applying the Proposed Method on IEEE 118-bus Modified Test System

In order to demonstrate the performance of the proposed method on systems with different scales, this section presents the results of applying the method on a larger network, i.e., the IEEE 118-bus test system where its topology depicted in Fig. 14. The data utilised in the examples below are available at [26]–[28].

a) Scenario 1: In this scenario, the load of bus 11 is gradually increased to its maximum at sample 808 as shown in Fig. 15(b). At this point, the magnitude of the TE impedance of the system seen from the bus 11 is equal to the magnitude of the load impedance connected to bus 11. This fact is observed in Fig. 15(d) which shows the curves of magnitude of load impedance and curves of magnitude of Thévenin impedance estimated by two-sample method and the suggested methods in this paper (proposed and modified methods). The corresponding TE parameters including the resistance and reactance estimated using the different methods are plotted in Fig. 15(c). The values of filtered and unfiltered correction coefficient α are depicted in Fig. 15(a). The estimated TE voltage source using two-sample method and the introduced

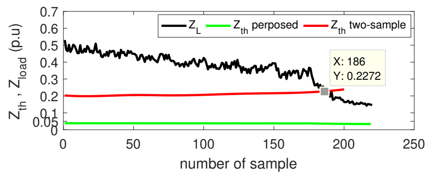


Fig. 12. Scenario 3: Magnitude of Thévenin impedance and of the load impedance as seen from bus 28.

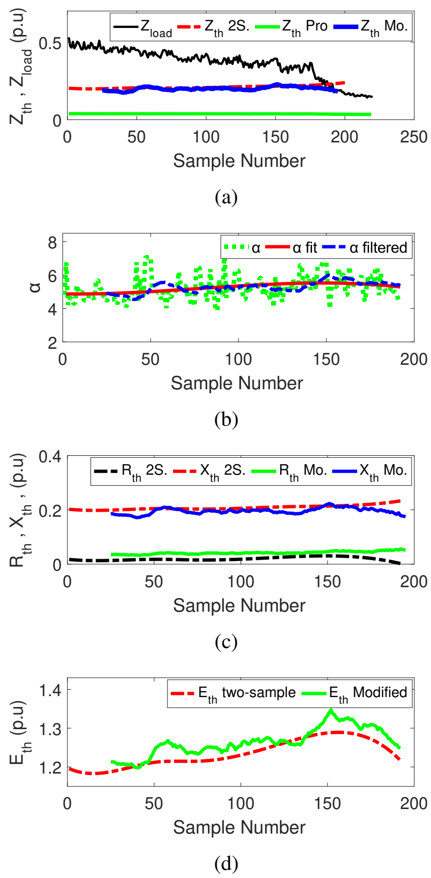


Fig. 13. Scenario 3: (a) Modified Thévenin and load impedance magnitudes; (b) α; (c) Modified Thévenin resistances and reactances; (d) Modified Thévenin voltage magnitudes.

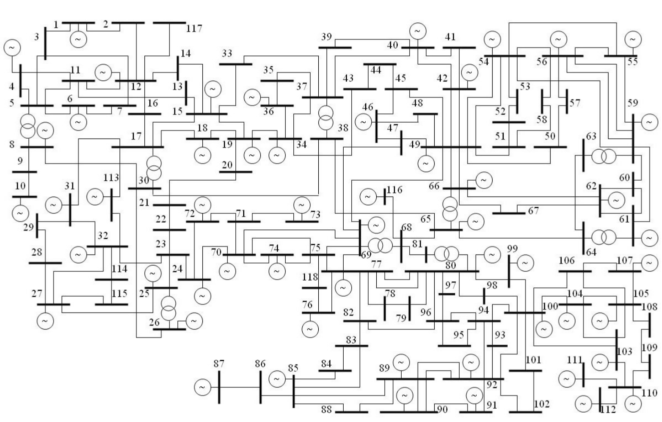


Fig. 14. IEEE 118-bus network.

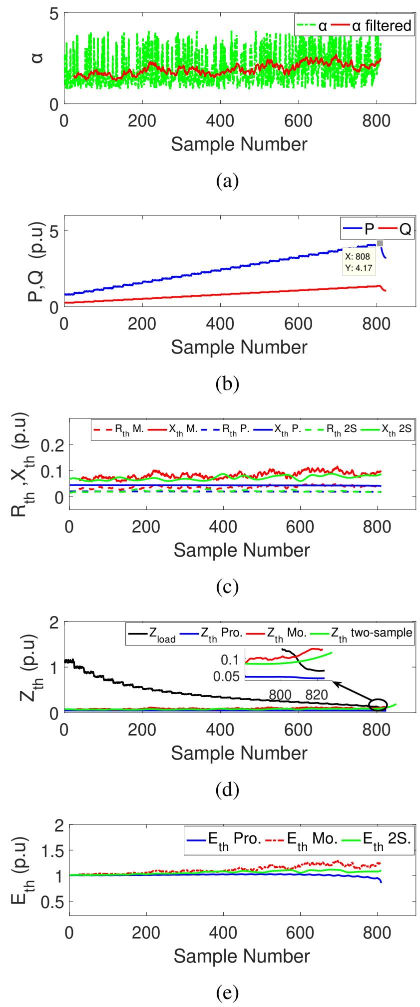


Fig. 15. Scenario 1 on IEEE 118-bus test system: (a) the correction coefficient α; (b) the active and reactive power of bus No. 11; (c) Thévenin resistance and reactance; (d) Thévenin and load impedance magnitudes; (e) Thévenin voltage magnitude values.

method are also given in Fig. 15(e).

The most critical operation situation of the system for estimating TE parameters is the case where the load connected to the bus of interest is a fixed power consuming or with minor load variation respect to side system variations. In such operation cases, the estimation methods based on multi-sample, e.g., two-sample and three-sample methods, can not accurately estimate the TE parameters. For example, In scenario 1 of the IEEE 118-bus modified system, the load connected to bus 3 is fixed. The results depicted in Fig. 16(a) show that the two-sample method is unable to estimate the TE impedance, where it gives the load impedance instead of the TE impedance seen from bus 3. On the other hand, the proposed method in this paper can approximately estimate TE impedance without using the correction factor α as shown in Fig. 16(b). If the algorithm uses the last computed α related to times in which load of bus 3 was varying (Fig. 16(c)), TE model can be corrected to a more accurate estimation as shown in Fig. 16(b). Unlike the two-sample method, the proposed method is also able to well-track the parameters of TE impedance, i.e., TE resistance and TE reactance and TE voltage source as shown in Fig. 16(d)

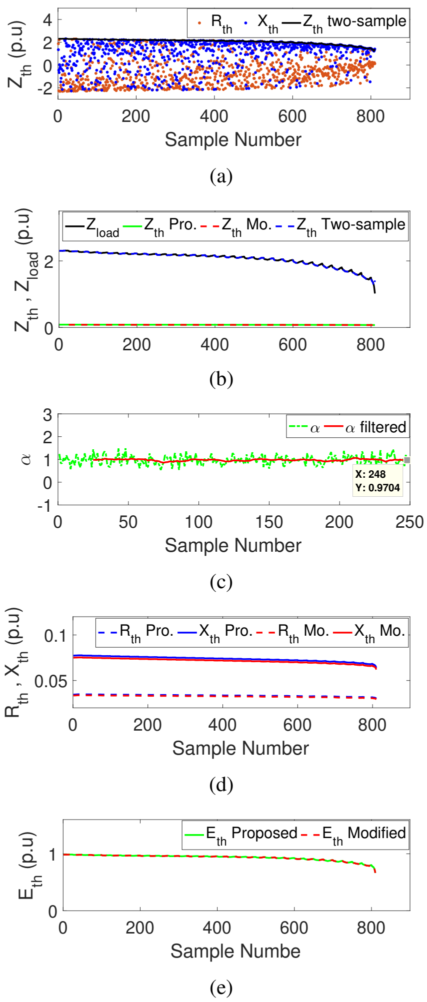


Fig. 16. Scenario 1 on IEEE 118-bus test system for load 3: (a) Thévenin resistance and reactance using two-sample method; (b) Thévenin and load impedance magnitudes; (c) the correction coefficient α; (d) Thévenin resistance and reactance; (e) Thévenin voltage magnitude values

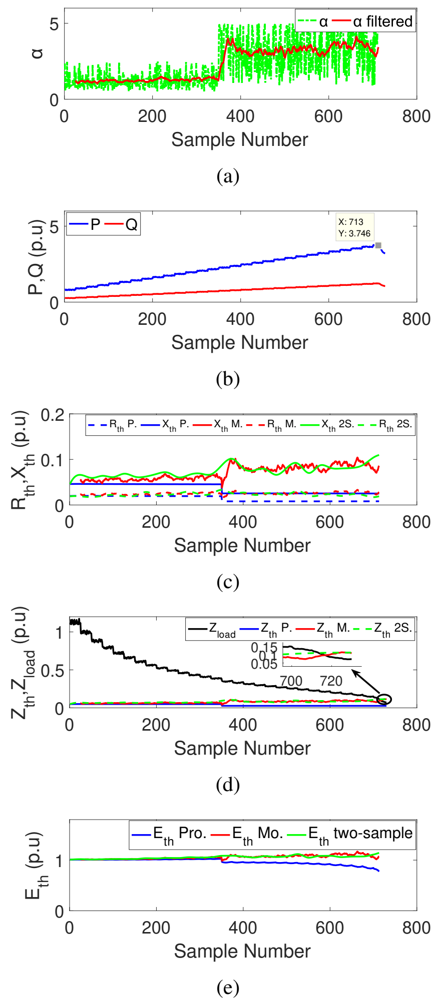


Fig. 17. Scenario 2 on IEEE 118-bus modified test system: corrected results. (a) α values; (b) the active and reactive power; (c) Thévenin resistances and reactances; (d) magnitude of the Thévenin impedances and the load impedances;(e) Thévenin voltage magnitudes.

and 16(e).

b) Scenario 2: In this second scenario, the active and reactive load powers connected to bus 11 increase monotonically as in Scenario 1, Moreover, at $t = 10$ s, the line 5-11 is disconnected. The maximum load point, in this case, is reached at the 713th as shown in Fig. 17(b). The magnitudes of the Thévenin impedances obtained with the proposed method and the two-sample method are shown in Fig. 17(d). Without applying the correction coefficient, the value of load and TE impedance intersect is inaccurate, while considering the proposed correction coefficient (shown in Fig. 17(a)) leading to an accurate intersect. The TE parameters, resistance and reactance, estimated using the proposed and twosample methods are depicted in Fig. 17(c) which shows the importance of the proposed correction coefficient for correcting TE parameters estimation based on the model of the network. The importance of this coefficient is also prominent in estimating the TE voltage as shown in Fig. 17(e). It also verifies the capability of the proposed method for online estimation of TE parameters especially in case of large disturbance such as a line outage.

c) Scenario 3: Similar to the scenario 3 applied to IEEE-39 bus system, the third scenario carried out over the IEEE-118 bus system considers ramps and random variations for all loads. For illustration, the trajectory of the active and reactive powers of the load at bus 11 are shown in Fig. 18(b), where the maximum load condition occurs in the 154th sample. Figure 18(a) shows the calculated correction coefficient and its filtered value for bus 11. By applying the proposed correction coefficient, the TE impedance estimated using the proposed method intersects the load impedance in an accurate point as shown in Fig. 18(d). It leads to a good accuracy in estimating TE parameters including TE resistance and reactance as shown in Fig. 18(c) and TE voltage source as shown in Fig. 18(e).

E. Comparison with Other Methods

As thoroughly illustrated in the simulations presented in Sections III.A-D, the proposed method shows a good performance for all features related to Thévenin estimation in comparison with other methods. In fact, the proposed method requires a low number of samples; its sensitivity to noise

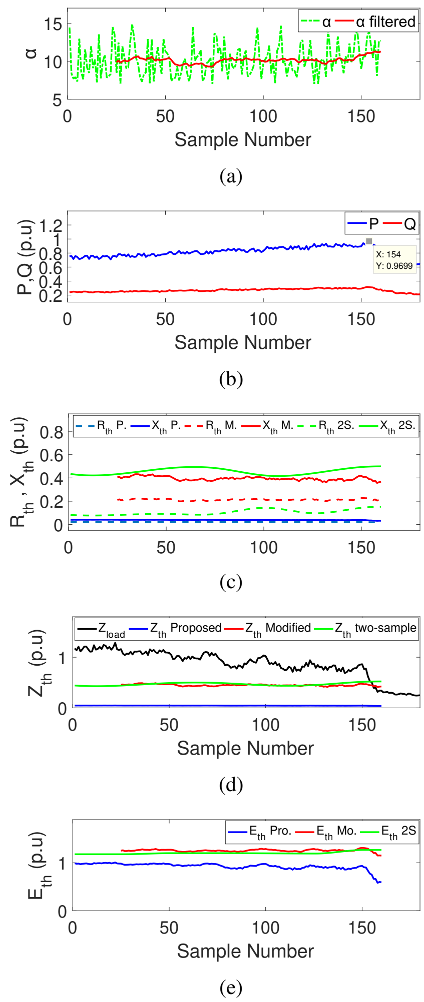


Fig. 18. Scenario 3 on IEEE 118-bus modified test system: (a) Modified Thévenin and load impedance magnitudes; (b) α; (c) Modified Thévenin resistances and reactances; (d) Modified Thévenin voltage magnitudes.

and transient conditions is negligible; and is not sensitive to the generator reactive power limits. The proposed method is also able to take into account system dynamics and prevent large pre-estimation data processing, does not show estimation delays, and does not depend on the amount of load variations.

Table II provides a qualitative comparison of the features and the performance of the proposed method with a variety of methods that have been proposed in the literature. In particular, the table considers the methods based on one sample, two samples, three samples, five samples, least squares and admittance matrix. The features and performance indexes that are compared are the number of required samples (NRS); sensitivity to the data noise and transients (SDNT); estimation delay (ED); required computation time and burden (CTB); estimation accuracy (EA), dependence on load variation amount (DLVA); sensitivity to the generator reactive power limits (SGRPL); and consideration of the system dynamics (CSD).

The proposed method performs better than least-squares methods, e.g., [3]–[7], with respect to indices NRS, ED, CTB, and EA. The proposed method is also exempt from the issues associated with SDNT, EA, and DLVA that are shown by methods based on one sample [16], two samples [8-10], three samples [11-13] and five samples [14]. Compared to the admittance-matrix methods [17-19], the proposed approach performs better in terms of SDNT, CTB, EA, and SGRPL since it does not require to compute the inverse of the admittance matrix nor supplementary and approximated computations to consider the reactive power limits of generators.

TABLE II
COMPARISON WITH STATE-OF-ART METHODS

Method	Reference	NRS	SDNT	ED	CTB
One sample	[16]	small	large	small	small
Two sample	[8]–[10]	small	large	small	small
Three sample	[11]–[13]	small	large	small	small
Five sample	[14]	medium	medium	medium	medium
Least squares	[3]–[7]	large	small	large	large
Admittance matrix	[17]–[19]	small	large	small	Large
Proposed		**small**	**small**	**small**	**medium**
Method	Reference	EA	DLVA	SGRPL	CSD
One sample	[16]	medium	small	✓	✓
Two sample	[8]–[10]	variable[1]	large	✓	✓
Three sample	[11]–[13]	medium	large	✓	✓
Five sample	[14]	medium	large	✓	✓
Least squares	[3]–[7]	variable[1]	large	✓	✓
Admittance matrix	[17]–[19]	medium	small	✗	✗
Proposed		**large**	**small**	✓	✓

[1] The accuracy depends on the tuning of the parameters.

F. Main Features and Applications of the Proposed Method

The main features of the proposed methods can be summarized as follows.

- It is able to accurately estimate not only the magnitude of the TE impedance but $R_{\rm Th}$ and $X_{\rm Th}$ separately.
- It is able estimate TE parameters even if the variations of either demand or network sides are small. this is an advantage with respect to existing methods that can return inaccurate or even negative TE resistance and reactance in same operating conditions.
- It is able to estimate TE parameters as seen at generation buses, not only at load buses as most existing methods.
- Unlike other methods such as two-sample & three-sample methods, the impact of noise of PMU data on the estimation results is negligible.

Based on the features above, the prposed appears to be relevant for assessing the short-, mid- and long-term voltage stability of power systems. The proposed method also allows reducing the complexity and computational burden of the algorithms solved by WAMS since it allows modelling portions of the systems through accurate TEs. Finally the proposed methods can be useful for the design and testing of control methods in large scale power systems.

IV. CONCLUSIONS

This paper proposes a novel technique to calculate the TE of the network as seen from given buses. The starting point is the conventional $\bar{Z}_{\rm bus}$ and the phasor measurements at the generator and load buses available from PMUs. Then, the conventional $\bar{Z}_{\rm bus}$ is adjusted to take into account uncertainty and the TE deduced based on the modified $\bar{Z}_{\rm bus}$. The proposed technique only requires present measurements for modified $\bar{Z}_{\rm bus}$ construction and two consecutive phasor measurements to calculate correction coefficients. Moreover, it is numerically efficient and can accurately estimate the TE of generation and load buses under any load-increase scenario. Simulation results shows the accuracy of the proposed method when the correction coefficients are taken into account. It is also important to observe that no relevant numerically issues were observed. This is due to the negligible variations of the measured variables of the system in the consecutive measurements. Finally, the proposed technique can run continuously and can be adapted to any measurement sampling time depending on the required calculation time. The results verify the superiority of the proposed technique over others which can take into account the technical and practical challenges.

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